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An Estimated DSGE Model of the Euro Area with Expectations about the Timing and Nature of Liftoff from the Lower Bound

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An Estimated DSGE Model of the Euro Area with Expectations about the Timing and Nature of Liftoff from the Lower Bound*

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Abstract

I investigate the implications of the zero lower bound (ZLB) in a structural New-Keynesian model for the euro area. The medium-scale DSGE model accommodates forward guidance by treating the expected durations of the ZLB constraint as free parameters in estimation. Incorporating professional forecasters' expectations about the future path of the policy rate provides well-identified estimates of the durations. These estimates indicate that unconventional monetary policy becomes increasingly important from 2018 on. Furthermore, when monetary policy is expected to be passive in its response to inflation after liftoff, forward guidance has weaker effects with deflationary pressures on the economy. Finally, including data from the Covid-19 pandemic in estimation leads to stable estimates and allows an assessment of monetary policy during that period.

Keywords: monetary policy; zero lower bound; forward guidance; liftoff; Covid-19

JEL classifications: E31; E52; E58; D84

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1 Introduction

In recent years, many central banks have lowered interest rates close to zero, including the US Federal Reserve, the Bank of England, and the European Central Bank. However, the New Keynesian framework poses challenges for models with zero lower bound (ZLB) periods due to indeterminacy. Early research proposed backward induction and the method of undetermined coefficients for solving linear rational expectations (LRE) at the ZLB (Eggertsson and Woodford, 2003; Svensson, 2000). Some studies, including Fernández-Villaverde, Gordon, Guerrón-Quintana, and Rubio-Ramirez (2015), argue that linearized solutions are not optimal for study at the ZLB, while others, such as Jones (2017) or Guerrieri and Iacoviello (2015), come to a different conclusion. Despite their differences in methodology, the latter studies propose occasionally binding constraint algorithms that provide linear solutions which are acceptable approximations for nonlinear models. Furthermore, regime-switching models, such as the Markov-switching approach by Bianchi (2013), allow multiple occurrences of a fixed-interest rate regime but assume constant expected durations.

In this paper, the approach I follow is that of Kulish, Morley, and Robinson (2017) (hereafter KMR (2017)), who propose to solve models containing both determinate and indeterminate regimes via backward induction, using a method derived by Kulish and Pagan (2017). An important feature of the KMR (2017) approach is that it treats the sequence of expected durations at the ZLB as free parameters in estimation. Due to the fact that estimation selects the duration sequence that best fits the data, these estimated durations may exceed the underlying policy constraints. Consequently, the model accommodates unconventional monetary policy in the form of forward guidance, whereby the central bank influences public expectations that the policy rate will be *lower for longer* than implied by the policy rule. This set-up allows me to (i) construct a new measure of expected durations for the euro area based on survey data, thereby providing accurate estimates of expected durations; and (ii) explain the apparent muted effects of forward guidance due to expectations regarding liftoff.

I estimate a medium-scale DSGE model for the euro area for the sample period from 2004-Q3 to 2019-Q4. As an extension, I also include the Covid-19 recession in the estimation sample and present a time series approach to deal with the large outliers in the data during the pandemic. Following Kulish and Pagan (2017) and KMR (2017), I incorporate a regime change into the well-known DSGE models of Smets and Wouters (2003, 2007) (hereafter SW (2007) and SW (2003)). Monetary policy in the model changes between a conventional policy regime in the form of a Taylor-type rule and an unconventional fixed-rate policy regime at the ZLB. Despite including the ZLB for a substantial part of the sample period, the structural parameters are consistent with those found in previous literature, indicating that the model is robust to changes in policy regime, the primary goal of any structural macroeconometric model seeking to address the Lucas Critique.

To expand the information set of the model, I propose the use of a constructed measure of expected durations obtained from the ECB's Survey of Professional Forecasters (SPF). This measure incorporates agents' subjective expectations about the future path of the policy rate, thus, aiding in identifying the sequence of expected duration at the ZLB. Adding subjective information to macroeconomic models has become increasingly popular (e.g., Del Negro and Eusepi, 2011; Müller, Christoffel, Mazelis, and Montes-Galdón, 2022). Nevertheless, to my knowledge, this is the first study to incorporate SPF survey data into a DSGE model to estimate the expected duration of the ZLB for the euro area.

Including the SPF survey data enhances the accuracy of the expected duration estimates. These estimates play a significant role in the non-linear model dynamics in the sense that even if the model can accommodate the policy change at the ZLB, the implications and responses to structural shocks and the micro foundation can be counter-intuitive. Earlier research, such as by Werning (2011), Cochrane (2017), and Diba and Loisel (2021), has identified some of those counter-intuitive properties as *the forward guidance puzzle*, *the paradox of toil*, and the *the fiscal multiplier puzzle*. These non-linearities are more apparent for estimates with a long expected duration, when the ZLB will bind for a longer period.

While the KMR (2017) approach and the SPF survey data deliver well-identified estimates for the sequence of expected durations, it is not immediately apparent from where these durations originate. Considering that the estimated durations can exceed the actual policy constraint, they may be at least partially the result of deliberate monetary policy. Therefore, following Jones, Kulish, and Rees (2022) and using the occasionally binding ZLB constraint algorithm proposed by Jones (2017), I decompose the duration estimates into durations originating from forward guidance policy and durations implied by the underlying constraint. Based on this decomposition, forward guidance appears to have become increasingly important from 2018 onwards.

However, these forward guidance effects are not only dependent on the expected duration of the ZLB but also on agents' expectations about the nature of liftoff from the lower bound. Since the paths of the endogenous variables are determined by backward induction through the structural equations (for instance, Gibbs and McClung, 2022), varying expectations about the nature of liftoff alter forward guidance effects. To examine the role of liftoff expectations, I assume agents either expect the central bank to respond actively or passively to inflation after liftoff. This requires a deviation from the KMR (2017) solution strategy that relies on an active post-ZLB regime in order to deliver a unique and stable solution. Following Gibbs and McClung (2022), I generalize the KMR (2017) approach and incorporate a convenient sunspot solution based on Bianchi and Nicolò (2021), which allows for a possibility of an extended period of passive monetary policy after liftoff. This alleviates the restrictive constraints on monetary policy modeling that arises from the determinacy conditions, known as the *Blanchard and Kahn* conditions, that most solution algorithms impose. The simple augmentation increases the possibilities of regimes that can be considered at the ZLB, and importantly, does not interfere with the remaining structural parameters or expected durations. When agents expect an active monetary policy regime after liftoff, forward guidance decreases expectations of shorter- and longer-term yields, thereby causing a positive response to inflation. In this case, the model exhibits large expansionary effects, which increase with the expected duration. Vice versa, when agents expect a passive monetary policy regime after liftoff, forward guidance effects are muted and deflationary pressures are observed. Additionally, a passive monetary policy regime after liftoff can increase uncertainty around interest rate setting and inflation, resulting in steeper Philips curves and larger inflation variations. When agents incorporate these expectations into their decision-making process, forward guidance announcements become less powerful.

As an extension, I propose a time-series approach to deal with the large data outliers observed during the Covid-19 pandemic. The large additional variation in the Covid-19 data and its low persistence presents challenges for the estimation of DSGE models. My approach, in the spirit of Lenza and Primiceri (2022) for VAR models, makes three adjustments to the standard estimation procedure without modifying the structural equations of the model. I introduce a time-varying innovation-covariance matrix that captures the heteroskedasticity in shock variances, an ARMA(1,1) correction for the dynamic effects of total factor productivity innovations, and a measurement correction for consumption data in the Kalman Filter. The estimation approach allows for an assessment of monetary policy during the Covid-19 pandemic despite being at the ZLB during the time. Adding Covid-19 data delivers robust parameter estimates compared to the pre-Covid-19 sample, but predicts longer implied forward guidance durations over recent years.

The remainder of this paper is structured as follows. Section 2 summarizes the adjustments to the SW (2007) model framework for the euro area. A discussion of the estimation methodology incorporating the ZLB is provided in the Section 3. Section 4 presents the estimation results of the benchmark model that assumes an active monetary policy regime after liftoff. Section 5 identifies the role of forward guidance in the euro area. Section 6, generalizes the KMR (2017) approach and allows for a prolonged period of passive monetary policy after liftoff. Section 7 extends the sample period to include the Covid-19 pandemic and assesses monetary policy during this period. Section 8 concludes.

2 Model

The model follows that for the US economy in SW (2007) and also their earlier version for the euro area in SW (2003), while incorporating adjustments to the monetary policy rule and yield curve as in KMR (2017). Variables are log-linearized around their steady-state balanced growth paths, with deviations from steady-state denoted by hats. Based on the standard SW (2007) framework, households choose consumption and hours worked to maximize utility over an infinite life horizon. The union's ability to differentiate labor over households introduces some monopoly power over wages. Goods producers in the intermediate goods market compete under monopolistic competition conditions, while final goods producers operate under perfect competition. Sticky prices in the goods and labor market are modeled with Calvo (1983) pricing. Ultimately, final goods producers purchase intermediate goods to transform and resell them to households. A representation of the complete model, including the sticky and flexible wage economies, and the shock processes can be found in the technical appendix.

2.1 Incorporating the Yield Curve

Following KMR (2017), I incorporate information from the yield curve into the model economy to help identify the monetary policy stance, and therefore the expected durations at the ZLB. This also accounts for the fact that short and medium-term (AAA-rated) government bond yields in the euro area were consistently negative at the ZLB. As in KMR (2017) and referring to the expectation hypothesis under which longer maturity bonds reflect agents' beliefs about the future path of the policy rate, interest rates are set according to:

$$\hat{r}_{j,t} = \frac{1}{j} E_t(\hat{r}_t + \hat{r}_{t+1} + \hat{r}_{t+j-1}) \qquad for\ j = 2, 3, ..., m \tag{1}$$

where $\hat{r}_{j,t}$ represents the log deviation of the yield from steady-state on a j-period bond. The observed counterpart of the yield curve becomes:

$$r_{j,t} = \hat{r}_{j,t} + r + c_j + \varepsilon_t^{\eta} + \epsilon_{j,t} \qquad for\ \ j = 2, 3, ..., m \tag{2}$$

where r is the steady-state of the one period nominal interest rate. Since, c_j is a maturity-specific time-invariant component, $r + c_j$ can be interpreted as the steady-state of a yield with j periods to maturity. The time-varying component to the term premia includes a persistent AR(1) shock ε_t^{η} for all observed maturities:

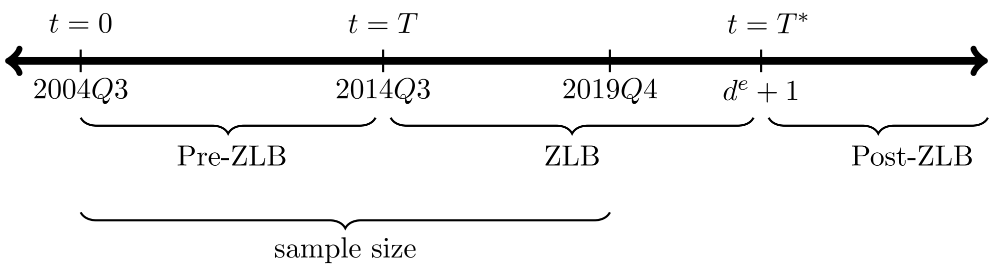
$$\varepsilon_t^\eta = \rho^\eta \varepsilon_{t-1}^\eta + e_{\varepsilon^\eta,t} \tag{3}$$

The use of yield curve information in estimation facilitates precise and robust (to different priors) estimates of expected durations since the concepts of forward guidance and yield curve are closely related. Forward guidance reduces short-term interest rates and investors might tend to shift to longer maturity assets without changing the interest path and thereby, lowering the term premia. Similarly, yield curve movements can help identify forward guidance announcements and pin down the estimates for the expected durations. For a closer discussion of how forward guidance can affect the term premia see, for instance, Hanson and Stein (2015) and de la Barrera, Falath, Henricot, and Vaglio (2017).

2.2 Implementing the ZLB

Following KMR (2017), monetary policy is modeled to operate in different regimes over time. In the first regime, henceforth *pre-ZLB*, monetary policy follows a standard feedback Taylor-type rule. Once the monetary authority enters a ZLB regime in $t = T$, the nominal policy rate is fixed and no longer responds to movements in output or inflation. At the ZLB, the central bank communicates a liftoff strategy including a post-ZLB monetary policy regime from $t = d^e + 1$. Assuming the central bank announcements are credible, the expected duration of the ZLB period in $t = T$ is given by d^e. After liftoff, henceforth *post-ZLB*, monetary policy can be active or passive depending on the central bank's responsiveness to inflation.

Figure 1: ZLB in the euro area



2.2.1 Active Monetary Policy

When the central bank is responding enough to changes in inflation to increase real interest rates after liftoff, monetary policy is assumed to be active. Specifically, the central bank will communicate to revert to its initial Taylor-type regime after T^*. Cagliarini and Kulish (2013) show the expectation of a final regime that satisfies the Taylor principle is sufficient to guarantee a stable and unique solution to a LRE model. The active monetary policy case will serve as the benchmark scenario:

$$\hat{r}_t \equiv \begin{cases} TR, & \text{for } t < T \\ -r, & \text{for } T \leq t < T^* \\ TR, & \text{for } t \geq T^* \end{cases}$$

where the Taylor rule (TR) is defined as the standard feedback rule in SW (2007):

$$\hat{r}_t = \rho_R \hat{r}_{t-1} + \Big(1 - \rho_R\Big)\Big(\phi_\pi \hat{\pi}_t + \phi_y(\hat{y}_t - \hat{y}_t^f)\Big) + \phi_{\Delta y}\Big(\hat{y}_t - \hat{y}_{t-1} - (\hat{y}_t^f - \hat{y}_{t-1}^f)\Big) + \varepsilon_t^m \tag{4}$$

2.2.2 Passive Monetary Policy When monetary policy is passive, the central bank is communicating to be less responsive to changes in inflation after liftoff compared to the pre-ZLB period. In other words, the central bank commits to a indefinite period of passive monetary policy and reacts only passively to changes in inflation:

$$\hat{r}_t \equiv \begin{cases} TR, & \text{for } t < T \\ -r, & \text{for } T \leq t < T^* \\ \hat{r}_t^{post}, & \text{for } t \geq T^* \end{cases}$$

where the Taylor rule (TR) is still defined as the standard feedback rule in SW (2007) but $\hat{r}_t^{post}$ corresponds to a new passive monetary policy regime, where $\phi_\pi^s < 1$.

