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Timing of entry with heterogeneous firm

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Timing of entry with heterogeneous firms*

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Abstract

We examine entry in a market-entry timing model. Early entry allows a firm to enjoy a higher instantaneous post-entry profit, while later entry has the benefit of lower entry costs. In our model, firms can be asymmetric in terms of costs. Specifically, a more efficient firm enters with lower present value of costs. First, we show that entry order is always efficient in the duopoly game while in the triopoly model an efficient entry order could be violated. Moreover, one of the most notable results is that in the triopoly model we generate the necessary condition for an efficient order of entry. In addition, we explore how the rents earned by duopolists relative to a monopolist (the structure of profits in the market) impact the order of entry. These results would be useful for future empirical studies of market entry. Furthermore, our paper investigates the welfare implications of the entry in equilibrium by exploring the dynamics of the initial entry time in duopoly and triopoly markets. Previous studies found that the leader's time of entry is typically inefficiently too late. Our results show that unlike in the symmetric case, in the presence of asymmetric firms, first entry is not necessarily inefficiently delayed, especially in markets with higher duopoly effects (which capture duopoly rents relative to those for a monopolist) and with firms that are more differentiated. This result implies that encouraging an extra firm to enter in an oligopolistic market could shorten the period consumers have to wait for new products, and potentially increase social welfare.

Key words: timing games, entry, leader, follower, process innovation, product innovation.
JEL classifications: C72, L13, O31, O33.

1 Introduction

Whether and when to enter a market is one of the most important and difficult decisions faced by a firm. For example, there is a trade-off between entering early and collecting

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rents for longer, with waiting, possibly coming into the market with either lower costs or a better product. As if this decision is not complicated enough for a firm considering entry on its own, the presence of a potential rival or rivals brings its own set of strategic issues. Should a firm try to preempt entry by its rival and enter the market early? Or, on the other hand, should a firm strategically wait and delay its entry in an attempt to free-ride on the investments of the market pioneers? In both scenarios, it is the interaction between technology (which determine entry costs) and strategic behaviour that jointly drive observed entry decisions. Furthermore, firms may strategically behave differently in the cases where they are facing symmetric and asymmetric rivals. Concerning the symmetric case, we could consider the situation where all potential entrants produce similar products and that these firms have similar costs and capabilities to exploit any profitable opportunities. In this case, it is difficult to rank one over another (in terms of efficiency and potential profits). For instance, consider Coke-Cola and Pepsis' introduction of low-sugar softdrinks or Cannon and Nikons' strategic interaction surrounding the enhanced technology related to megapixels. In both cases, neither firm has an obvious advantage. Previous literature has found that with symmetric firms the incentive to preempt a rival can result in rents be equalized in equilibrium. In some cases, clustering in terms of entry can be observed in equilibrium in markets with more than two firms when rival's entry or technology decision accelerate the action of other firms.

Importantly, moreover, Mankiw and Whinston (1986) argue that entry behaviour of the oligopolists is typically not efficient because firms normally do not take consumer surplus into account and tend to enter earlier in the presence of a 'business stealing effect' (i.e. additional entry causing incumbents to reduce sales). While the first market entry is normally later than social optimal as the initial entrant is not impacted by the 'business stealing effect', this is not true for subsequent entry. Therefore, as a general rule, in equilibrium, first entry typically occurs too late whereas subsequent entries by later followers occur too early. Extending on this theme, Argenziano and Schmidt-Dengler (2013) further show that in the market preemption game with symmetric firms, there is a non-monotone relationship between the number of firms and the initial time of entry and that entry occurs earlier in the duopoly market than in any other market, implying extra competition in duopoly market further ruins social welfare. These results rely on symmetry between firms. However, there are many industries, indeed most, where some firms are superior to their rivals. In this context, rent equalization will no longer be the necessary condition to eliminate preemption incentives given that superior firms in terms of product and costs have greater potential to exploit market opportunities than their inferior rivals. It is an open question as how differences in firms will influence the equilibrium outcome. One possibility is that entry order in equilibrium may reflect the efficiency ranking of firms. It has been observed that Apple, which could be typified as one of the most efficient firm in the smart-phone industry is the pioneer to apply mature facial recognitial technology. But will it always be the case that most efficient firm moves first? Riordan (1992) shows that in equilibrium the order that firms enter reflects their relative cost efficiency in duopoly market. This framework provides the microfoundation for the recent empirical analysis of market-entry, see also Argenziano and Schmidt-Dengler (2012). The assumption that entry in market occurs in the order of profitability is commonly used in the empirical studies to address the inherent multiplicity problem of equilibria in modelling. However, following the intuition of Argenziano and Schmidt-Dengler (2012), this assumption of efficient order may not be credible in that the equilibrium entry order when more than two firms are competing depends on the primitives of the model.

Developing a new approach, this paper explores how the equilibrium outcome is affected by the differences in efficiency or costs between firms and the number of potential entrants. Specifically, we explore how entry time and efficiency are impacted by alternative market structures (duopoly and triopoly) and market characteristics (profit structure) in a preemption game when firms are heterogeneous in terms of cost.

By adapting the equilibrium formulation of Argenziano and Schmidt-Dengler (2014) and completely describing the pure-strategy subgame perfect equilibria, we find several important new results. We find that in duopoly model, firms enter the market in the order of their cost efficiency. In contrast, in the triopoly model, the entry order depends on the relative efficiency among firms and the profit structure in the market. This is a new result, which has important implications for future empirical studies of market entry.

Secondly, unlike in the symmetric game, the time of first entry in a duopoly game is not necessarily earlier than in the game with three asymmetric firms. The occurrence of delay in the initial entry is related to the properties of both the market and the firms in the game. To be more specific, first entry is more likely to be delayed in the market where firms are less asymmetric to one another and in the market with smaller total rents available to the duopolists relative to monopolist in the market.

The theoretical literature of market preemption games has mostly focused on identical firms and their choices as to when to optimally enter a market so as to maximize profits typified by Fudenberg and Tirole (1985).[1] They study the preemption game with observable technology adoption between two firms and show that in such a game, rent equalization must hold in equilibrium. Taking a different approach, Reinganum (1981) focuses on the analysis of equilibrium of the games of technology adoption with unobservable actions in different market structures and derives the equilibrium conditions. Extending on these two perspectives regarding observability, some papers investigate the preemption game with variation in the payoff structures. Park and Smith (2005) analyse a timing game with a general payoff structure for games with both observable and unobservable actions. Their paper explores how equilibrium in these games, in which there is shifting between phases of slow and explosive (positive probability) stopping and captures many economic and social timing phenomena. In addition, some models focus on the role of uncertainty (real-options framework). Smit and Ankum (1993) study the timing of investment with consideration of option to defer investment in production facilities analogous to a call option on dividend-paying stock. Bouis et al. (2009) study investments oligopoly market with more than two (anticipated) identical competitors. With a real-options approach, their model allows aggregate uncertainty in the payoff process, restricted to a specific payoff growth. More recently, Argenziano and Schmidt-Dengler (2014) extend on the previous study by exploring the general N-player investment timing game with certain payoff structure. Relying on the equilibrium property of rent equalization, they summarize the conditions required for investment clustering.