3 Methods

The model equations are linearized around the steady-state and cast into state-space form. The model is estimated using Bayesian estimation techniques. The proposal for the MCMC algorithm is based on the posterior mode obtained by minimizing the negative log posterior of the pre-ZLB sub sample. The likelihood for the data sample is defined as $\mathcal{L}(Z \mid \theta, d)$, where θ denotes the set of structural parameters, Z the data, and d the set of expected durations at the ZLB:

$$p(\theta, d \mid Z) \propto \mathcal{L}(Z \mid \theta, d)p(\theta)p(d)$$

Using the Metropolis-Hastings algorithm, draws are generated from the posterior proposal of the pre-ZLB sub-sample to get full marginal posterior distributions for each parameter. A detailed description of the posterior sampler can be found in the technical appendix.

3.1 Solution Strategy at the ZLB

Most of the existing LRE solution strategies, such as the Sims (2002) and the Binder and Pesaran (1995, 1997) algorithms, assume a time-invariant structure, in which the structural parameters of a system remain constant over time. Therefore, any future (un-)anticipated changes in the model structure are not included in standard algorithms. Nevertheless, in recent years, different approaches have emerged for addressing these structural changes. Cagliarini and Kulish (2013) propose a rational expectation solution for linear stochastic models with predictable structural variations, building on the solution of Sims (2002). Jones et al. (2022) present an iterative application of the solution for anticipated structural changes which due to its nature allows checking the conditions of uniqueness and determinacy. Kulish and Pagan (2017) allow for structural changes that are either anticipated or unanticipated and most importantly their solution strategy takes the form of a time-varying coefficient vector autoregression (VAR) model, which can be used to calculate a likelihood. Similarly, Jones (2017) and Guerrieri and Iacoviello (2015) propose occasionally-binding constraint algorithms, that can serve as a linear approximation for a non-linear model. I follow KMR (2017), whose generalized method expands the Binder and Pesaran (1995, 1997) algorithm to capture the anticipated structural change of monetary policy at the ZLB.[1] Contrary to other solution methods, such as Guerrieri and Iacoviello (2015) or Jones (2017), the expected durations of the ZLB are not computed as a function of the ZLB constraint, which allows for the possibility that the expected durations exceed the duration of the underlying constraint on policy, accommodating forward guidance as a *lower for longer* policy into the model.

Before the anticipated structural change occurs at $t = T$, the structure is stable in the standard Binder and Pesaran (1995, 1997) form:

$$A_0 y_t = C_0 + A_1 y_{t-1} + B_0 E_t y_{t+1} + D_0 \varepsilon_t \tag{5}$$

where y_t is the $n \times 1$ state vector of forward looking variables and ε_t the $l \times 1$ of structural shocks. The standard rational expectation solution in the pre-ZLB regime is given by:

$$y_t = C + Q y_{t-1} + G \varepsilon_t \tag{6}$$

[1] In the pre- and post ZLB regime, the model is solved by the Sims (2002) algorithm due to its favorable properties in terms of accuracy and computational speed. For a detailed comparison see, for instance, Anderson (2007). However, the ZLB period is solved using the Binder and Pesaran (1995, 1997) algorithm due to its iterative nature. To provide a solution for the entire model, the Sims (2002) solution matrices are subsequently transformed into Binder and Pesaran (1995, 1997) form.

The ZLB is a time-varying constraint that hits the economy in period $t = T$ and is assumed to be lifted at time $t = d + 1 = T^*$ where d is the expected duration. At the ZLB, the nominal policy rate is pegged, so the real policy rate can be expressed in terms of deviations from steady-state $r_t = -r$.[2] The time-varying solution is built on the assumption that the central bank communicates a clear liftoff strategy to a determinate solution. Hence, at the ZLB, the solution to the model follows a time-varying coefficient VAR process according to:

$$\{C_t\}_{t=T}^{T+d} + \{Q_t\}_{t=T}^{T+d} y_{t-1} + \{G_t\}_{t=T}^{T+d} \varepsilon_t \tag{7}$$

3.2 Policy Expectations after Liftoff

In the KMR (2017) solution strategy, the model selects the duration sequence that best fits the data, allowing for expected durations that exceed the underlying policy constraint. As a result, the model can accommodate unconventional policies such as forward guidance. Forward guidance corresponds to announcements by the central bank about the expected future path of the policy rate and can be seen as a powerful tool aiming to influence expectations about the duration of the ZLB. When these announcements are credible, agents revise their expectations about the future path of the policy rate and subsequently their expectations about longer-term yields. The revision of these expectations affects monetary policy today in forward-looking models. Thus, forward guidance can be used to stimulate aggregate demand when interest rates are expected to remain *lower for longer* in the future (Campbell, Fisher, Justiniano, and Melosi, 2017; Del Negro, Giannoni, and Patterson, 2012). In addition to the expected length of the ZLB, the expected path of the future interest rate is also dependent on the monetary policy regime after liftoff. Therefore, assuming the central bank communicates its liftoff strategy during the fixed interest rate regime (figure 2), agents expect monetary policy to either be active or passive in its response to inflation.

3.2.1 Active Monetary Policy

A sufficient condition to guarantee a unique solution of the KMR (2017) approach is that the central bank commits to a determinate post-ZLB monetary policy regime that is credibly communicated to the economy during the ZLB.[3] In this case, the monetary policy rule reverts back to the Taylor-type rule regime and, given there are no future structural changes, will hold into the infinite future. The solution of this model, henceforth referred to as the benchmark model,

[2] The variables are expressed as deviations from steady-state. Hence, $r_t = -r$ corresponds to the level of the interest rate being zero $r_t + r = 0$.

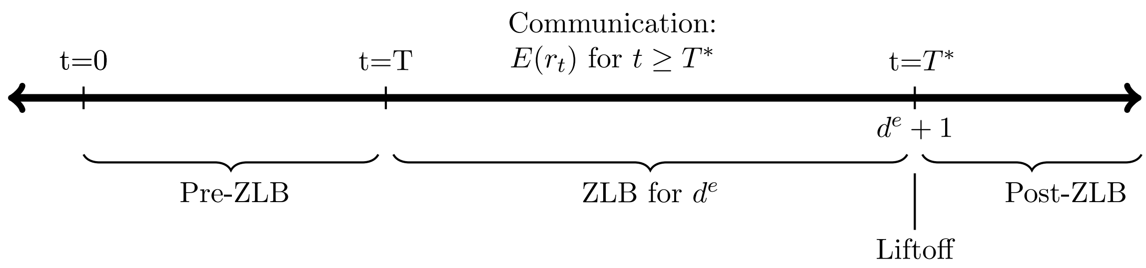
[3] For proof of this condition see Cagliarini and Kulish (2013).

is:

$$y_t = \begin{cases} C + Qy_{t-1} + G\varepsilon_t, & \text{for } t < T \\ \{C_t\}_{t=T}^{T+d} + \{Q_t\}_{t=T}^{T+d} y_{t-1} + \{G_t\}_{t=T}^{T+d} \varepsilon_t, & \text{for } T \leq t < T^* \\ C + Qy_{t-1} + G\varepsilon_t, & \text{for } t \geq T^* \end{cases}$$

A deliberate forward guidance policy has similar effects to conventional expansionary monetary policy when the central bank is expected to resume active monetary policy after liftoff. If agents expect an active regime in the future, keeping interest rates lower for longer lowers their expectations about the future path of the policy rate. Consequently, the yield curve shifts down, putting pressure on inflation and stimulating the economy today despite no actual policy changes. The expansionary effects increase over-proportionally in the length of the expected duration, which was first identified as *forward guidance puzzle* by Del Negro et al. (2012).

Figure 2: Monetary Policy after Liftoff



3.2.2 Passive Monetary Policy In the event that the central bank decides to maintain passive monetary policy after liftoff and only reacts sluggishly to inflation changes, forward-looking agents will adjust their optimization choices based on this new set of information. As mentioned previously, KMR (2017) assume a determinate regime following liftoff in order to guarantee a unique solution. To ease this restriction, I generalize their approach as proposed by Gibbs and McClung (2022) and incorporate a sunspot solution based on Bianchi and Nicolò (2021) that allows for an extended period of passive monetary policy. With this modification, monetary policy will not automatically revert to its Taylor-type rule after liftoff, but instead enter a new regime.

If monetary policy is modeled passively, the model might not have a unique and stable solution even if its parameter space is conceptually and intuitively reasonable. This is particularly true for assumptions made by a standard Taylor rule. Taylor (1993) argues that the inflation coefficient in a policy rule must be greater than one, a requirement known as the *Taylor Principle*. Inflation-induced increases in nominal interest rates will also increase the real interest rate when the central bank responds sufficiently to inflation to increase the nominal interest rate by more, $\phi_\pi > 1$. As a result, inflation can be disciplined and pinned down as a function of the real interest rate. Vice versa, when the central bank is less aggressive, $\phi_\pi < 1$, the nominal interest rate is increasing by less than inflation and subsequently the real interest rate is decreasing. In this case, changes in the nominal interest rate cannot pin down inflation as a function of the real interest rate but rather inflation becomes expectational. As a result, the number of expectational variables does not match the number of explosive roots and the system becomes explosive with multiple solutions each defined by the expectations about future inflation. More generally, these uniqueness problems, known as indeterminacy, occur if the number of unstable roots (i.e., eigenvalues > 1) is smaller than the number of non-predetermined variables. Therefore, the forecast errors are not solely determined by the fundamental shocks in the model, but rather can be decomposed into a fundamental shock and a sunspot shock component (see, for instance, Lubik and Schorfheide, 2004). Lubik and Schorfheide (2003) were among the first to present an algorithm that computes the complete set of indeterminate equilibriums. Cagliarini and Kulish (2013) show that a monetary policy rule that violates the *Taylor principle* does not necessarily lead to multiple equilibria and indeterminacy if it eventually returns to a determinate regime. Moreover, Farmer, Khramov, and Nicolò (2015) and Bianchi and Nicolò (2021) present sunspot solution techniques to solve LRE model solutions under indeterminacy.

To ensure a solution to an indeterminate model when allowing for an extended period of passive monetary policy, I augment the system by a sunspot solution based on Bianchi and Nicolò (2021). The sunspot provides the missing explosive root to ensure a unique and stable solution to the model:

$$s_t = \varphi s_{t-1} + \mu_t - \nu_t, \qquad \varphi \equiv \frac{1}{\varrho} \tag{8}$$

where s_t is an autoregressive parameter, ν_t is a sunspot shock with standard deviation σ^ν, and μ_t is a one-period ahead forecast error associated with the forward-looking variable π_t, such that $\mu_t = \pi_t - E_{t-1}\pi_t$. The matrix φ $(m \times m)$ augments the system by the missing unstable roots. In the case considered here, φ is a 1×1 matrix, providing a single missing root in case of indeterminacy when $\phi^s_\pi < 1$. The solution to the indeterminate system depends on the parameter ϱ. When setting $\varrho < 1$, the sunspot solution provides the necessary explosive root to guarantee a unique and stable solution to the system. The augmented sunspot equations constitute a separate model block and are therefore not interfering with the endogenous variables in the model $y(\theta)$[4]. Therefore, the sunspot has no impact on the LRE solution when $\varrho > 1$. To ensure a unique solution in both cases, the parameter ϱ is set equal to the inflation response parameter, so that it can deliver the missing explosive root if and only

[4]Referring to Bianchi and Nicolò (2021), I provide proof for this equivalence in the technical appendix.

if $\phi_\pi \leq 1$:

Table 1: Sunspot Solution

Case	Parameter Choice	Unique Solution
$\phi_\pi > 1$	$\frac{1}{\varrho} \equiv \frac{1}{\phi_\pi} < 1$	Sunspot inactive
$\phi_\pi \leq 1$	$\frac{1}{\varrho} \equiv \frac{1}{\phi_\pi} \geq 1$	Sunspot active

If the central bank does not revert to Taylor-type rule policy, but rather chooses passive monetary policy, the LRE model can be described as follows:

$$y_t = \begin{cases} C + Qy_{t-1} + G\varepsilon_t, & \text{for } t < T \\ \{C_t\}_{t=T}^{T+d} + \{Q_t\}_{t=T}^{T+d} y_{t-1} + \{G_t\}_{t=T}^{T+d} \varepsilon_t, & \text{for } T \leq t < T^* \\ C^s + Q^s y_{t-1} + G^s \varepsilon_t, & \text{for } t \geq T^* \end{cases}$$

3.3 Data

The model is estimated using nine observables from the euro area: log difference of real GDP, real consumption, real investment, and real wages, log and demeaned hours worked, log differences of the GDP deflator as the measure for inflation, the main operating financing rate (MOFR), and one and five-year maturity bond yields. The first seven observables are constructed as in SW (2007) where all real observables are expressed in period-on-period log growth rates. The additional two interest rates correspond to the yield curve augmentation of the model as in KMR (2017). The sample period for the benchmark model runs from 2004-Q3 to 2019-Q4.[5] The data is drawn from two databases, *Eurostat* and *ECB's Statistical Data Warehouse* (SDW). Over the sample period, the data account for changes in the euro area's composition.[6] Since 2015, a number of outliers have appeared in the investment data series, causing large spikes in the data followed by largely offsetting levels in adjacent quarters. This increase in volatility can be attributed to R&D capitalisation reporting reforms, which include the transfer of intellectual property assets to affiliated entities and re-domiciled PLCs, particularly in Ireland.[7] In order to eliminate distortions in estimation, I linearly interpolate the investment data to remove the signalling power from the impacted quarters.

[5]Data on yields for the one and five-year maturity bonds are not available before 2004-Q3.

[6]Changing composition data is available for each observable except hours worked which is only available for a fixed composition of all 19 current member states.

[7]The effects cause large movements in national account data, but have little impact on underlying domestic activity (see, for instance, Avdjiev, Everett, Lane, and Shin, 2018; Osborne-Kinch, Mehigan, and Woods, 2020). Despite the Central Statistics Office and Ireland's national statistical agency recommending modified indicators, the European Commission is legally bound to adhere to ESA10 reporting standards which are subject to these distortions (Fitzgerald, 2016). Further discussions of reporting standards for national accounts are given in Lynch and Thage (2017) and Rassier (2017).

For this analysis, the ZLB period captures both the time when the MOFR was strictly zero, and the period during which above-zero effective lower bound without further movement could be assumed. In September 2014, the ECB reduced the MOFR to 0.05% by 10bp, which was referred to at the time as the effective lower bound. Despite the ECB's announcement to reduce the MOFR by another 5 bps to 0.00% in March 2016, based on its communication, I assume that the ZLB constraint effectively binds from 2014-Q3. The euro area yield data is proxied by an artificial data series constructed by the ECB and includes AAA-rated euro area central government bonds, which are updated every TARGET business day at noon. Two additional observables can be obtained from the constructed data, which are zero-coupon par yields for bonds with a maturity of one year and five years.

3.4 Priors

Structural Parameters $p(\theta)$ Priors for the structural parameters are set in accordance with SW (2003) and SW (2007). To capture the high volatility of the sample period due to the GFC and European sovereign debt crisis, the standard deviations of structural socks follow a slightly looser prior similar to KMR (2017). Following SW (2007), five parameters are fixed in the estimation. The depreciation rate is set to 10% annually and the exogenous spending-GDP ratio is fixed at 0.18. The steady-state labor market mark-up (λ_w) is set at 1.5, and the Kimball aggregators ε_p (goods market) and ε_w (labor market) are set equal to 10. Besides common boundaries of autoregressive parameters or the Taylor rule parameter for inflation, the lower bound for the labor elasticity parameter is set to 0.5 to ensure a model solution within the determinate parameter space. A summary of all prior and posterior distributions can be found in table 2.

Prior Expected Durations $p(d)$ The data for the informative prior of the sequence of durations is constructed from the ECB's quarterly Survey of Professional Forecasters (ECB SPF, 2013). To my knowledge, this is the first study that utilizes the SPF survey for estimating expected durations at the ZLB for the euro area. The survey measure not only aids with the identification of the expected durations but also includes information about agents' expectations of the future path of the policy rate in the estimation process.[8] Manski (2004) was among the first to argue in favor of including subjective information to better predict expectation formation in economics models. Since then, including subjective information has become a widely accepted strategy to improve the measurement of expectations in macroeconomic models. Baele, Bekaert, Cho, Inghelbrecht, and Moreno (2015) find a strong role for survey expectations in regime-switching models with rational expectations. Del Negro

[8] While it is possible to estimate the sequence of expected durations without the informative prior, robustness checks show that using the survey data enhances the accuracy of the expected duration estimates.

and Eusepi (2011) propose a measure for observed inflation expectations to improve the fit of a SW (2007)-style model in terms of model-implied expectations. KMR (2017) use a survey measure to identify the expected duration of the ZLB in the US. And more recently, Müller et al. (2022) use SPF survey data about inflation expectations to assess the effects of forward guidance in the euro area taking into account earlier work by Campbell et al. (2017), but they do not directly consider expected durations.

Even though the SPF focuses on inflation expectations, it also collects data on real GDP growth, unemployment, and most importantly, the MOFR in the euro area for multiple horizons. Participants are professionals affiliated with financial or non-financial institutions within the euro area. The informative prior is based on answers to survey questions such as the following:

Figure 3: 2013 Q1 SPF: Forecasting the Interest Rate

Please report selected other information underlying your forecasts (average over the period):

	2013 Q1	2013 Q2	2013 Q3	2013 Q4	2014	2015
ECB's interest rate (main refinancing operations)						

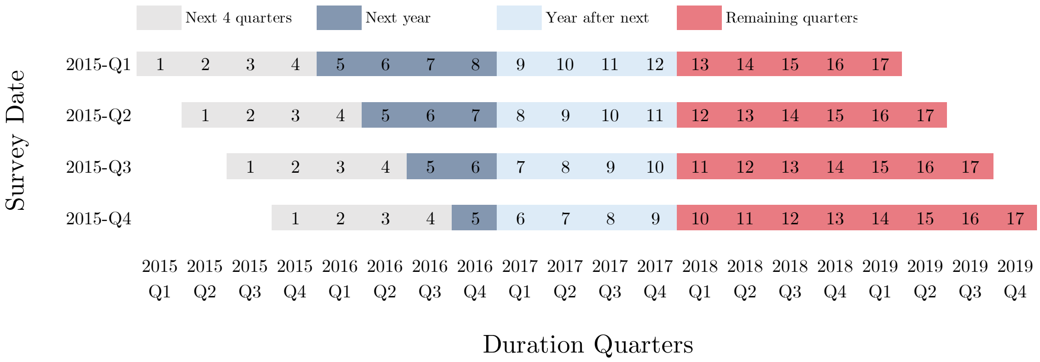
Note: Example questionnaire for the 2013 Q1 Survey of Professional Forecasters of euro area macroeconomic variables conducted by the European Central Bank. Source: ECB SPF (2013)

In the survey, agents forecast the future path of the MOFR over the next four quarters, the average over the next year, and the average over the following year. Forecasters' expectations of liftoff are pinned down by positive non-zero interest rate forecasts, which, in turn, determine the ZLB period's duration. Using these beliefs, I construct a probability distribution for each quarter's expected duration at the ZLB. There are no ambiguities in answers in the first quarter of each year, which includes forecasts for the upcoming 12 quarters respectively. In the remaining quarters, the survey answers may overlap due to the wording of the question. For instance, the survey conducted in 2015-Q2 asks for forecasts for 2015-Q2, 2015-Q3, 2015-Q4, 2016-Q1, 2016, and 2017, thus, asking for 2016-Q1 twice. In order to avoid unequal weighting on the survey answers, if a respondent expects the MOFR to be zero in 2016-Q1 but believes a non-zero average for the rest of the year, the expected duration will be equally allocated between 2016-Q2, -Q3, and -Q4. Figure 4 visualizes the overlapping responses using 2015 as an example year.

Because the question is open-ended, there are no point estimates for expected durations beyond the forecasting period of the survey. In surveys in which respondents expect the interest rate to be zero across the entire survey period, there is no indication of when they expect the liftoff to occur. Thus, I introduce an upper bound of 17 quarters for expected durations.[9] When a respondent forecasts a zero MOFR over the entire survey period, I assign equal probabilities to the following quarters up to

[9]The MOFR is strictly zero for 16 periods at the end of the data sample. I vary the upper bound of the estimates to different values and find results are robust to changes.

Figure 4: Overlapping Responses across different quarters of 2015



Note: The figure visualizes the overlapping responses in the SPF results and the respective duration quarters.

a maximum of 17 quarters. Moreover, some respondents to the 2014-Q3 ZLB survey may not have predicted a drop in MORF at quarter end. It is therefore likely that higher interest rate forecasts are not associated with beliefs of a short duration for the ZLB, but with outdated beliefs from the previous quarter. Due to the limited informative power of 2014-Q3, I reuse the responses for 2014-Q4, assuming these are responses for which all respondents have anticipated a fixed interest rate regime. Accordingly, a prior distribution for the expected durations is constructed based on the SPF survey data and a uniform distribution in which each element is equally weighted.[10]

4 Results for Benchmark Model

Following KMR (2017), I jointly estimate the set of structural parameters and expected durations.[11] The posterior distribution is based on 500,000 chains where the first half of the draws are discarded as burn-in to allow the sampler to converge. The convergence of the parameters is supported by the Brooks, Gelman and Rubin test (Brooks and Gelman, 1998; Gelman and Rubin, 1992) with results based on two parallel chains of 500,000 draws, each with different starting values.

4.1 Structural Parameters and Fit of the Model

The posterior estimates of the structural parameters, presented in table 2, are broadly in line with estimates by KMR (2017), SW (2003), and SW (2007). This is especially encouraging given the different data and the length of the ZLB period (22 quarters) which makes up a considerable share of the sample period for the euro area.

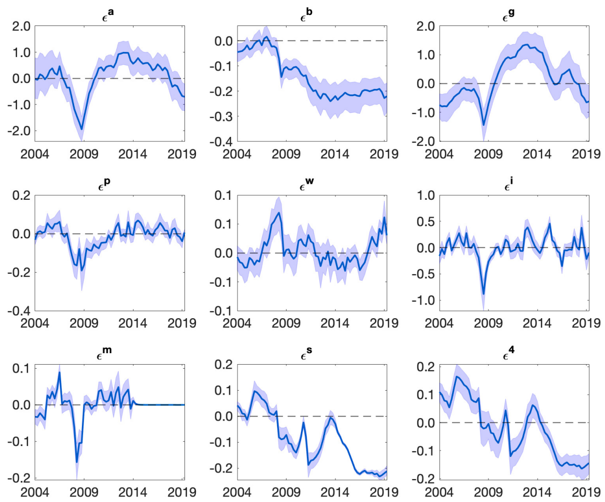
As anticipated, the shock variances are larger than previously estimated by SW (2003) and SW

[10] I discuss the sensitivity of the results to different weights on the survey to emphasize how important the survey measure is in the estimation process.

[11] The posterior mode, used to tailor the proposal density in the sampler is computed using a sub-sample that ends before the euro area hits the ZLB. In other words, as in KMR (2017) I optimize over the joint likelihood for pre-ZLB to get the proposal for the structural parameters.

(2007) for the euro area and the United States respectively. The posterior mean of the inflation trend parameter ($\bar{\pi}$) implies an annualized inflation target of 2.04% which nearly matches the official ECB target of 2%. The posterior mean estimate for the steady-state interest rate implies an annual steady-state nominal interest rate of 3.78% and an annual steady-state real interest rate of 1.74%. Trend growth (γ) is estimated to be around 0.25% on a quarterly basis, which is a bit higher than the average growth rate in the data (0.11%) but in line with estimates of trend growth in other settings. The relative risk aversion parameter ($\sigma^c = 0.93$) is estimated to be smaller than 1, implying an inter-temporal elasticity of consumption that is larger than unity. While this is substantially lower than estimates by SW (2007) for the US ($\sigma^c = 1.38$) or by SW (2003) for the euro area ($\sigma^c = 1.61$), more recent studies that include the ZLB period find similarly low estimates (for the US: Jones (2017): $\sigma^c = 0.98$ and KMR (2017): $\sigma^c = 1.05$).[12] Consumption habits are more important ($\lambda = 0.74$) compared to SW (2003) and KMR (2017) for the US. Turning to the Philips curve estimates; wages are less flexible ($\xi_w = 0.93$) than previously estimated for the euro area (SW (2003): $\xi_w = 0.76$) and the US (KMR (2017): $\xi_w = 0.86$). The degree of price stickiness ($\xi_p = 0.77$) is estimated to be lower than in SW (2003). Finally, in the pre- and post-ZLB regime, the monetary authority responds aggressively enough to inflation to fulfil the Taylor principle ($\phi_\pi = 1.49$). Reactions to the output gap ($\phi_y = 0.06$) and output growth ($\phi_{\Delta y} = 0.02$) are small, which is consistent with the idea that the ECB is primarily focused on its price stability mandate.

Figure 5: Smoothed Shocks



Note: The figure plots a simulated Kalman-smoothed time series for the shock processes: ε_t^a (technology), ε_t^b (risk premium), ε_t^g (government spending), ε_t^p (price mark-up), ε_t^w (wage mark-up), ε_t^{qs} (investment technology), ε_t^m (monetary policy), ε_t^4 (1Y Term Premia), and ε_t^{20} (5Y Term Premia), using 100 randomized posterior MCMC draws $\hat{theta}_j$, and the 10^{th} and 90^{th} percentile confidence band.

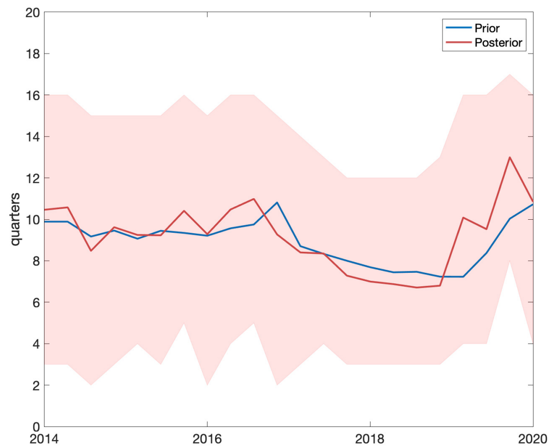
[12]Assuming log utility in consumption (i.e., $\sigma^c = 1$) does not qualitatively or quantitatively affect the remaining structural parameter estimates in the model.