[1] See, for example, Dutta et al. (1995), Hoppe and Lehmann-Grube (2005) and Smirnov and Wait (2015).

Basic features of my model follow from the classic literature on two-player preemption games. As in the model of technology adoption games developed by Fudenberg and Tirole (1985), it is assumed that firms take irreversible and observable actions. Additionally, at the early stage of the game, a second-mover advantage exists due to the higher cost of market entry. But as costs falls, the benefits of entering first increase, and at some point in time both firms prefer to be the market leader rather than a follower. In their model, Fudenberg and Tirole (1985) show that the incentive for both firms to preempt their rival can lead to rent equalization. The motives to preempt are driven by the higher profits flow due to the fixed reduction in production cost brought by technology implementation. But this is not the only possible outcome. Alternatively, it is also possible that players have an incentive to wait for the reduction in implementation cost, which is assumed to decrease in time at a reducing rate. In this case, both firms might have an incentive to wait, so that entry is delayed. In equilibrium, Fudenberg and Tirole (1985) also show that following entry by the leader, there could be a period delay until the next firm adopts the new technology (costs need to be sufficiently lower to account for the lower flow of rents that accrues to the second entrant). With extension on the basic model, Argenziano and Schmidt-Dengler (2014) apply the same technique of backward induction for equilibrium derivation in the investment preemption game with more than two players. In this case, profit is a flow and is decreasing in the number of firms that have entered. This provides an incentive for firms to try to preempt each other in the hope for the higher post-investment profits. In addition, the cost of investment decreases with the time of entry into the market, providing some incentive for investment delay. In contrast with the basic duopoly game, Argenziano and Schmidt-Dengler (2014) show that in equilibrium, the incentive to preempt can lead to clustering in investment times and derive the condition for clustering in the more complicated market structure. My model also relies on this extension of Fudenberg and Tirole (1985), including assumptions and payoff structure. Furthermore, I use backward induction as the technique for equilibrium derivation, which is also applied in the previous literature.

Following the mechanism of determining the equilibrium in the N-player investment game, Argenziano and Schmidt-Dengler (2013) later examine the effect of the number of firms on the first market-entry time with careful consideration of possibility of clustering. As suggested by the symmetric duopoly game of Fudenberg and Tirole (1985), the incentive to preempt induces early entry into the market when there are two potential entrants, as compared with the situation when there is only one firm considering its optimal time of entry (the monopoly case). Extending this line of analysis, Argenziano and Schmidt-Dengler (2013) consider how the first entry time is related to the number of competitors involved in the game. Critically, however, they show that despite the fact that there is no monotonic relationship between the number of firms and the time of first entry, the first entry is the earliest in a duopoly market. Based on the result, Argenziano and Schmidt-Dengler (2013) further state that given linear demand, the last $(N-1)$ entries occur earlier than socially optimal and the first entrant occurs later than socially optimal. This result shares the same spirit with the research of Mankiw and Whinston (1986) on the excessive entry in the scale economies when considering social optimality and welfare. In addition, expanding on theme that scale economies distort the timing, as well as the number of the entrants, Mills

(1991) investigates a fully dynamic model to explore sequential entry timing in a growing industry and shows that, despite the earlier entry of the later entrants, the first entry occur not necessarily premature by analyzing models and providing numerical examples to support this proposition. His line of research suggests that regulatory policy should place obstacles in the path of potential entrants to enhance social welfare. Argenziano and Schmidt-Dengler (2013)'s research has similar welfare implication. Their result that under linear demand initial entry in duopoly market is later than socially optimal and it is earlier than in any other market also suggests potential welfare gains from regulatory restrictions.

As noted above, major focus of the existing timing game literature is on symmetric games. However, we know that firms are often different. Firms differ in all sorts of ways such as their technology (and their ability to use it), their set of complementing products, their customer network and reputation, just to name a few. Considering heterogeneity in firms, Riordan (1992) introduces asymmetry in the profits flow for firms into the duopoly's innovation adoption game. In his model, price and entry regulation alter firms' profit to a different extent, resulting in asymmetry in post-innovation profits among firms. In this context, Riordan (1992) shows that firms enter the market in the order of their efficiency (costs). Alternatively, Pawlina and Kort (2006) study the asymmetric case in which the entry costs of firms are exogenously different from each other.[2] By identifying three classes of equilibria, where the lower cost firm always invests weakly earlier than their rival, they provide evidence for the entry in the order of efficiency. These theoretical results build a micro-foundation for the assumption of consistency between order of market entry and efficiency ranking, which is widely adopted in empirical analysis of market-entry game. For instance, in modelling the entry in the airline industry, Berry relies on the assumption that entry is in order of efficiency to address the problem of inherent multiplicity of equilibria. His paper studies the issue of airline's airport presence by treating the entry decisions of airlines as an indication of underlying profitability. Although he points out that further proof needs to be provided for the assumed entry order, the estimates obtained are in fact consistent with the large literature that indicates an important role for the proximity of alternative airport presence in determining airline profitability. Meanwhile, extending the duopoly models cited above, Argenziano and Schmidt-Dengler (2012) argue that with more than two players, the order of entering may not be consistent with the efficiency order. They construct a market-entry preemption game model with two identical inefficient players and one efficient player, where the post-entry profit is much higher for the efficient player. In this environment, they provide the necessary parameter range that results in inefficient entry order. The intuition for this result is that competition between the two inefficient firms dampens the preemption incentive for the efficient firm to such an extent that it may not even play the role as the first entrant in equilibrium. This evidence provides a precautionary note regarding the need to carefully consider the effect of some primitive parameter values when deriving the equilibrium entry order and entry time, and when making statements with regards to optimality and social welfare. With additional firms, the decrease in the entry cost and intensity of competition in the follower's subgames shortens the period of earning higher profits, which will dampen the incentive of preemption and thus influence the strategic entry choices of early entrant. In the symmetric model, this trade-off between

[2] See also Smirnov and Wait (2018).

benefits of preempting and waiting has been proved to result in a delay in initial entry in market with more than two firms thereby resulting in further deterioration in social welfare. However, this theme is under-researched in the asymmetric preemption game. In the presence of firm asymmetry, this issues is further complicated by the possibility that the market pioneer need not be the most efficient (lowest cost) potential entrant. Focusing on asymmetric market preemption game in duopoly and triopoly market, our analysis shows that delay is not necessarily a feature in equilibrium when there are more than two players in the market. This provides implication that social welfare might enhance in the presence of more competition in the market.