Using randomized draws from the posterior chains, figure 5 shows a simulated Kalman-smoothed series for the shock processes. As indicated by the posterior estimates of the autoregressive parameters (compare table 2), the technology shock (ε^a), the preference shock (ε^b), the government spending shock (ε^g) and the term-premia shocks (ε^4 and ε^{20}) are persistent. As in KMR (2017), the preference shock is estimated to be persistently negative throughout the sample period, absorbing some of the expected duration impact. A negative and persistent risk premium shock mitigates potential expansionary effects of forward guidance policies.[13] The monetary policy shock (ε^m) fluctuates around zero, until at the ZLB when, by assumption, the shock is set to zero.

4.2 Expected Durations

The joint estimation allows me to estimate the expected durations of the ZLB in each quarter as independent parameters. The posterior estimates are based on a prior that assigns 50% weight to the SPF survey data and 50% weight to a uniform distribution. At the beginning of the sample period, the ZLB is expected to bind for around 2.5 years (10-11 quarters). From 2017 to 2018, the expected durations steadily decrease, reaching their lowest estimate of 6.67 quarters in 2018-Q3. During 2019, estimates increase sharply, exceeding an expected duration of over three years (12.95 quarters) in 2019-Q3.

Figure 6: Expected Durations of the ZLB in the euro area



Note: The figure shows the prior (blue line) and posterior mean (red line) and the equal-tailed 80% posterior bands of the expected durations.

According to Figure 6, there is considerable variation in the estimates of expected duration over time. Compared to the KMR (2017) US results where estimates peaked around 2012 and declined steadily thereafter, the estimates for the euro area are consistently longer. The increase in durations

[13] Draws with a lower persistence parameter as well as tightening the prior around a lower value, results in a higher volatility of the remaining shocks at the ZLB.

towards the end of the sample period is consistent with developments in the euro area that gave no suggestion of an imminent liftoff.

The estimates are robust to changes in the survey weight, changes in the upper bound limit and against market implied durations derived from the forward curve. Despite strong robustness across varying survey weights, adding more weight to the survey measure increases the accuracy in estimating the expected duration, suggesting that survey data are informative. The duration estimates are similar to market expectations of the forward curve in the euro area. The forward curve and the corresponding ZLB duration are constructed according to the methodology provided by the ECB and include a term structure and show the different rates offered at different maturities. Encouragingly, the market expectations show a similar trend pattern as the SPF survey measure, but the forward curve durations are longer for most of the sample period reflecting the captured term premium. Finally, the expected duration estimates are robust against variation in their upper bound limit. When increasing this limit from 17 to 25 quarters, the duration estimates remain stable, although slightly above the benchmark model.

4.3 Model Dynamics with a pegged interest rate

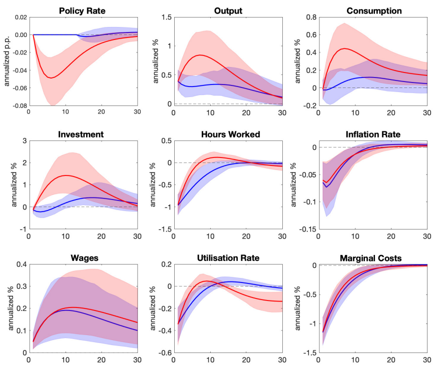
In LRE models, forward-looking agents incorporate their expectations of the ZLB duration into their decision-making process. The qualitative and quantitative effects of structural shocks will, therefore, depend on the estimated expected durations. It is well known that models with a fixed interest rate respond differently to structural shocks and effects may seem paradoxical and counterintuitive to standard New-Keynesian theory (see, for instance, Cochrane, 2017; Diba and Loisel, 2021; Werning, 2011). To investigate these responses and account for nonlinearities at the ZLB, the dynamics of the model are based on a Generalized Impulse Response Function (GIRF) algorithm similar to the one proposed by Koop, Pesaran, and Potter (1996). Future shocks are conditioned on only the history and/or shock, and therefore, are averaged out. The subsequent GIRF is an average of what happens given an information set from the present and past based on the conditional expectation on the history ω_{t-1}:

$$GIRF(N, \vartheta, \omega_{t-1}) = E[\tilde{y}_{t+N} \mid \vartheta^j(\varepsilon_t^j = \tilde{\varepsilon}_{t|T}^j), \mathbf{d}_{13}, \omega_{t-1}] - E[\tilde{y}_{t+N} \mid \vartheta, \mathbf{d}_{13}, \omega_{t-1}]$$

To illustrate the effects of expected durations on the model dynamics, consider the following two scenarios: a duration of 13 quarters and an imminent liftoff after one quarter where the steady-state serves as history ω_{t-1} before the respective shock hits the model. Until liftoff, the expected durations decline continuously by one period at a time. The responses are then based on random draws from the joint posterior distribution where the duration is estimated to be 13. The shocks ϑ are randomly drawn from the Cholesky-factorized variance-covariance matrix at the respective draw of structural parameters. To model the effects of a particular shock, the random draws of this shock are replaced by a one-standard-deviation shock based on of the smoothed empirical innovation when the interest rate is fixed ($\varepsilon_t^j = \tilde{\varepsilon}_{t|T}^j$). A detailed description of the GIRF algorithm can be found in the technical appendix.

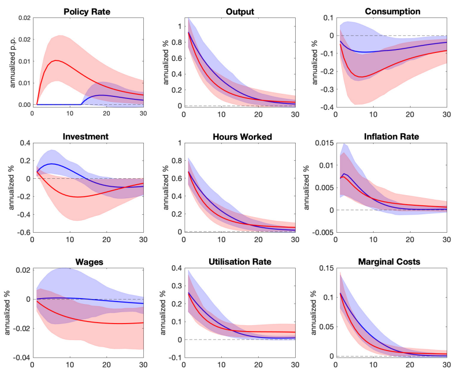
Figure 7 shows the GIRFs for a positive shock to productivity. In both scenarios, the dynamics are similar to those in standard New-Keynesian theory. The shock directly increases output and reduces the marginal costs of production. Higher levels of income boost consumption and investment in the economy. Revised labor-leisure optimization decreases hours worked, and lower marginal costs reduce inflation. As production becomes more efficient, the rental rate of capital and capacity utilization decrease. Since the central bank cannot immediately stabilize inflation by conducting expansionary monetary policy, the effects on the real economy are less pronounced when the interest rate is pegged for longer periods of time. The missing monetary transmission channel is particularly notable for investment and consumption.

Figure 7: Technology Shock



Note: GIRF for a positive technology shock. Blue line: initial duration=13, Red line: initial duration=1

Figure 8: Exogenous Spending Shock

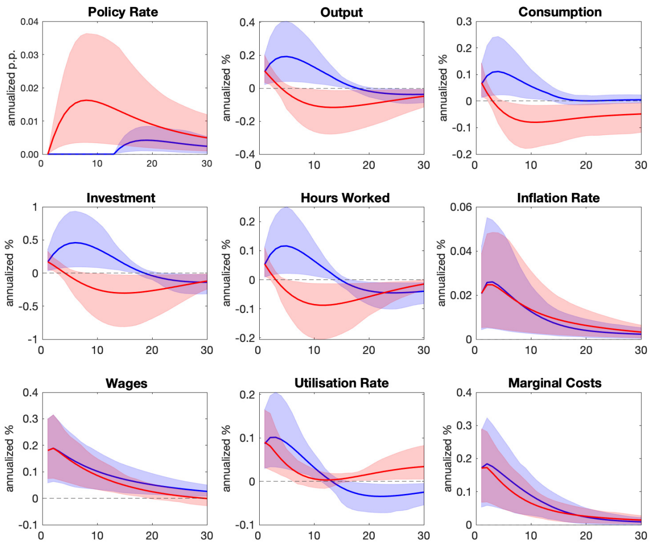


Note: GIRF for a positive spending shock. Blue line: initial duration=13, Red line: initial duration=1

Figure 8 plots the GIRFs for a positive exogenous spending shock, such as a government spending shock. Crowding-out effects and overshooting are only visible in the imminent liftoff scenario. This paradox is commonly known as the *fiscal multiplier puzzle*. According to Christiano, Eichenbaum, and Rebelo (2011), the government-spending multiplier is not only higher at the ZLB, but it also increases with the expected length of the ZLB. In addition, Cochrane (2017) shows that spending multipliers increase exponentially with increasing expected durations. When the policy rate is assumed to be fixed for an extended period, the boost in output combined with falling wages increases hours worked. In the absence of interest rate responses, investment increases. Upon lifting the interest rate peg, the policy rate increases marginally, accelerating the return to steady-state. The lack of conventional monetary policy options, leaves fiscal policy as an important policy tool.[14]

Figure 9 plots the GIRFs for a supply shock in the form of a wage mark-up shock. In a standard NK-setting, a positive wage mark-up shocks exhibit contractionary effects on the economy. The initial shock increases the real wage and, in turn, raises real marginal costs and inflation. Higher wages increase the return on capital and capital utilization. When the liftoff is modeled immediately, the central bank responds to rising inflation by increasing the policy rate which contracts the economy. The real variables (output, investment, and consumption) and hours worked decline. During an interest rate peg, this effect is reversed, resulting in the wage mark-up shock having expansionary effects. If the interest rate is pegged, the central bank is not responding to inflation by raising interest rates and the shock turns expansionary. This paradox, also known as the *Paradox of Toil* (Eggertsson, 2010), has a key policy implication that is that lowering firms' marginal costs, for example, by reducing wage mark-ups or the minimum wage, is not stimulatory to the model economy when the nominal interest rate is fixed.

Figure 9: Wage Mark-up Shock



Note: GIRF for a positive wage mark-up shock. Blue line: duration=13, red line: duration=1

For a positive shock to investment technology effects on inflation, wages, and marginal costs are small. Perhaps surprisingly, the negative effects on consumption are marginally smaller when the liftoff is expected in one quarter. However, the overall effects are qualitatively and quantitatively

[14]While the fiscal multiplier puzzle is present in models with a binding ZLB constraint, there is an increasing literature about how history dependence can resolve this puzzle. For instance, Hills and Nakata (2018) show that the presence of a lagged shadow policy rate in the interest rate feedback rule reduces the government spending multiplier at the ZLB. Furthermore, Kiley (2016) finds the government spending multiplier at the ZLB to be small in a sticky information model and Cochrane (2017) argues that the multiplier can be small in an alternative backward-stable equilibria.

similar in both cases. Both scenarios result in rising inflation on impact, while output, consumption, and investment decline. As a response, hours worked and the real wage are decreasing which triggers a drop in the real marginal costs and subsequently a fall in inflation back to steady-state.

A price mark-up shock has similar qualitative effects for both scenarios. When expected durations are assumed to be long, the effects of the price mark-up shock are amplified because the central bank is not counteracting rising inflation levels.

Turning to the risk premium shock, as found in KMR (2017) and Jones (2017), this shock interacts with the expected durations in the model. Its high persistence offsets the possibly large expansionary effects of long expected durations partially. Responses to the shock are amplified the longer the fixed-interest constraint remains in place. This accentuates the non-linearities in the dynamics of the model at the ZLB. The magnitude of the shock response is sensitive to the prior of the shock persistence ρ^b, but a lower persistence affects the remaining structural shocks considerably.

5 The Role of Forward Guidance in the euro area

In themselves, the estimated durations for each quarter of the ZLB do not reveal why the duration is expected. There are two possible determinants for of expected length of the ZLB: the underlying policy rule and deliberate monetary policy. In a recent paper, Jones et al. (2022) build on KMR (2017) and Jones (2017) and propose a method to identify forward guidance shocks during the ZLB and thereby uncover the nature of the expected duration.

In the absence of further shocks, the expected duration $\{\mathbf{d}_t\}_{t=1}^{T}$, is assumed to fall by one quarter in each period t:

$$E_t[\mathbf{d}_{t+1}] = \mathbf{d}_t - 1 \quad \text{for} \quad \mathbf{d}_t > 0 \tag{9}$$

The actual expected duration $\mathbf{d}_t$, as estimated following KMR (2017), can differ from the expected duration implied by the policy rule and the structural shocks. Throughout each period t, the duration which coincides with the underlying policy rule will be denoted by *lower bound duration*. This type of duration can be obtained with the occasionally binding constraint algorithm of Jones (2017). The algorithm finds the lower bound duration based on a smoothed state estimation, a smoothed structural shock, and a fixed interest rate regime.[15] In absence of further shock the lower bound duration is also

[15] A detailed description of the algorithm is provided in the technical appendix.

expected to fall by one quarter in every period t:

$$E_t[\mathbf{d}_{t+1}^{lb}] = \mathbf{d}_t^{lb} - 1 \quad \text{for} \quad \mathbf{d}_t^{lb} > 0 \tag{10}$$

The central bank can, however, intentionally extend or reduce the expected duration of the ZLB through credible announcements. The resulting deviations from conventional monetary policy correspond to forward guidance.[16] Hence, unconventional monetary policy is assumed to be powerful enough to extend the expected duration of the ZLB beyond the constraint and the structural shocks. The *forward guidance duration* is then defined as the share of expected duration not explained by the lower bound duration:

$$\mathbf{d}_t^{fg} \equiv \mathbf{d}_t - \mathbf{d}_t^{lb} \tag{11}$$

As in Jones et al. (2022), forward guidance shocks ε_t^{fg} capture unexpected changes of the forward guidance duration, $\Delta\mathbf{d}_t^{fg}$, beyond the anticipated one quarter decrease in each period. A forward guidance shock in period t is defined as:

$$\varepsilon_t^{fg} = \begin{cases} \mathbf{d}_t^{fg} - \mathbf{d}_{t-1}^{fg}, & \text{if } \mathbf{d}_t^{lb} \geq 1 \\ \mathbf{d}_t^{fg} - \mathbf{d}_{t-1}^{fg} + 1, & \text{if } \mathbf{d}_t^{lb} = 0 \end{cases}$$