2 Model

In this section, we provide a sketch of the framework analyzing preemption games adopted by Argenziano and Schmidt-Dengler (2014) in which they explore the conditions necessary for clustering of entry. Section 2.1 gives a general description of the market preemption game. Section 2.2 outlines Argenziano and Schmidt-Dengler (2014)'s equilibrium derivation mechanism in symmetric duopoly and triopoly market preemption game, respectively.

2.1 Market entry preemption game

Firms are engaged in an infinite-horizon dynamic market-entry competition in continuous time, in which they have to optimally decide whether and when to enter a market. More precisely, at each instant in time each firm that has not yet entered the market observes the number and the identity of the firms already present and chooses one of the two actions 'enter' or 'wait'. Following the framework of Fudenberg and Tirole (1985), entry is irreversible and profit before entry is normalized to zero to ensure the unique outcome in each subgame.

Denote the set of firms to be $N = \{1, 2, 3\}$ and a single firm $i \in N$. The instantaneous post-entry profits $\pi(m)$ is received by each of m firms present in the market. Define $\pi = \{\pi(1), \pi(2), \pi(3)\}$. The entry incurs a cost for an individual firm i, which has a present value of $c_i(t)$. The outcome of the game is a vector of entry times T^j, for $j = 1, 2, 3$. If firm i is the j-th entrant, it will receive a payoff of

$$V_i^j(T^1, T^2, T^3) = \sum_{m=j}^{N} \pi(m) \int_{T^m}^{T^{m+1}} e^{-rs} ds - c_i(T^j), \tag{1}$$

where r denotes the common discount rate.

We make the following standard assumptions:

Assumption 1. $\pi(m)$ is strictly positive for any m and is strictly decreasing in m.

This assumption means that entry into the new market always yields a positive payoff for a firm. This ensures that all firms will enter the market at some time, as entry is more profitable than the option of not entering at all. Meanwhile, competition becomes more intense as more firms are present in the market, resulting in less profits earned by individual firms. Hence, it is reasonable to assume that $\pi(m)$ is strictly decreasing in m.

We make the following definition.

Definition 1. $s = 2\pi(2) - \pi(1)$ is a duopoly effect in the market.

A higher value of s indicates higher total profitability under duopoly competition. This notion was introduced by Mills (1991), who studied the impact of growth of certain market on the distortion in entry time. Note that following Assumption 1, $s < \pi(2)$. This will be useful later in characterizing equilibrium in the entry game.

Assumption 2. The present value cost function of individual firm i is assumed to be in the form $c_i(t) = ke^{-(\alpha+r)t}$, where k is the instantaneous current value of cost of entry at time zero. α denotes the rate at which the current value of entry cost declines and r denotes the time discount rate of entry cost.

This assumption indicates that the current value of the cost of entry at time zero is decreasing at a rate of $(\alpha + r)$ which is higher than the discount rate for the instantaneous profits. As explained by Argenziano and Schmidt-Dengler (2014), this captures the effect of upstream process innovation or economy of learning and scale that develop over time.

Assumption 3. Immediate entry at time zero is not profitable as costs are higher than the discounted monopoly profits: $c_i(0) > \frac{\pi(1)}{r}$. Furthermore, the value of entry is strictly positive in finite time τ such that entry is eventually profitable for all firms.

These assumptions are essential to guarantee that (i) there is no immediate entry at time zero in which case the first entrant can enjoy the monopoly profits forever; and (ii) all firms enter in the market eventually.

Assumption 4. Time is continuous with an infinitely fine grid.

This assumption follows Simon and Stinchcombe (1989), who develop a framework to address the problem of associating the strategy profiles to outcomes in continuous-time games. As a result a continuous-time game could be interpreted as a game played on discrete-time grid. This allows us to apply such a concept as backward induction to solve the game.

Assumption 5. In the situation in which n firms enter at the some time, only one firm is allowed to enter with probability $\frac{1}{n}$ successfully.

This assumption introduces a randomization device as in Katz and Shapiro (1987) to rule out the possibility of coordination failure to ensure the existence of equilibrium in the continuous-time preemption game. Following the explanation of Argenziano and Schmidt-Dengler (2014), we can consider the scenario where two firms find it optimal to invest at some time t. Without the randomization device, it is possible for both of them to enter the market at t successfully. In this case, each firm earns a profit less than what they could earn otherwise and this results in a coordination failure. However, with this randomization device, only one firm can successfully enter the market at any point in time with the other firms entering whenever they find optimal.

2.2 Equilibrium Derivation

2.2.1 Stand-alone time

Following Katz and Shapiro (1987), consider the hypothetical scenario where firm i is able to act as a sole decision-maker (monopolist) and enter the market at some optimal time t to earn a guaranteed permanent payoff $\pi(m)$ minus present value cost $c_i(t)$. In this case, firm would rationally choose an entry time $t = T_i^*(m)$, which maximizes its lifetime payoff,

$$V_i(t) = \pi(m) \int_t^{\infty} e^{-rs} ds - c_i(t). \tag{2}$$

$T_i^*(m)$ is the solution to

$$V_i'(t) = 0. \tag{3}$$

Therefore, we have

$$T_i^*(m) = \frac{\ln \frac{k(\alpha+r)}{\pi(m)}}{\alpha}. \tag{4}$$

$T_i^*(m)$ is defined as a market stand-alone entry time with m firms. Observe that $T_i^*(m)$ is decreasing in the value of $\pi(m)$. Hence in a hypothetical monopoly case, the higher the post-entry profit is the earlier a firm will enter the market. The following lemma was introduced by Argenziano and Schmidt-Dengler (2014).

Lemma 1. *In any pure-strategy SPNE, if firm i is the last to enter, the entry time will be no later than $T_i^*(N)$.*

As the last entrant, firm i acts as a sole agent deciding when to enter and receive an infinite profit flow $\pi(N)$. It will thus optimally enter at its stand-alone time $T_i^*(N)$ to maximize its profits.

2.2.2 Equilibrium analysis in benchmark case, symmetric duopoly

This section formally provides an illustration of equilibrium formation in the symmetric duopoly-model, following Argenziano and Schmidt-Dengler (2014).

From Lemma 1, both firms invest no later than $T^*(2)$. Utilizing the logic of backward induction, each firm will anticipate that if it invests first at some time $t < T^*(2)$, its rival will follow and enter at $T^*(2)$. Therefore, the payoff for the leader is:

$$L(t) = \pi(1) \int_t^{T^*(2)} e^{-rs} ds + \pi(2) \int_{T^*(2)}^{\infty} e^{-rs} ds - ke^{-(\alpha+r)t}. \tag{5}$$

The payoff received by the follower is:

$$F(t) = \pi(2) \int_{T^*(2)}^{\infty} e^{-rs} ds - ke^{-(\alpha+r)T^*(2)}. \tag{6}$$

To aid the intuition for the equilibrium outcome, construct function $D(t)$ to represent individual's preemption incentive as follows:

$$D(t) = L(t) - F(t). \tag{7}$$

This equation captures the difference in payoffs earned by the leader and the follower. If the incentive function $D(t)$ is positive, there is no equilibrium as the follower could always profitably deviate and preempt to become the leader. Therefore, equilibrium initial entry t_1 can only occur when $D(t)$ is non-positive. As a consequence, in equilibrium, the leader will enter at some time when $L(t)$ and $F(t)$ cross (t_1) for the first time and the follower enters at the stand-alone time $T^*(2)$.

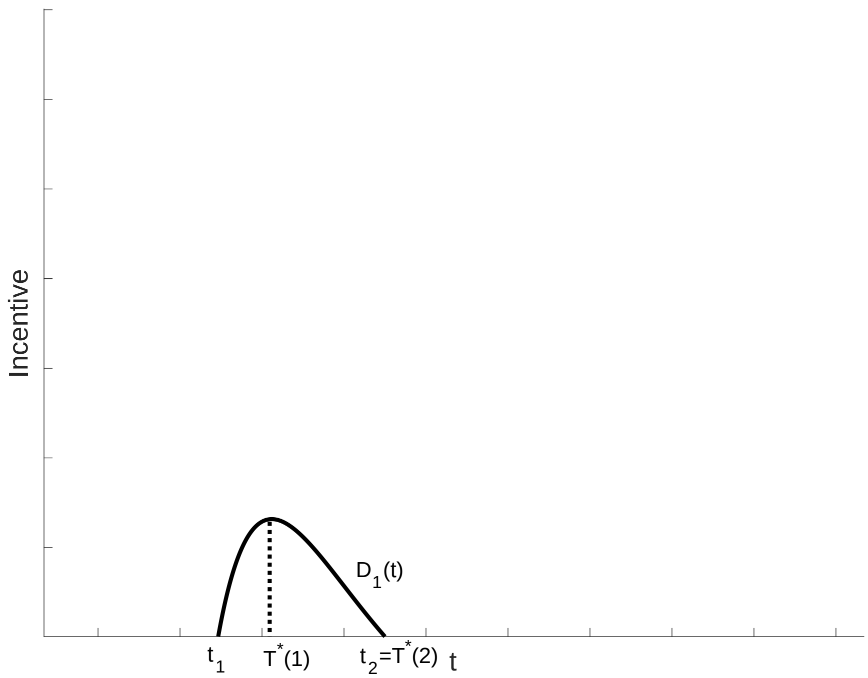


Figure 1: symmetric duopoly market

Argenziano and Schmidt-Dengler (2014) introduce a concept of Leader Preemption Constraint (LPC) to intuitively illustrate the mechanism of the equilibrium formation. In the absence of a rival, a single firm would like to invest at the stand-alone time $T^*(1)$. However, in the presence of an opponent, who would follow at $T^*(2)$, the leader cannot enter at $T^*(1)$ in equilibrium if $D(t)$ is positive. Otherwise the follower would always profitably deviate and preempt the leader by entering earlier at some time $T^*(1) - \epsilon$ as illustrated in Figure 1. Therefore, the second player introduces a Leader Preemption Constraint (LPC) on leader's entry decision, bringing forward the first entry time until a time when earlier preemption is not profitable. In other word, the first entry must occur weakly earlier than either $T^*(1)$ or the first intersection of the Leader and Follower payoff functions.

2.2.3 Equilibrium analysis in benchmark case, symmetric triopoly

In this market structure, there are two preemption races to be considered, namely: the race to be the first entrant and the race to be the second entrant. In this case, t_1 is not only determined by the Leader Preemption Constraint but also identified by the Follower Preemption Constraint (FPC), which is derived in the subgame following the first entry. As in the duopoly case, the second entrant's incentive for preempting the leader introduces the LPC with respect to the initial entry time. Similarly, the follower preemption race determines the second entry time and implicitly defines the upper bound on the time of the first entry. Applying backward induction, we firstly consider the game between the second and third entrants. This subgame is analogous to the duopoly game. As in the previous two-firm market, we derive the leader and follower's payoff respectively, as

$$L_2(t) = \pi(2) \int_t^{T^*(3)} e^{-rs} ds + \pi(3) \int_{T^*(3)}^{\infty} e^{-rs} ds - k e^{-(\alpha+r)t} \tag{8}$$

and,

$$F_2(t) = \pi(3) \int_{T^*(3)}^{\infty} e^{-rs} ds - k e^{-(\alpha+r)T^*(3)}. \tag{9}$$

The preemption incentive function which is also an indicator for the strength of FPC, is

$$D_2(t) = L_2(t) - F_2(t). \tag{10}$$

In equilibrium, FPC forces the second entry to take place at t_2, where $D_2(t)$ is zero. This result implicitly assumes that under FPC the first entry must occur weakly before t_2.

We now illustrate the LPC for the three-player game by analyzing the first-entry preemption subgame. The payoffs for the leader and follower are

$$L_1(t) = \pi(1) \int_t^{t_2} e^{-rs} ds + \pi(2) \int_{t_2}^{T^*(3)} e^{-rs} ds + \pi(3) \int_{T^*(3)}^{\infty} e^{-rs} ds - k e^{-(\alpha+r)t} \tag{11}$$

and,

$$F_1(t) = \pi(2) \int_{t_2}^{T^*(3)} e^{-rs} ds + \pi(3) \int_{T^*(3)}^{\infty} e^{-rs} ds - k e^{-(\alpha+r)(t_2)}. \tag{12}$$

Following the intuition outlined above, the incentive to preempt the leader, resulting in an LPC can be described by the function:

$$D_1(t) = L_1(t) - F_1(t). \tag{13}$$

In this subgame, the incentive captured by the LPC brings the first entry time earlier to some time when $D_1(t)$ turns positive for the first time.