5.1 Identifying Forward Guidance

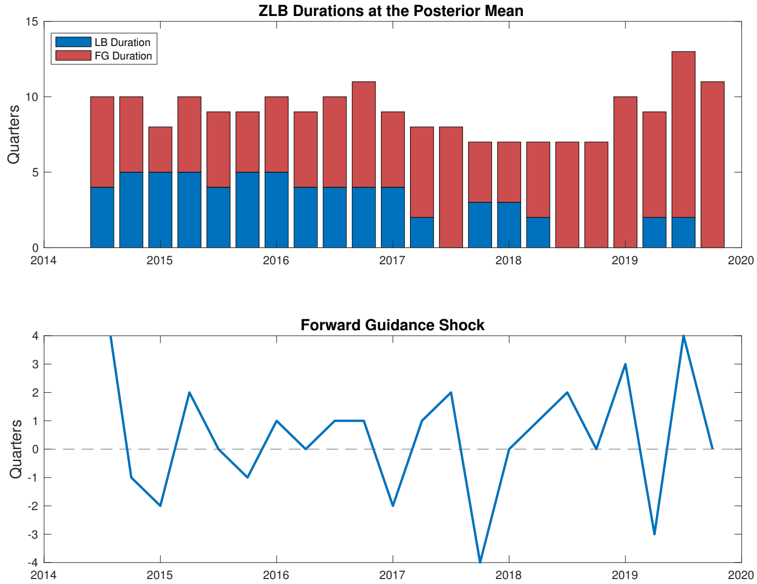
Figure 10 shows the (rounded) mean expected duration estimates of the ZLB as lower bound durations and forward guidance durations for the euro area based on the posterior estimates of the structural parameters and expected durations.[17]

Approximately half of the total estimated duration, between 4 and 6 quarters, is accounted for by the lower bound duration within the first half of the ZLB period. Consequently, forward guidance is assumed to account for the other half of the expected durations. Beginning in early 2017, forward guidance takes over and the lower bound durations disappear. Apart from two periods in 2018 and the peak of expected durations towards the end of the sample in 2019, the policy constraint itself is not binding in the second half of the ZLB period. This is supported by the forward guidance shock,

[16] In addition to providing insight into calendar-based forward guidance policies, the duration measure also offers insights into state-contingent forward guidance policies. When the central bank adopts a state-contingent forward guidance strategy based on macroeconomic indicators, the model implies a rational expectation path for each of these indicators. Whenever these expectations differ from those of the occasionally binding constraint, state-contingent forward guidance is equivalent to calendar-based guidance in the model.

[17] Note that, the fixed interest rate at the ZLB, $r_t = r$, changes from 0.05% to 0% in 2016 and hence the steady-state value of the interest rate $-r$ as well.

Figure 10: Lower Bound and Forward Guidance Duration



Note: The upper graph shows the decomposition of expected durations into *lower bound duration* and *forward guidance duration*. The lower graph shows the respective forward guidance shock over the sample period.

depicted in the second graph in figure 10. When forward guidance assumes greater importance in the second half of the ZLB period, the forward guidance shock is largely positive. The increase in forward guidance durations, particularly from 2019 onwards, coincides with the ECB's shift to an *enhanced forward guidance* strategy in June 2018.[18] This emphasizes the role of unconventional monetary policy in the euro area.

5.2 Forward Guidance Dynamics

As forward guidance has become an important component of monetary policy in the euro area, I will investigate how the model dynamics are affected by deliberate changes in forward guidance. To illustrate the effects of central banks changing expected durations in non-linear models, consider the following GIRF:

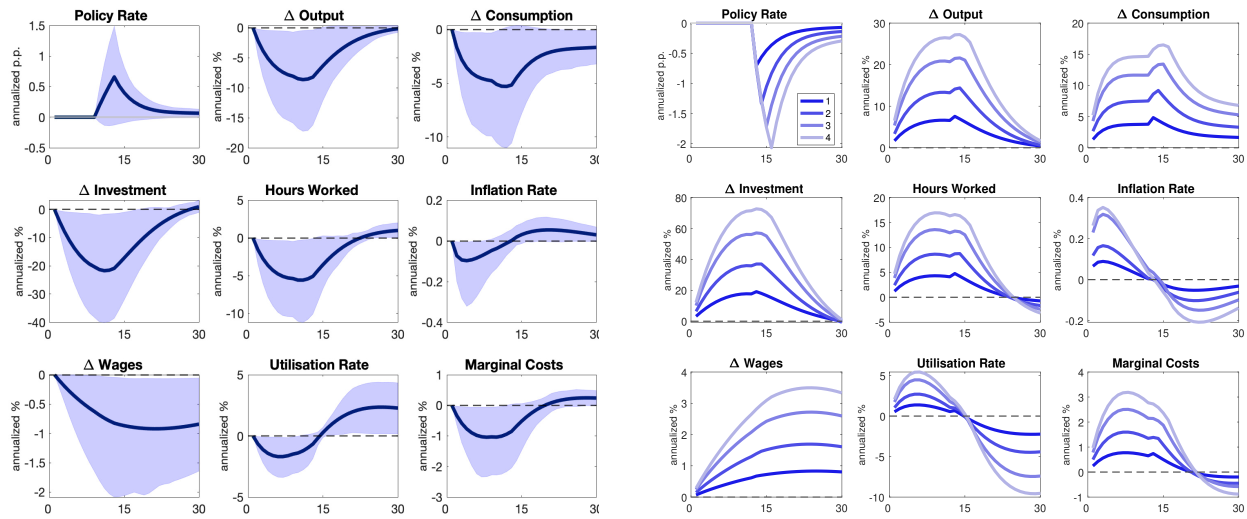
$$GIRF(N, \vartheta, \omega_{t-1}) = E[y_{t+N} \mid \vartheta, \mathbf{d}^*, \omega_{t-1}] - E[y_{t+N} \mid \vartheta, \mathbf{d}^{12}, \omega_{t-1}] \tag{12}$$

With the exception of changing the expected duration, the calculation follows that in section 4.3. Suppose the central bank announces a liftoff after two years (8 quarters), instead of the previously

[18]The ECB's Governing Council adopted a more calendar-based approach by stating that key interest rates will not be increased for at least another year. In previous announcements, it was simply announced that interest rates were staying at their levels for an extended period of time.

expected duration of three years (12 quarters). Thus, consider a fall in expected durations by 4 quarters, similar to the *taper tantrum* exercise in KMR (2017). Because agents expect the liftoff to occur sooner than expected, they will revise expectations about the future path of the policy rate and change their optimization behavior. Consequently, real variables in the model contract on impact similar to conventional monetary policy tightening. By keeping the policy rate pegged for a longer period of time, the expected path of future interest rates will also be lowered, which will have an expansionary effect on the economy. In addition, the quantitative effects increase with lengthening of the expected duration.

Figure 11: Forward Guidance Dynamics



11.1. 4 quarters decrease in d **11.2. 1-4 quarters increase in d**

Note: The figure shows the dynamics of forward guidance in the model. Figure 11.1 displays a 4 quarter decrease in expected duration similar to a taper tantrum scenario. Figure 11.2 shows the dynamics for a 1 to 4 quarter increase to expected durations.

Figure 11 plots GIRFs for increases in expected durations of one quarter, two quarters, three quarters, and one year. Because agents now expect the liftoff later than previously anticipated, real variables expand on impact. A forward guidance announcement of one additional quarter leads to an increase in output by approximately 8%, in consumption by 5%, and in investment by 18%. With an expected duration shock of four quarters, output increases by 28%, consumption increases by 16%, and investment increases by 70%. With longer forward guidance extensions, these effects become more pronounced and are non-linear in the sense that increasing expected durations by 2 quarters might not necessarily exhibit the same qualitative or quantitative effects as increasing expected durations by 4 quarters (Campbell et al., 2017; Del Negro et al., 2012). Overpredicting the expansionary effects of forward guidance announcements is widely known as *the forward guidance puzzle* (Del Negro et al., 2012). Rational agents will base their optimal choices on their expectations about future policy. Thus, the path of endogenous variables, which is determined by backward induction, depends on these beliefs (see, for instance, Gibbs and McClung, 2022). The fixed interest rate period and expectations of active policy after liftoff create a feedback loop to inflation that leads to the large expansionary effects of real variables. The longer the interest rate is pegged, the stronger these effects become.

6 Nature of Liftoff

6.1 Estimating the Model under Indeterminacy

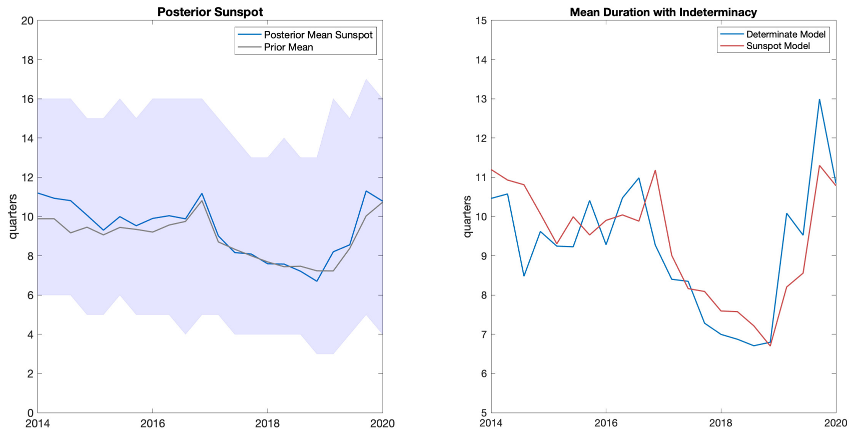
Next, I consider the possibility of a passive monetary policy regime after liftoff. In order to estimate the model under indeterminacy, five new parameters are added. The Taylor-type rule in the second regime is similar to that in the pre-ZLB regime, except for the inflation reaction parameter, which is assumed to be strictly smaller than that consistent with passive monetary policy. The prior for ϕ_π^s follows a uniform distribution over the interval [0,1]. The priors for the remaining Taylor rule parameters are the same as in the determinate case and the sunspot standard deviation follows an inverse gamma distribution with a mean of 0.42 and a standard deviation of 0.29.[19]

The structural parameters are similar to those for the determinate case (compare table 2). Among the more notable differences are the long-term yield curve standard deviations which are estimated to be lower, as well as a higher price mark-up shock persistence. Prices are estimated to be more flexible in the sunspot model ($\xi_p = 0.65$) than in the determinate model ($\xi_p = 0.77$). Monetary policy in the liftoff regime reacts passively to changes in inflation $\phi_\pi^s = 0.59$.

It appears that even though passive monetary policy is extended, the estimated expected durations of the ZLB period remain stable compared to the benchmark model. The left-hand plot of figure 12 shows the mean posterior estimates of the expected durations and the corresponding 80% confidence band and the prior mean with equal weight on the survey measure and a uniform distribution. On the right, the estimates from the determinate model are plotted against those from the sunspot model which shows that inferences about the expected duration of the policy regime are unaffected by agents' expectations after liftoff. As a result, even if the central bank engages in passive monetary policy for a prolonged period, the expected durations of ZLB regimes do not change dramatically.

[19] In order to avoid dual-modal chains, Taylor rule reaction parameters on output growth and the output gap are bounded to be strictly positive.

Figure 12: Expected Durations and Passive Monetary Policy



Note: The figure shows the mean prior, the mean posterior estimates for the determinate model and the durations when monetary is expected to be passive after liftoff at prior weights: $\omega = 0.5$.

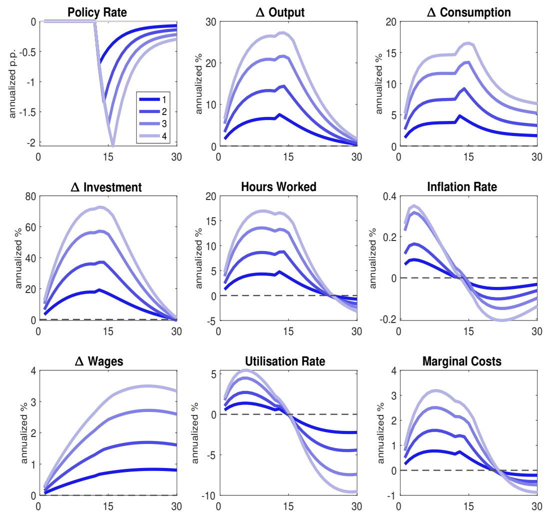
The model fits the data fairly well, with a few differences in the smoothed shock estimates for the mark-up shocks. This is not surprising since output and inflation are both forward-looking and are therefore linked to monetary policy. Intuitively, forward guidance lowers uncertainty and aids in forming expectations about future inflation. By reducing the inflation reaction parameter, the central bank is also allowing future inflation to be more volatile. When inflation is allowed to fluctuate, prices become less sticky, shifting the Phillips curve. This will in turn translate to more volatile mark-up shocks and higher variability of inflation and output.

6.2 Effectiveness of Forward Guidance

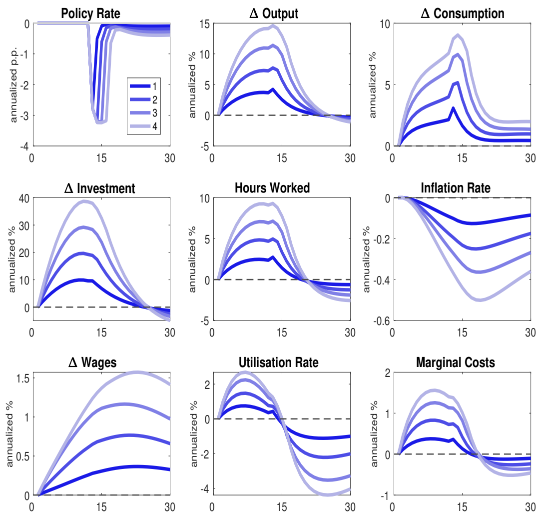
Forward-looking agents update their beliefs about the future stance of monetary policy if the central bank is not expected to return to an active monetary policy regime after liftoff. As a result of revised expectations, unconventional monetary policy effects are muted. Thus, when agents expect the central bank to commit to unconventional monetary policy for an extended or indefinite horizon, the expansionary effects of forward guidance are much less pronounced.