As could be seen from the above analysis, both the LPC and FPC influence the time of the first market entry by any of the firms. Therefore we need to analyze the relative intensity of the follower preemption race and the leader preemption race to derive equilibrium entry time in the game. Note that the intensity of the constraints introduced by these two races can be quantified by the preemption incentive of the follower in that subgame. Therefore, the analysis of t_1 can be simplified by considering the leader preemption incentive after anticipating the second entry, which is determined by the last's entrant preemption incentive. To be more precise, if the sign of $D_1(t)$ is positive when $t < t_2$, we could conclude that under the threat of FPC, the first entry is still desirable so that firms have positive incentive for preempting the leader. In this case, the LPC brings the first entry strictly earlier than t_2 to some time when $D_1(t) = 0$. In contrast, a negative value of $D_1(t)$ when $t < t_2$ indicates that the FPC is so strong that no firm has incentive in participating in the leader preemption race. In this case, only the FPC determines the first entry, resulting in a cluster in the first and second entry time at t_2. The following three graphs provide an illustration of the impact of the relative strength of LPC and FPC as post-entry monopoly profit decreases.

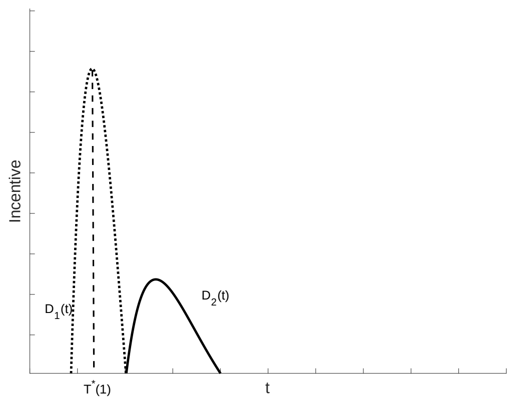
The outcomes in graphs (a) and (b) in Figure 2 indicate that when monopoly profits are sufficiently large, the LPC effectively determines the first entry time alone (not the FPC). In addition, the difference in $D_1(t)$ and $D_2(t)$ between these two graphs reveals that a decline in monopoly profits weakens the intensity of the leader preemption race. In graph (c), monopoly profits are sufficiently small such that LPC has no effect in determining the time of first entry. In this case, FPC results in a cluster with two firms entering at the same time.

The above examples show that FPC and LPC are not independent and are influenced by the primitive parameter values. Argenziano and Schmidt-Dengler (2014) approach the relationship between strengthes of the two constraints via the analysis of the sign of the following expression:

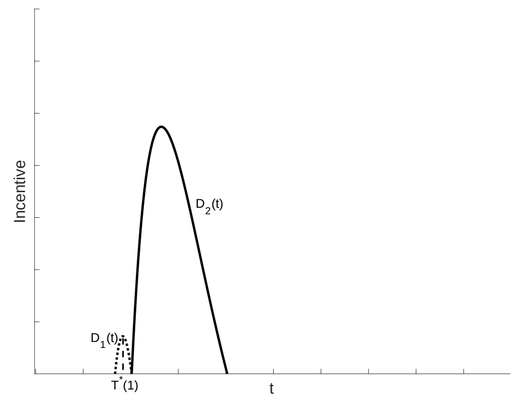
$$D_2(T^*(1)) = \frac{\pi(2)}{r}(e^{-rT^*(1)} - e^{-rT^*(3)}) - (c(T^*(1)) - c(T^*(3))). \tag{14}$$

$D_2(T^*(1))$ represents the firm's incentive for being the second entrant at the monopoly stand-alone time. A negative $D_2(T^*(1))$ would be observed in the above graphs (a) and (b) with no clustering in entry when $T^*(1)$ is earlier than t_2. This intuitively demonstrates that entering as a monopoly is still tempting despite the rent equalization in equilibrium. While a positive value in $D_2(T^*(1))$ signals clustering in entry since the monopoly stand-alone time is later than the second entry. In other words, no firm would have positive incentive for entering earlier than t_2 if they anticipate that there is no opportunity for them to obtain a maximum profit.

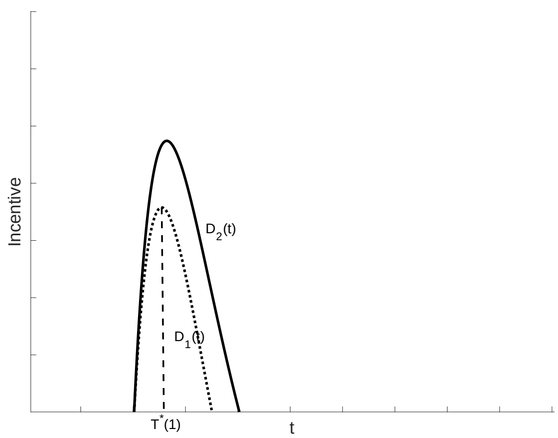
Observe that $D_2(T^*(1))$ is increasing in $\pi(2)$ and decreasing in $\pi(1)$ and $\pi(3)$. Therefore, with any certain pair of $\pi(2)$ and $\pi(3)$, starting from the cutoff case where $D_2(T^*(1)) = 0$, a decrease in $\pi(1)$ would make $D_2(T^*(1))$ positive, resulting in clustering of entry. In contrast, if we increase $\pi(1)$, $D_2(T^*(1))$ becomes negative, corresponding to the scenario where first entry is strictly earlier than t_2. The intuition is that as $\pi(1)$ becomes more attractive, firms' incentive for preempting the leader increases, thereby intensifying the leader preemption race. As a result, the relative strength of the LPC over FPC enhances. The first entry occurs strictly earlier than t_2. Alternatively, we could interpret the result in terms of the concept of duopoly effect. A decrease in $\pi(1)$ is equivalent to an increase in the duopoly effect s given a fixed $\pi(2)$. Therefore, the above result intuitively tells us



(a) $\pi(1) = 390$, no clustering



(b) $\pi(1) = 300$, no clustering



(c) $\pi(1) = 210$, clustering

Figure 2: Triopoly symmetric model with $\pi(2) = 200$; $\pi(3) = 100$

that first entrant would be more willing to wait for the benefits brought by the duopolistic competition in the market and thus enter at the same time with the second entrant. While in the opposite case, the initial entry would be strictly earlier than the second entry in that the potential first entrant is more willing to enjoy the higher monopoly profits for some period instead of waiting for duopoly benefits.