Figure 13 compares the GIRFs for a deliberate increase in expected durations by one, two, three, and four quarters for the determinate model (left-hand side) and the sunspot model (right-hand side). When expected durations increase by four quarters, output increases by 28% and investment increases by 70%. A passive monetary policy regime following liftoff reduces the expansionary effects for the same increase in expected duration. In this case, a four-quarter increase in expected duration leads to an increase in output by approximately 14% and a 39% increase in investment. The liftoff regime cannot eliminate forward guidance completely, but it does change the inflation response to announcements. When the central bank announces that monetary policy will return to active policy, forward guidance appears to re-inflate the economy, but otherwise demonstrates deflationary effects.

Figure 13: Increase in Forward Guidance Durations



13.1. Active Liftoff regime



13.2. Passive Liftoff regime

Note: The figure shows the dynamics of a 1 to 4 quarter increase to expected durations in the model. Figure 13.1 shows the dynamics when monetary policy is active after liftoff and figure 13.2 when it assumed to be passive.

6.2.1 Expectation Channel Empirical evidence has been inconclusive regarding the effectiveness of forward guidance in different settings. Although theory predicts a strong effectiveness of forward guidance (Eggertsson and Woodford, 2003; Krugman, Dominquez, and Rogoff, 1998), empirical evidence is less consistent. Moessner, Jansen, and de Haan (2017) argue that there is a disconnect between the same theory and its practical implementation due to a lack of commitment on the central bank's side. Filardo and Hofmann (2014) find mixed evidence of the systemic effects of forward guidance other than a reduction in the volatility of near-term expectations about the future path of policy interest rates. Del Negro et al. (2012) find positive and meaningful effects dependent on the type of forward guidance announcements. Similarly, Campbell et al. (2012) show that the effectiveness of forward guidance is dependent on the communication of the central bank and the resulting formation of private expectations. They tie their counter-intuitive findings around forward guidance to expectations of the *delphic* content of announcements regarding non-disclosed future economic conditions which are not necessarily based on economic theory.[20]

By applying this logic to monetary policy expectations after liftoff, agents may assume that the central bank's commitment to the future regime is *delphic* in nature, since the central bank's non-disclosed expectations about economic conditions do not seem promising. This explanation might not

[20] Campbell et al. (2012) show that while news of monetary tightening raises interest rates, inflation forecasts raise as well whereas unemployment forecasts decrease.

be sufficient, however, since effects on the real economy, except for inflation, are still consistent with an expansionary shock. Passive monetary policy in the liftoff regime can, however, lower market expectations of future interest rates, which could have the same effect as forward guidance announcements, keeping rates expected longer lower and lowering the term premia. This might result in the yield curve shifting or flattening and consequently, expectations of future inflation decline, pushing down actual inflation. Expectations of a return to a passive regime could, therefore, explain the relatively muted effects of forward guidance in practice compared to the theoretical effects.

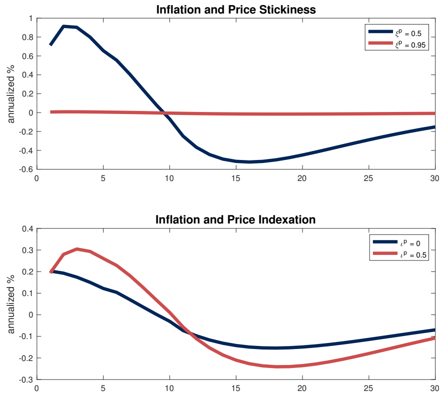
6.2.2 Inflation Feedback Loop Another explanation is the inflation feedback loop that is prominent in a model with active policy after liftoff. The coordination on a sunspot solution in the liftoff regime exhibits similar effects as introducing history dependence at the ZLB. In the inflation-sunspot, equilibrium inflation expectations are no longer purely forward-looking in response to forward guidance announcements, but are pre-determined (see, Gibbs and McClung, 2022). Several studies have shown that introducing history dependence can anchor expectations at the ZLB and eliminates the New-Keynesian puzzles. Cochrane (2017) demonstrates that equilibrium selection at the ZLB can mitigate these puzzles, particularly those which involve expectations of active fiscal policy and passive monetary policy. Alternatively, Kiley (2016) and Carlstrom, Fuerst, and Paustian (2015) show that sticky information models can resolve these paradoxes. Other papers, such as Diba and Loisel (2021) and Gibbs and McClung (2022), and Cole (2018), assess the mitigation of these puzzles by utilizing history-dependent policies such inflation targeting, price targeting, or the endogenization of bank reserves.

While the inflation-sunspot solution does not introduce history-dependent policy, it causes similar effects by pre-determining inflation expectations at the ZLB. Kiley (2016) shows that under a pegged interest rate regime, forward guidance effects are amplified by increased price flexibility and a steeper Phillips curve; the so-called *paradox of volatility*. Furthermore, a forward guidance announcement leads to a reversal in inflation when monetary policy is modeled actively in the liftoff regime, jumping on impact and then turning negative, known as the *reversal puzzle*. Carlstrom et al. (2015) explain this puzzle by showing that if there is a degree of inflation indexation, lagged inflation becomes endogenous at the ZLB; future inflation depends on past inflation, while past inflation depends on future inflation.

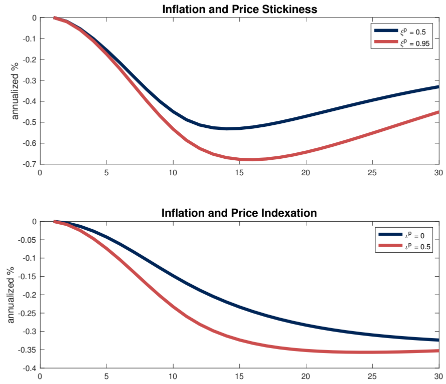
With monetary policy modeled actively upon liftoff, inflation enters a feedback loop that is amplified by the degree of price stickiness. Passive monetary policy upon liftoff, however, results in inflation outcomes that are not influenced by forward guidance, thus removing the feedback loop.

Figure 14 illustrates that inflation dynamics are less affected by price stickiness under passive monetary policy. Indeed, the sunspot model reveals that higher degrees of flexibility are associated

Figure 14: Expected Durations and Phillips Curve Dynamics



14.1. Active Monetary Policy



14.2. Passive Monetary Policy

Note: The figure shows inflation responses to a 5 quarter increase in expected durations at different Phillips curve parameter values (price stickiness ξ_p and price indexation ι_p).

with lower inflation responses. If a central bank reacts less aggressively to inflation changes, it is similarly allowing more variability in the price setting and might not be as committed to managing inflation. This is supported by the estimation results, which show that the indeterminate model exhibits a steeper Philips curve. Thus, if the central bank signals future passive monetary policy, agents will update beliefs to lower future interest rates, increasing inflation variation and sluggishly adjusting interest rates. As agents incorporate these newly formed inflation expectations into their optimization behavior, forward guidance announcements become less expansionary.

6.2.3 Communication of the Central Bank Based on the results, it appears that the communication strategy of the central bank around liftoff, as well as the type of forward guidance, matters for it to be effective. Forward guidance is more effective when the central bank communicates a return to active monetary policy after liftoff. This is consistent with different experiences with forward guidance among central banks. While in the US calendar-based forward guidance was generally effective (Campbell, Evans, Fisher, Justiniano, Calomiris, and Woodford, 2012; Feroli, Greenlaw, Hooper, Mishkin, and Sufi, 2017), forward guidance had only limited success in inflating the economy in Japan or the euro area despite having long expected durations at the ZLB. While Gertler (2017) argues that, for Japan, the absence of an inflation target might explain why individuals have adaptive rather than rational expectations, this is not applicable in the euro area. Instead, ECB policy focused on an upper-bound asymmetric inflation target that might have prevented inflation expectations from rising. In addition, central banks in the euro area and Japan used an open-ended forward guidance strategy that does not clearly communicate a potential liftoff strategy. When comparing different types of forward guidance, Ehrmann, Gaballo, Hoffmann, and Strasser (2019) find that open-ended forward guidance increases uncertainty and maintains market responsiveness rather than reducing it.

The model extension provides a theoretical basis for this distinction. A clear liftoff strategy matters for the effectiveness of monetary policy at the ZLB in forward-looking models. When agents expect the future path of monetary policy to be less active, a *lower for longer* policy will not necessarily succeed in inflating the economy. The sunspot model appears to match the experience of relatively muted effects of forward guidance compared with what theory predicts in the determinate case.[21]

7 Estimating the Model with Covid-19 Data

In recent history, the Covid-19 pandemic was the largest contractionary shock to the euro area, greater than both the GFC and the sovereign debt crisis. The Covid-19 shock is characterized by a very sharp fall in output, consumption, and investment, followed by an almost immediate recovery. Output fell by approximately 10% in the 2020-Q2, contracting twice as much as during the GFC. Given its volatile nature, investment fell by even more than output, approximately three times more compared to the GFC. Unlike the U-shaped recovery following the GFC, investment and consumption had an immediate rebound in 2020-Q3, giving the shock a V-shaped appearance. This can be attributed to the unique nature of the shock which was not caused by underlying structural issues but rather by intentional supply and demand constraints such as lockdowns and social distancing measures.

In the context of the pandemic as an unprecedented historic event, the question arises whether the Covid-19 data can be used to make inferences about economic models or if the large variation in the data will distort the estimates for the structural parameters. In fact, the latter has been shown for the US by Schorfheide and Song (2021), Lenza and Primiceri (2022) and Carriero et al. (2021) and for the euro area by Bobeica and Hartwig (2021) and Morley, Palenzuela, Sun, and Wong (2022). Common strategies to handle outliers, such as the use of time-varying volatility models or autoregressive heteroskedasticity models might not be desirable for estimating the Covid-19 period for two reasons. First, the change in the shock volatility appears transitory and not persistent. Most periods with high shock volatility develop continuously over time, but the Covid-19 shock occurs only in selected quarters without a transition period. Second, it is exactly known when the change in volatility occurred, which makes it relatively straightforward to account for it in estimation. Traditional stochastic volatility models, Markov switching models, or conditional heteroskedasticity models, such as ARCH or GARCH models, are not suited for large volatility changes with low persistence. A simple strategy to deal with outliers is to exclude Covid-19 data from estimation and treat all

[21] To compare the log marginal data density (log-MDD) of the benchmark and the sunspot model, I calculate a modified harmonic mean estimator based Geweke (1999). The log-MDD marginally supports the sunspot model indicating a slightly better fit respective to the benchmark model.

Covid-19 observations as non-informative. Schorfheide and Song (2021) show that omitting the recent data from estimation delivers a relatively good forecasting performance in a mixed-frequency VAR except for 2020-Q2. Alternatively, Carriero et al. (2021) find that a stochastic volatility model with Student-t distributed innovations and allowing for outliers prevents a persistent increase in volatility when estimating single-frequency Bayesian vector autoregression (BVAR) with stochastic volatility.

The approach used in this paper is closely related to that in Lenza and Primiceri (2022) and Morley et al. (2022). Lenza and Primiceri (2022) re-scale the innovation-covariances to capture the increased volatility during the Covid-19 months by estimating scaling factors for each month of the pandemic which decay over time. Morley et al. (2022) use the Lenza and Primiceri (2022) approach and estimate scaling factors for the covariance matrix of a large BVAR for the euro area without the volatility decay, finding that the data variation post 2020-Q3 appears to be close to its original level. Building on Lenza and Primiceri (2022) and Morley et al. (2022), but allowing for heterogeneity in the change in shock volatility, I propose three adjustments to an otherwise standard DSGE model estimation. First, the transitory change in shock volatility is modeled using a time-varying innovation-covariance matrix that estimates heterogeneous additive shock variance terms across the Covid-19 shock quarters. In addition, the total factor productivity (TFP) shock and the TFP component in the exogenous spending shock will follow an autoregressive moving average (ARMA) process in 2020-Q2 and 2020-Q3 in order to reduce the signal coming from the negative TFP innovations in 2020-Q1 and 2020-Q2 and to allow for a negative autocorrelation during the Covid-19 period. Third, I add a measurement correction for consumption data in the Kalman Filter due to the lack of informative power about economic decisions given variation due to lockdowns and social distancing measures.

7.1 Methodology

7.1.1 Change in Shock Volatility By adding additive variance terms for Covid-19 shock periods to the innovation-covariance matrix, I account for transitory changes in shock variances during the pandemic. Recall, that all shock processes follow a first-order autoregressive process with an iid-normal error term:

$$\varepsilon_t = \rho\varepsilon_{t-1} + \eta_t$$

where $\eta_t \sim iid(0, \sigma^2)$ with $E(\eta_t'\eta_t) = \Omega_t$ and $E(\eta_t'\eta_{t-j}) = 0$, $\forall j > 0$.

The data volatility is absorbed by additive variance terms that are estimated for the Covid-19 pandemic quarters. Instead of scaling being common across all shocks as in Lenza and Primiceri (2022), individual standard deviations are estimated for each shock and quarter. As in Morley et al.