3 Duopoly market preemption game with asymmetric players

As noted previously, in most markets, firms are not identical. Firms that have a better reputation or better facilities might be better at exploiting opportunities or might be able to produce the same product with much lower costs. To capture possible firm asymmetries, we adjust the cost function by adding an efficiency parameter to reflect the differences between firms, as described in the following assumption.

Assumption 6. The present value cost function of individual firm i is assumed to be in the form $c_i(t) = ke^{-(\alpha+r)(t+\Delta_i)}$, with Δ_i denoting each firm's efficiency level.

With such a cost function, firms with a higher value of Δ are more efficient due to the fact that their costs decrease from a smaller value at a higher rate. In the duopoly model, A corresponds to a more efficient firm with an efficiency parameter Δ_A, while B is a less efficient firm with Δ_B, where $\Delta_A > \Delta_B$.

3.1 Equilibrium derivation

As in the benchmark case, the presence of the second player introduces a Leader Preemption Constraint (LPC) on whoever takes the role of the leader and the strength of this constraint can be quantified by the last entrant's preemption incentive. The key difference from the benchmark case is that the strength of the constraint in this case not only depends on the profits structure but also on the relative efficiency between firms involved in the preemption race. This indicates an asymmetry in the impact of constraints each firm imposes on its rival, which in turn determines the equilibrium entry order and entry time.

In the following lemma, we show that the entry order in the asymmetric duopoly game is always efficient.

Lemma 2. *In the asymmetric duopoly game, entry order will always be efficient. In equilibrium, the more efficient firm A firstly enters at* $t_1 = min\{T_A^*(1), T_B^{AB}\}$ *and the less efficient firm B follows and enters at* $t_2 = T_B^*(2)$.

Proof. See Appendix A.

Following Argenziano and Schmidt-Dengler (2012), we apply the backward induction technique to derive the equilibrium. The mechanism is analogous to the symmetric game. Each firm will estimate both its payoff of being the leader by anticipating that the follower will enter at duopoly stand-alone time and its payoff of being a follower. As long as the leader's payoff outweighs the follower's payoff, firm has inventive to preempt its rival to be the first entrant. Define T_i^{AB} to be the earliest time that entering as a leader is more profitable than waiting. A comparison of T_A^{AB} and T_B^{AB} indicates the order of entry. More precisely, if i takes the role of the first entrant and j follows later, i's payoff will be,

$$L_i^{AB} = \pi(1) \int_t^{T_j^*(2)} e^{-rs} ds + \pi(2) \int_{T_j^*(2)}^{\infty} e^{-rs} ds - ke^{-(\alpha+r)(t+\Delta_i)}. \tag{15}$$

Instead, if firm i takes the role of a follower, its payoff is,

$$F_i^{AB} = \pi(2) \int_{T_i^*(2)}^{\infty} e^{-rs} ds - ke^{-(\alpha+r)(T_i^*(2)+\Delta_i)}. \tag{16}$$

Note that under the above assumptions, L_i^{AB} is strictly quasi-concave and maximized at $T_i^*(1)$. Comparing the leader and the follower payoffs for each player, we evaluate each firm's incentive for preemption. Given that one firm will always attempt to enter earlier as long as they are better off being a leader, we can construct the preemption incentive function for each firm i as

$$D_i^{AB}(t) = L_i^{AB}(t) - F_i^{AB}(t). \tag{17}$$

A positive $D_i^{AB}(t)$ corresponds to the situation in which player i has an incentive to enter earlier. Constructing the graphs of functions (17) and comparing the earliest times of their intersections with the horizontal axis (T_i^{AB}) reveals the rank of each firm's incentive for preempting. The firm with earlier T_i^{AB} enters first.

Figure 3 below presents observations for efficient entry order in two cases where A is significantly more efficient than B and where A is weakly more efficient than B. Graph (a) in Figure 3 depicts a scenario where firm A will enter at its stand-alone time $T_A^*(1)$ as a monopoly. Following the terminology in Riordan (1992), such firm A is defined as a strong leader whose entry decision is not threatened by rival's preemption incentive. There is a significant difference in the efficiency parameters of the two firms in this case. In contrast, graph (b) in Figure 3 depicts a scenario of a weak leader where the difference in Δ_A and Δ_B is smaller and the leader A's entry decision is affected by the intensity of the preemption race. As demonstrated in these two graphs, in both cases, $D_A^{AB}(t)$ is higher than $D_B^{AB}(t)$, indicating that A always has a stronger preemption incentive than B. In other words, at some time t when $D_B^{AB}(t)$ is weakly above zero, $D_A^{AB}(t)$ will always be strictly positive. Therefore, in equilibrium A will always take the role of the leader and B enters as a follower. The time of the first market entry in asymmetric duopoly game is also illustrated in the above two examples. In the strong-leader's situation, $T_A^*(1) < T_B^{AB}$, ensuring A will enter at its profit maximizing time $T_A^*(1)$. In other words, firm B induces such a weak LPC that firm A's entry decision is not influenced by the presence of B. On the other hand, in the weak-leader example, where $T_A^*(1) > T_B^{AB}$, in equilibrium, A can only enter between T_A^{AB} and T_B^{AB} when its rival does not have preemption incentive. Given that in this range the potential profits for A are the highest at T_B^{AB}, A will enter weakly earlier than T_B^{AB}.

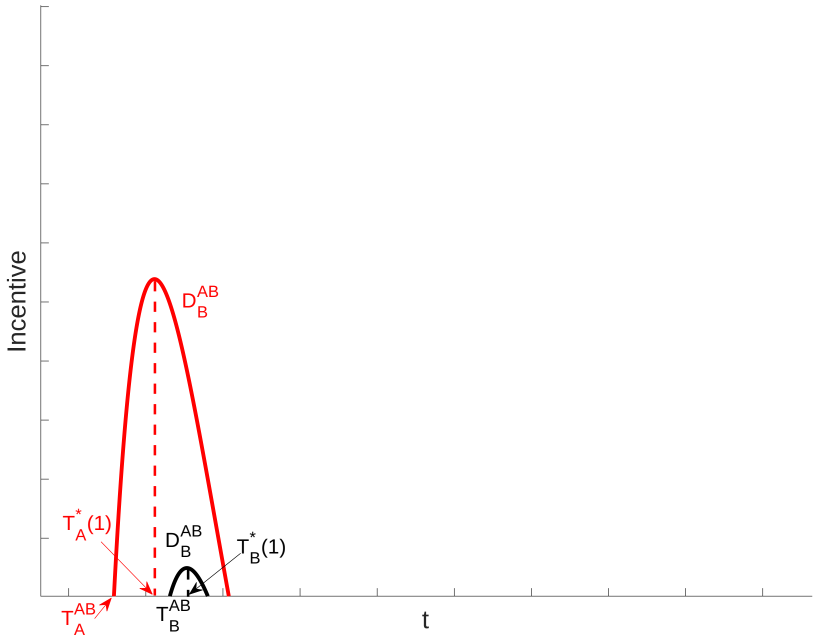
These examples indicate that in a duopoly market, entry order will always be efficient and that the first entry time will be the minimum between the monopoly stand-alone time of the more efficient firm and the zero preemption incentive time of the less efficient firm.

To prove this results formally, we introduce the preemption incentive function for both firm A and firm B:

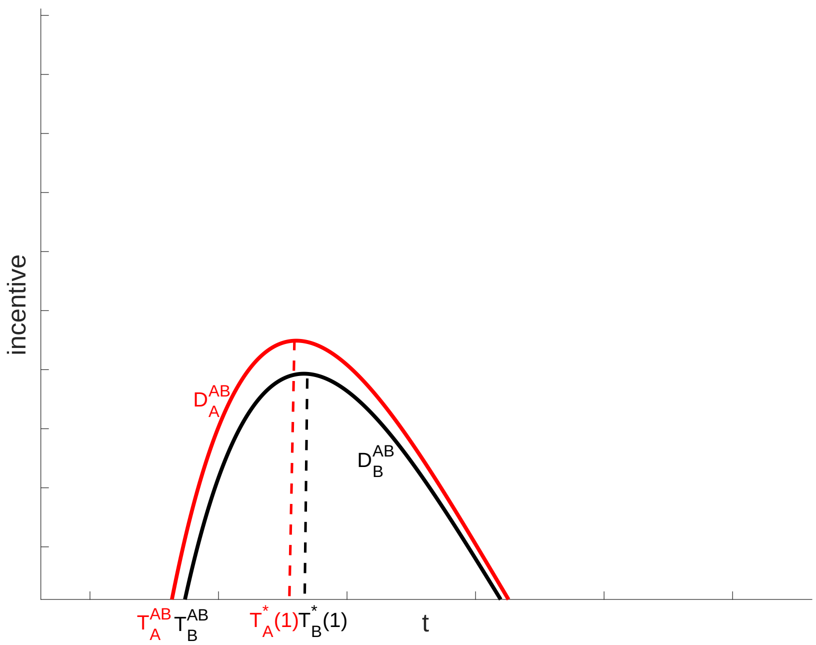
$$D_A^{AB}(t) = \pi(1)\int_t^{T_B^*(2)} e^{-rs}ds + \pi(2)\int_{T_B^*(2)}^{T_A^*(2)} e^{-rs}ds + (ke^{-(\alpha+r)(T_A^*(2)+\Delta_A)} - ke^{-(\alpha+r)(t+\Delta_A)}) \tag{18}$$

and

$$D_B^{AB}(t) = \pi(1)\int_t^{T_A^*(2)} e^{-rs}ds + \pi(2)\int_{T_A^*(2)}^{T_B^*(2)} e^{-rs}ds + (ke^{-(\alpha+r)(T_B^*(2)+\Delta_B)} - ke^{-(\alpha+r)(t+\Delta_B)}). \tag{19}$$



(a) A is a strong leader, $\Delta_A = 0.1$



(b) A is a weak leader, $\Delta_A = 0.01$

Figure 3: Efficient entry order in asymmetric duopoly model with $\pi(1) = 300$, $\pi(2) = 200$ and $\Delta_B = 0$

In the Appendix A, we show that because of $T_A^*(2) < T_B^*(2)$, $D_A^{AB}(t) > D_B^{AB}(t)$ always holds. Therefore, the efficient firm A will always preempt firm B in equilibrium.

To capture the intuition for the relationship between these two firms' incentive functions we compare these two functions term by term. The first term in each incentive function $\pi(1) \int_t^{T_i^*(2)} e^{-rs} ds$ measures the monopoly profit for firm i before the second entry. This term is bigger in $D_A^{AB}(t)$ because if firm A enters first in the market, it earns monopoly profits longer than B would (if it enters as the first entrant) due to the fact that $T_B^*(2)$ is later than $T_A^*(2)$. With regards to the third term $(ke^{-(\alpha+r)(T_i^*(2)+\Delta_i)} - ke^{-(\alpha+r)(t+\Delta_i)})$, it has a negative value and captures the extra entry cost for firm i when its entry time is brought forward from a duopoly stand-alone entry time $T_i^*(2)$ to t. Given that A has an earlier $T_A^*(2)$ and lower cost than B if they both enter at t, it is less costly for A to enter earlier. Therefore, the third term is larger in $D_A^{AB}(t)$. To complete the argument, it is essential to show that monopoly profits for A earned from $T_A^*(2)$ to $T_B^*(2)$ and the duopoly profits not earned by A over the same period do not offset the previous two effects. The reason is that by preempting B, firm A delays the time at which it earns duopoly profits from $T_A^*(2)$ to $T_B^*(2)$ and replace these profits with the higher monopoly profits during this interval. Hence for firm A the total effect is positive. In contrast, for firm B, preempting A would bring forward the time at which it earns duopoly profits. By definition of $T_B^*(2)$, bringing forward duopolists profits reduces B's payoff; extra duopoly profits are more than offset by the extra cost in entry. Given that $D_A^{AB}(t)$ is bigger than $D_B^{AB}(t)$, we can conclude that in equilibrium B cannot enter first.

3.2 Dynamics of initial entry in the market

As noted above, the equilibrium entry is related to both firms' preemption incentives and thus is influenced by factors, including characteristics of both firms and the market they are competing in. In this section, we explore the comparative statics of how the initial market entry time in equilibrium alters with changes in parameters. In particular, we attempt to investigate how the potential entrants' efficiency levels and the duopoly effect affect the initial entry. To approach the dynamic patterns of interest, we conduct analysis in two markets with different duopoly effects, $s = 50$ and $s = 100$. In each of the markets, we fix $\Delta_A = 0$ and let Δ_B alternate between $(-0.15, 0.15)$. Observe that this means, when $\Delta_B \in (-0.15, 0)$, firm A is the more efficient firm in the market. In the case where $\Delta_B \in (0, 0.15)$, firm B is the more efficient firm in the market. From Lemma 2, the initial entry time t_1 is $min\{T_A^*(1), T_B^{AB}\}$ when A is the more efficient firm and t_1 is $min\{T_B^*(1), T_A^{AB}\}$ if B is the more efficient firm in the market. The figure below demonstrates the pattern of the dynamics of t_1 in the two cases.