(2022), no decay of the additional shock variances is estimated.[22] I take this approach to address not only shock volatility heteroskedasticity, but also its heterogeneity across structural shocks. For each quarter of the Covid-19 shock; 2020-Q1, 2020-Q2, and 2020-Q3, an additional shock variance term $\sigma_{i,j}^2$ is estimated:

$$\sigma_{i,t}^2 = \sigma_i^2 + D_t \sigma_{i,j}^2$$

where $D_t = [0, 1]$ is a dummy-vector that denotes periods, j, with higher volatility and i identifies each of the shocks in the system $i = [a, b, g, qs, p, w]$. When $D_t = 0$, the variance-covariance matrix is identical to the one from the pre-Covid-19 sample.[23] When $D_t = 1$, an additional shock variance parameter is estimated to reflect the increased volatility during this period.

$$\sigma_{i,t}^2 = \sigma_i^2 + \begin{cases} 0, & \text{if } D_t = 0 \\ \sum_{j=1}^{\Upsilon} \sigma_{i,j}^2, & \text{if } D_t = 1 \end{cases}$$

where $\Upsilon = 3$ is the number of additional shock variances for each of the three high-volatility quarters. As a result of the transitory nature of the shock, these additional variances are not persistent and have no effect on future periods. To capture these effects, Ω_t becomes a time-varying innovation-covariance matrix:

$$\Omega_t = \begin{pmatrix} \sigma_1^2 + D_t \sigma_{1,j}^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 + D_t \sigma_{2,j}^2 & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \dots & 0 & \sigma_l^2 + D_t \sigma_{l,j}^2 \end{pmatrix}$$

Each of the additional shock variances is estimated using a prior that follows an inverse gamma distribution with a mean of 1 and a standard deviation of 2.

7.1.2 ARMA (1,1) correction Aside from the temporary change in the innovation-covariance matrix, I propose an Autoregressive Moving Average (ARMA) correction for the total factor productivity (TFP) innovations in the technology shock and the exogenous spending shock to account for

[22] Including shock variances for the post-Covid-19 era does not significantly affect estimates of structural parameters. However, some observables, foremost consumption, exhibit increased volatility until the end of the extended sample period in 2021-Q1. This is addressed by a correction in the Kalman filter, discussed in section 7.1.3.

[23] During the entire pandemic period, monetary policy was at the ZLB, so no additional shock variances are estimated for monetary policy or yield curve shocks. By construction, the monetary policy shock is zero at the ZLB, and therefore also the yield curve shocks exhibit no additional volatility in this period.

the transitory nature of real activity dynamics during the pandemic.[24] The exogenous spending shock includes net exports, which are subject to domestic TFP movements, and therefore, are impacted by TFP innovations. Both shocks, the TFP shock (ε_t^a) and the exogenous spending shock (ε_t^g), follow first-order autoregressive processes with iid-normal TFP error terms:

$$\varepsilon_t^a = \rho^a \varepsilon_{t-1}^a + \eta_t^a$$
$$\varepsilon_t^g = \rho^g \varepsilon_{t-1}^g + \eta_t^g + \rho^{ga} \eta_t^a$$

where $\eta_t^a \sim iid(0, \sigma_{a,t}^2)$. Restricting ρ^a and ρ^g to be smaller than one guarantees that the process is stationary as well as that the consequences of the innovation η_t^a die out eventually. The Covid-19 pandemic, however, is characterized by a very low shock persistence in the sense of a large contraction followed by an immediate rebound before returning more to its usual dynamics. The quick rebound effect of the shock cannot be captured by estimating the technology Covid-19 shock using an AR(1) process. To account for this, a Moving Average (MA) term is modeled for the TFP innovations in 2020-Q2 and 2020-Q3:

$$\varepsilon_t^a = \begin{cases} \rho^a \varepsilon_{t-1}^a + \eta_t^a - \theta^a \eta_{t-1}^a, & \text{if } t\text{=2020-Q2 or } t\text{=2020-Q3} \\ \rho^a \varepsilon_{t-1}^a + \eta_t^a, & \text{for all other } t \end{cases}$$

In the case of the exogenous spending shock, the MA component is adjusted by the parameter that indicates how important the TFP innovation is to the shock process ρ^{ga}:

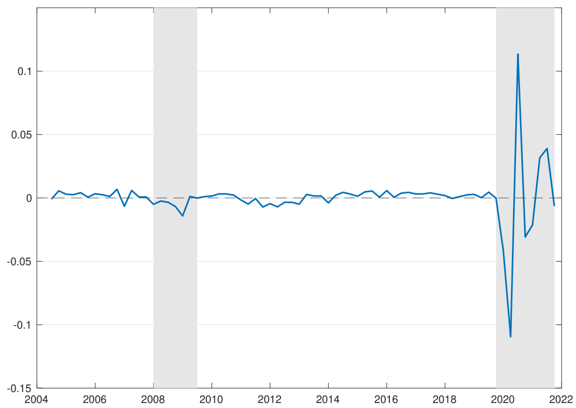
$$\varepsilon_t^g = \begin{cases} \rho^g \varepsilon_{t-1}^g + \eta_t^g + \rho^{ga} \eta_t^a - \theta^a \rho^{ga} \eta_{t-1}^a, & \text{if } t\text{=2020-Q2 or } t\text{=2020-Q3} \\ \rho^g \varepsilon_{t-1}^g + \eta_t^g + \rho^{ga} \eta_t^a, & \text{for all other } t \end{cases}$$

The MA dynamics cause the signal from 2020-Q1 and 2020-Q2 to be less persistent compared to other innovations that follow autoregressive dynamics. The newly introduced negative parameter θ^a will be estimated to be between zero and one, which induces a temporary negative autocorrelation into the series. The larger θ^a is estimated, the greater the reduction in persistence of the innovations and the stronger the negative autocorrelation. Thus, adding the MA term accounts for the unusual V-shaped dynamics of Covid-19.

[24] In a preliminary analysis I considered an ARMA correction for all structural shocks in the model, the technology shock appears to absorb most of the unusual dynamic around the Covid-19 pandemic. For most other shocks the additional MA parameter is not well-identified and significantly lowers the shock persistence.

7.1.3 Measurement correction in the Kalman filter As opposed to the GFC, where consumption dropped marginally, social health measures, lockdowns, and social distancing led to large drops during the Covid-19 pandemic with higher volatility in the following quarters. As shown in Figure 15, the Covid-19 pandemic caused consumption to fall by approximately 10%, compared to only 2% during the GFC. Despite the fact that the later lockdowns and social restrictions in 2021 did not trigger a similar response to the first lockdown in 2020-Q2, consumption volatility remained substantially higher than at any other time during the sample period.

Figure 15: Consumption Observations the Pandemic



Note: The consumption data is expressed in period-on-period log growth rates. The sample period runs from 2004:Q3 to 2021:Q4. *Source: Eurostat*

The model has difficulties accommodating these prolonged large fluctuations. The measured consumption data does not reflect the behavioral responses at other times and so is not informative about structural parameters related to consumption behavior. As opposed to traditional consumption theory that suggests households prefer a smooth path of consumption, the increased volatility during the pandemic and the months following distorts consumption Euler equation estimates, such as habit formation and intertemporal substitution elasticity.[25] Thus, along with shock adjustments, consumption observations are also dropped from the Kalman filter throughout the post-Covid-19 sample period, while policy rate observations are still excluded to avoid singularity. As a result, the new measurement equation is:

$$\check{z}_t = \check{H} y_t + \check{\nu}_t$$

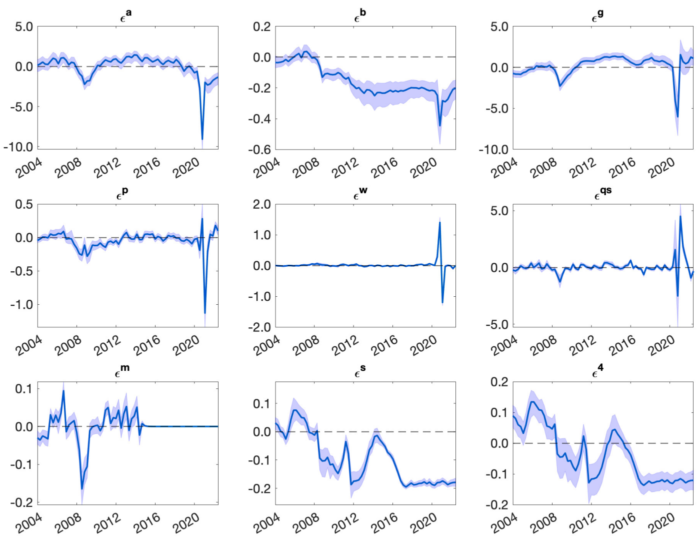
where the $(n^{obs} - 2) \times n$ matrix $\mathcal{C}$ reduces the number of observables to seven, $\check{z}_t \equiv \mathcal{C} z_t$, $\check{H} \equiv \mathcal{C} H$, and $\check{\nu}_t \equiv \mathcal{C} \nu_t$.

[25]Estimation experiments indicated a significantly smaller parameter for consumption habits and the elasticity of intertemporal substitution both of which interact with the forward guidance dynamics in the model.

7.2 Results

Based on the aforementioned adjustments, the model is re-estimated over a longer data sample period, from 2004-Q3 to 2021-Q4. Even with the large increase in volatility in the Covid-19 data, most structural parameter estimates remain stable compared to the pre-Covid-19 sample. The additional variance terms absorb most of the increased volatility in the data and the usual shock standard deviations are estimated almost identical to those pre-Covid-19, except for the exogenous spending shock (ε^g) and the price-mark up shock (ε^p) where the standard deviation is estimated to be marginally higher. In addition, differences can be observed for the estimates of the wage Phillips curve, where a lower wage indexation (ι^w) and a lower persistence of the wage mark-up shock (ρ^w) are estimated. The MA parameter for the technology shock (θ^a) is estimated to be 0.72 which is smaller than the autoregressive parameter of the technology shock. In the exogenous spending shock process, the MA parameter is adjusted by the parameter ρ^{ga}, and is thereby also reducing the signal from prior TFP innovations. As a result, the TFP innovation persistence from 2020-Q1 and 2020-Q2 is significantly reduced, despite not being completely offset.

Figure 16: Smoothed Structural Shocks during the Pandemic



Note: The figure plots a simulated Kalman-smoothed time series for the shock processes for the Covid-19 period.

Moving on to the additional parameters in the Covid-19 sample. Although these estimates are designed to control for heteroskedasticity rather than estimate the variances precisely, they confirm the substantial increase observed throughout Covid-19.[26] Considering that each estimate is based on

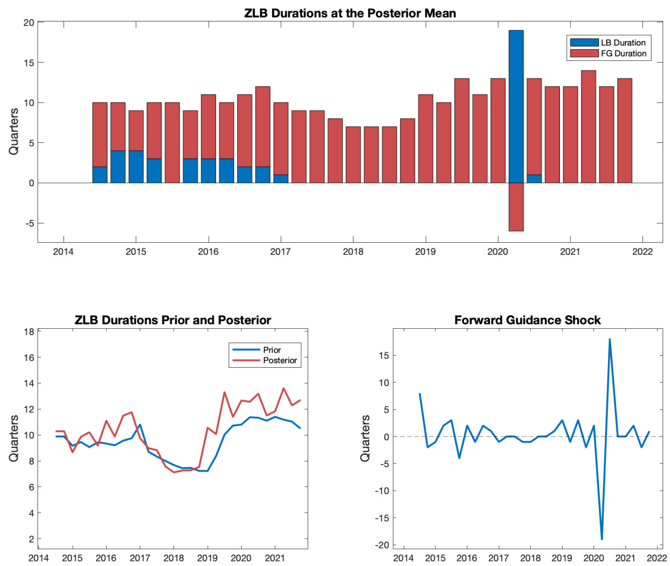
[26] As an independent exercise, I estimated scaling factors that are common to all shocks, similar to Lenza and Primiceri (2022), and found scaling factors between three and five. Estimates for other parameters were quantitatively similar to those obtained using additive variance terms. However, I make the additive variance approach the baseline because the scaling factors do not allow for what appears to be some degree of heterogeneity in how much shock volatilities change.

data that is only available for one quarter, the high variance of these estimates is not surprising.

In figure 16, a smoothed series of the structural shock visualizes how the pandemic impacted the model. The volatility of all shocks increases significantly during the Covid-19 shock quarters. In the first two quarters of 2020, the technology shock has a strong negative impact before quickly returning to pre-Covid levels. The Phillips curve shocks follow inflation and wages data movements and exhibit large and V-shaped fluctuations. Additionally, the investment-specific technology shock is characterized by a strong rebound effect. By construction, the monetary policy shocks is strictly zero at the ZLB, which translates into very little movement of the yield curve shocks.

As shown in figure 17, the expected durations follow a similar pattern than those in the pre-Covid-19 period. Due to the large fluctuations in the data, estimated durations remain high until the end of the sample period. Moreover, from late 2018 to the end of 2021, the posterior estimates exceed the prior estimates continuously. As in the pre-Covid-19 sample, the expected duration estimates are

Figure 17: Duration estimates during the Pandemic



Note: The upper graph shows the decomposition of expected durations into *lower bound duration* and *forward guidance duration* during the entire sample period. The lower right graph shows the respective forward guidance shock. The lower left graph plots the expected duration estimates against the prior with as survey weight of 50%.

restricted by an upper bound of 17 quarters, but the bound is only reached in the second quarter of 2020. An increase in the upper bound implies longer durations for the 2020-Q2 quarter, but also induces a trade-off with identifying the remaining durations.[27] As expected the duration estimates remain at a high level during the Covid-19 shock with visibly higher volatility. The forward guidance dynamics are similar, but overall lower than when estimated using data from the pre-Covid-19 period,

[27]The higher the upper bound, the weaker the other durations are identified. As a result, long durations in all periods of the ZLB have been given a lot more prior weight.

now accounting for just below half of the estimated durations initially and expanding throughout the ZLB period. The level shift in lower bound duration reflects how different the structural shocks transmit through the economy. The large volatility in the Covid-19 quarters might not make the usual drivers of the expected durations appear as negative as before.

The lower bound and forward guidance durations, however, follow the same qualitative dynamics as those in the pre-Covid-19 sample. Except for the large contraction in 2020-Q2, forward guidance is also the main driver behind durations throughout the Covid-19 pandemic. Accordingly, the lower bound duration in this quarter exceeds the estimated parameter, indicating negative forward guidance dynamics.[28] The long durations in the following quarters were not caused by economic conditions, but rather by intentional policy, which was subsequently abandoned in mid-2022 earlier than anticipated. Moreover, the durations seem to be largely influenced by inflation throughout the Covid-19 period. The lower bound durations are determined by the implicit policy rule responses which are constrained at the ZLB. However, according to the policy rule estimates, most of the policy function reaction is due to inflation rather than output growth or the output gap. Since inflation picked up soon after the onset of the pandemic, the underlying policy rule might not have been constrained even if the output gap remained negative.

8 Conclusion

This paper estimates a DSGE model for the euro area at the zero lower bound (ZLB). Monetary policy switches between a conventional policy regime in the form of a Taylor-type rule and an unconventional policy regime at the ZLB along with forward guidance. By expanding the information set of the model with survey expectations about the expected durations at the ZLB, following Kulish et al. (2017), the structural parameters and a sequence of expected durations can be jointly estimated. Despite including a policy change for a substantial period of the sample into the well known model of Smets and Wouters (2007), estimates for the structural parameters are consistent with previous literature. The estimates for the sequence of expected durations at the ZLB are robust against varying priors and financial market expectations in the form of forward curve durations. In addition, putting more weight on the survey measure improves estimation accuracy, reinforcing its significance. Compared to the US results by Kulish et al. (2017), the estimated durations are consistently longer and increase toward the end of the sample period. As a main contribution, this paper provides a robust estimate of the expected durations at the ZLB in the euro area that can conveniently be included in a wide range of model

[28]The ARMA correction for the TFP shock reduces negative innovation persistence and the volatility of ZLB durations during the pandemic. This results in an expansionary monetary policy stance despite negative forward guidance durations.

settings.

It appears that the ZLB's non-linear dynamics are largely determined by its expected length and nature. Furthermore, it is increasingly unconventional policy rather than the structural shocks that determine the duration of the ZLB in the euro area. Forward guidance, thus, extends the duration beyond the underlying policy rule, implying expansionary effects due to increases in expected duration caused by forward guidance announcements. The non-linearities at the ZLB substantially influence the model behavior and emphasize the role of forward guidance. At the ZLB, non-linear dynamics highlight the importance of estimating models with a reasonable measure of expected durations.

This paper also provides empirical evidence that forward guidance dynamics depend on expectations about monetary policy after liftoff. Following Gibbs and McClung (2022), I generalize the KMR (2017) approach by proposing a sunspot solution à la Bianchi and Nicolò (2021) that accounts for the possibility that the central bank communicates not to return to its original Taylor rule. Despite not being fixed, the nominal interest rate follows unconventional or passive monetary policy in the liftoff regime. Although the sunspot solution does not affect estimates of structural parameters or expected durations, it substantially reduces the effects of forward guidance. The monetary policy choice after liftoff does not affect the expected duration of the ZLB period, however, it influences the implied effects of forward guidance. In forward-looking models when agents incorporate expectations about an unconventional future path of the policy rate, a deliberate increase in the expected duration of the ZLB in the sunspot model mitigates the expansionary effects on the real economy substantially and exhibits deflationary pressures.

In addition, I propose a time-series approach to deal with the large data outliers observed during the Covid-19 pandemic. In the spirit of Lenza and Primiceri (2022), the approach makes three adjustments to the standard estimation procedure: a time-varying innovation-covariance matrix that captures the heteroskedasticity in shock variances, an ARMA(1,1) correction for the technology shock, and a measurement correction for consumption data in the Kalman Filter. I observe stable structural parameter estimates when Covid-19 data is included in estimation. Despite slightly shorter *lower bound durations*, qualitative expected durations dynamics are similar to those pre-Covid-19. With the exception of the 2020-Q2 lockdown quarter when the lower bound constraint on the policy rate was strongly binding, forward guidance remains the primary driver behind the estimated durations for most of the pandemic period.

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A Appendix

Table 2: Posterior Estimates

Parameters		**Prior**			**Posterior**			**Posterior**			**Posterior**		
					Determinate Model			**Sunspot Model**			**Covid Model**		
		Distribution	**Mean**	**Std. dev.**	**Mode**	**Mean**	**Std. dev.**	**Mode**	**Mean**	**Std. dev.**	**Mode**	**Mean**	**Std. dev.**
Shock Standard Deviations													
Technology	σ^a	IG	0.42	0.29	0.66	0.66	0.05	0.65	0.67	0.04	0.67	0.67	0.21
Risk premium	σ^b	IG	0.20	3.00	0.15	0.16	0.01	0.17	0.18	0.01	0.16	0.16	0.08
Fiscal	σ^g	IG	0.42	0.29	0.50	0.51	0.03	0.50	0.51	0.02	0.58	0.59	0.20
Investment	σ^q	IG	0.42	0.29	0.61	0.60	0.06	0.58	0.59	0.06	0.63	0.65	0.25
Monetary	σ^m	IG	0.20	3.00	0.30	0.30	0.01	0.29	0.30	0.01	0.29	0.30	0.11
Price mark-up	σ^p	IG	0.20	3.00	0.34	0.35	0.02	0.33	0.34	0.02	0.41	0.41	0.14
Wage mark-up	σ^w	IG	0.42	0.29	0.30	0.31	0.01	0.31	0.31	0.01	0.32	0.33	0.11
1 year	σ^4	U	1.00	0.58	0.34	0.39	0.10	0.22	0.35	0.34	0.35	0.39	0.31
5 year	σ^{20}	U	1.00	0.58	1.09	1.14	0.46	0.77	0.91	0.16	1.13	1.12	0.65
Term premia	σ^η	U	2.50	1.44	0.55	0.61	0.16	0.66	0.65	0.09	0.48	0.53	0.37
Sunspot	σ^ν	IG	0.42	0.29	-	-	-	0.37	0.38	0.03	-	-	-
Shock Persistences													
Technology	ρ^a	B	0.50	0.20	0.92	0.87	0.05	0.94	0.86	0.07	0.85	0.87	0.05
Risk premium	ρ^b	B	0.50	0.20	0.97	0.96	0.01	0.95	0.95	0.01	0.96	0.96	0.01
Fiscal	ρ^g	B	0.50	0.20	0.89	0.90	0.04	0.91	0.89	0.05	0.86	0.86	0.05
Fiscal technology	ρ^{ga}	N	0.50	0.20	0.51	0.50	0.08	0.49	0.5	0.07	0.53	0.55	0.10
Investment	ρ^q	B	0.50	0.20	0.38	0.39	0.13	0.40	0.41	0.13	0.34	0.37	0.09
Monetary	ρ^r	B	0.50	0.20	0.24	0.22	0.08	0.23	0.25	0.09	0.21	0.21	0.08
Price mark-up	ρ^p	B	0.50	0.20	0.18	0.27	0.15	0.60	0.52	0.17	0.10	0.24	0.15
Wage mark-up	ρ^w	B	0.50	0.20	0.10	0.13	0.07	0.10	0.15	0.08	0.03	0.06	0.03
Term premia	ρ^s	B	0.71	0.16	0.97	0.95	0.03	0.97	0.95	0.03	0.98	0.96	0.02
Structural Parameters													
Investment Adj. Costs	φ	N	4.00	1.50	5.96	6.12	1.24	6.27	6.31	1.17	6.15	6.25	1.33
CRRA	σ^c	N	1.50	0.375	0.94	0.93	0.09	0.89	0.93	0.08	0.91	0.90	0.10
Habits	λ	B	0.70	0.10	0.74	0.74	0.05	0.77	0.75	0.04	0.76	0.73	0.06
Degree of Wage Stickiness	ξ_w	B	0.50	0.10	0.93	0.93	0.02	0.92	0.92	0.02	0.93	0.93	0.01
Elasiticity of Labor Supply	σ^l	N	2.00	0.75	1.46	1.51	0.52	1.58	1.75	0.51	1.72	1.70	0.49
Degree of Price Stickiness	ξ_p	B	0.50	0.10	0.80	0.77	0.06	0.65	0.65	0.08	0.69	0.70	0.06
Indexation Wages	ι_w	B	0.50	0.15	0.39	0.39	0.13	0.35	0.35	0.12	0.15	0.18	0.07
Indexation Prices	ι_p	B	0.50	0.15	0.27	0.35	0.15	0.14	0.15	0.06	0.22	0.27	0.12
Capacity Utilisation	ψ	B	0.50	0.15	0.59	0.62	0.12	0.55	0.60	0.12	0.62	0.63	0.12
Fixed Costs	ϕ_p	N	1.25	0.125	1.62	1.63	0.09	1.61	1.59	0.09	1.57	1.60	0.11
Monetary Policy: Inflation	ϕ_π	N	1.50	0.25	1.52	1.49	0.22	1.29	1.38	0.20	1.40	1.50	0.24
Monetary Policy: Smoothing	ρ_R	B	0.75	0.10	0.87	0.86	0.03	0.85	0.84	0.03	0.87	0.87	0.03
Monetary Policy: Output Gap	ϕ_y	N	0.125	0.05	0.05	0.06	0.02	0.05	0.05	0.02	0.07	0.08	0.03
Monetary Policy: Output Growth	$\phi_{\Delta y}$	N	0.125	0.05	0.02	0.02	0.02	0.05	0.04	0.02	0.03	0.04	0.02
Constant Inflation	$\bar{\pi}$	G	0.63	0.10	0.51	0.51	0.06	0.55	0.56	0.06	0.48	0.50	0.06
Discount Factor	$\beta^{-1} - 1$	G	0.25	0.10	0.22	0.2	0.07	0.14	0.18	0.07	0.24	0.24	0.08
Constant Labor	$\bar{l}$	N	0.00	2.00	3.71	3.85	0.96	3.78	3.67	0.86	3.23	2.91	0.87
Trend Growth	γ	N	0.40	0.10	0.24	0.25	0.03	0.22	0.22	0.03	0.20	0.20	0.02
Capital Share	α	N	0.30	0.05	0.26	0.26	0.02	0.25	0.26	0.02	0.22	0.21	0.03
Term Premia Constant	c_4	N	1.20	0.25	1.38	1.41	0.23	1.52	1.49	0.20	1.45	1.47	0.20
Term Premia Constant	c_{20}	N	2.35	0.25	2.10	2.09	0.21	2.03	2.02	0.22	2.08	2.02	0.19
Monetary Policy Sunspot: Inflation	ϕ^s_π	N	1.50	0.25	-	-	-	0.76	0.56	0.25	-	-	-
Monetary Policy Sunspot: Smoothing	ρ^s_R	B	0.75	0.10	-	-	-	0.74	0.74	0.10	-	-	-
Monetary Policy Sunspot: Output Gap	ϕ^s_y	N	0.125	0.05	-	-	-	0.21	0.21	0.04	-	-	-
Monetary Policy Sunspot: Output Growth	$\phi^s_{\Delta y}$	N	0.125	0.05	-	-	-	0.12	0.10	0.05	-	-	-

Chains have 500,000 draws with the first half discarded as a burn-in.

N= Normal Distribution; IG= Inverse Gamma Distribution; U=Uniform Distribution; B= Beta Distribution; G=Gamma Distribution

Table 3: Additional Posterior Estimates for Covid-19

Additional Covid Parameters		**Prior**			**Posterior**		
		Distribution	**Mean**	**Std. dev.**	**Mode**	**Mean**	**Std. dev.**
Shock Standard Deviations 2020-Q1							
Technology	σ^{a1}	IG	1	2	1.06	1.28	0.90
Risk premium	σ^{b1}	IG	1	2	0.54	0.73	0.59
Fiscal	σ^{g1}	IG	1	2	0.71	1.47	1.27
Investment	σ^{q1}	IG	1	2	0.66	1.17	1.01
Price mark-up	σ^{p1}	IG	1	2	0.55	0.77	0.65
Wage mark-up	σ^{w1}	IG	1	2	0.63	0.80	0.75
Shock Standard Deviations 2020-Q2							
Technology	σ^{a2}	IG	1	2	2.36	2.10	1.10
Risk premium	σ^{b2}	IG	1	2	0.54	0.76	0.61
Fiscal	σ^{g2}	IG	1	2	0.63	1.11	1.09
Investment	σ^{q2}	IG	1	2	0.75	1.48	1.25
Price mark-up	σ^{p2}	IG	1	2	0.62	0.86	0.86
Wage mark-up	σ^{w2}	IG	1	2	0.87	1.09	0.73
Shock Standard Deviations 2020-Q3							
Technology	σ^{a3}	IG	1	2	0.62	0.84	0.69
Risk premium	σ^{b3}	IG	1	2	0.56	0.77	0.64
Fiscal	σ^{g3}	IG	1	2	1.52	1.59	1.06
Investment	σ^{q3}	IG	1	2	1.76	1.95	1.31
Price mark-up	σ^{p3}	IG	1	2	0.89	1.13	0.96
Wage mark-up	σ^{w3}	IG	1	2	0.91	1.10	0.82
ARMA Correction							
MA Parameter technology	θ^{a}	B	0.71	0.16	0.74	0.72	0.10

Chains have 100,000 draws, with the first half discarded as a burn-in.
IG= Inverse Gamma Distribution; B= Beta Distribution