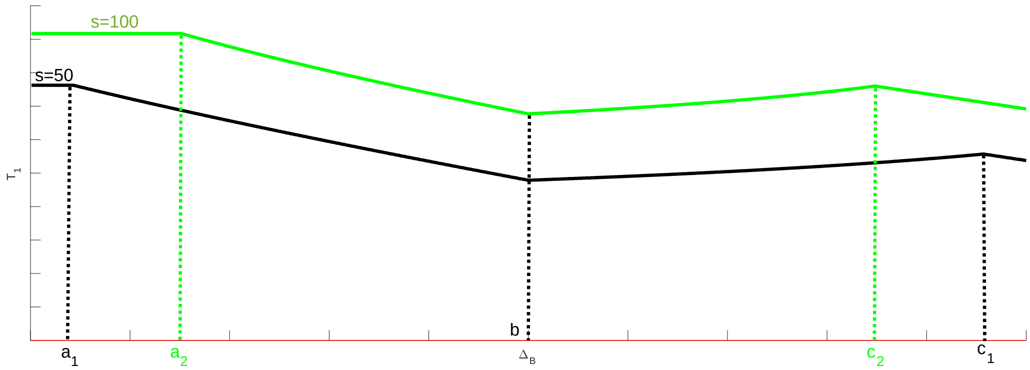


Figure 4: Dynamics of first entry in duopoly

As shown in Figure 4, there is a general dynamic pattern of t_1 with respect to changes in Δ_B. To be more specific, t_1 is constant when Δ_B is small. This is followed by a range in which t_1 decreases when Δ_B is between a and b. Next there is an upward trend range between b and c. Finally, it linearly decreases as Δ_B increases.

The intuition can be captured by looking at how significant the threat of preemption is from the less efficient firm. When firm B is very inefficient (Δ_B smaller than a), firm A is unperturbed by its rival and enters at its monopoly stand-alone time $T^*_A(1)$. By the same logic, if firm B is significantly more efficient than its rival A (Δ_B bigger than c), B takes the role of the strong leader and enters at $T^*_B(1)$ to obtain maximum profits as a monopoly. Observe that $T^*_B(1)$ is a decreasing function in Δ_B, firm B will enter earlier as its efficiency level improves over time. In terms of changes observed when Δ_B is in between a and c, rivalry between the two is more intense and the threat of preemption cannot be ignored. Therefore, t_1 is determined by the preemption incentive of the follower. This is analogous to the weak-leader's situation described above. Entry is brought forward by the constraint of preemption to T^{AB}_i. In particular, in the range from a to b, firm B is the less efficient firm and thus T^{AB}_B determines the initial entry time. As the less efficient firm B becomes more efficient, its incentive for preempting firm A increases, thereby resulting in earlier T^{AB}_B. In the parameter range between b and c, firm B acts as the weak leader in the market-preemption game. In this case, $t_1 = T^{AB}_A$, which is an increasing function in Δ_B. The intuition for this result is that the less efficient firm A would anticipate an earlier second entry by rival B as B becomes more efficient. This contributes to a shorter period for A to generate monopoly profits if it preempts. Therefore, firm A's incentive for preempting decreases with Δ_B and thus results in a delay in $t_1 = T^{AB}_A$. In addition, this pattern also shows that the initial entry time is always earlier in the duopoly than in the monopoly ($t_1 = T^*_A(1)$ when $B \in (-0.15, a)$), which is consistent with the result from Argenziano and Schmidt-Dengler (2012).

Moreover, Figure 4 indicates the impact of the duopoly effect on both firms' entry decisions. Two main changes are observed as the duopoly effect increases. Firstly, the dynamic pattern overall shifts up. Secondly, the interval from a to c narrows as s increases. The intuition for the first change observed is that both firms would be more willing to delay entry in the market with higher duopoly effect. While the move from a_1 to a_2 and b_1 to b_2

intuitively suggests that the preemption incentive of follower is smaller in the market with higher duopoly effect. In such a market, the leader preemption race is less intense and thus the period when market leader's entry decision is unperturbed by the follower is longer.

Following the analysis of Argenziano and Schmidt-Dengler (2013), monopoly stand-alone entry time is usually later than the socially optimal time as it does not take consumer surplus into account and there is no 'business stealing effect'. Our result implies that in market with higher duopoly effect where it is easier for the more efficient firm to enter at its stand-alone time, initial entry is more socially sub-optimal.

4 Triopoly market preemption game with asymmetric players

In the triopoly market-entry model, we have three firms, namely A, B and C with $\Delta_A > \Delta_B > \Delta_C$. Following the logic of backward induction, the focus is first drawn on the subgame after the first entry, which determines the equilibrium time of the second and the third entry. Analysis then moves to the preemption game between the first and the second entrant. In the presence of asymmetry, firm that holds lower preemption incentive or imposes weaker preemption constraint to its rivals would enter as a follower in the subgame that it competes in.

4.1 Last entrant and entry time

We first generalize Lemma 1 for heterogeneous firms.

Lemma 3. *In any SPNE, all firms enter the market in finite time, the least efficient firm C enters last at $T^*_C(3)$.*

Lemma 3 specifies that the last entry time is the last entrant's triopoly stand-alone entry time. This is because without preemption threat from later entry, the last entrant will rationally enter at some time when its lifetime wealth is maximized. In addition, this lemma also specifies that firm C is the last entrant. This follows from the fact that there is an efficient entry order in the duopoly model. In the earlier two-firm case, we have proved that the firm with later stand-alone entry time will have less leader preemption incentive than its rival and thus enter later. The subgame after the initial entry in the triopoly model is analogous to the duopoly game where follower in this subgame also enters at its triopoly stand-alone time $T^*_i(3)$. Given the fact that $T^*_i(3)$ decreases with Δ_i, firm C will have the latest stand-alone entry time. As a result, C will have the lowest preemption incentive and thus it will enter the last.

In terms of the entry order of the remaining two firms as we show in the following, it depends on the primitive parameter values. More specifically, it depends on the relative strength of the leader preemption constraint that each firm is able to inflict on other rivals.

4.2 FPC and second entry time

Following the analysis in the benchmark case, in the subgame following the first entry, the last entrant firm C will introduce a FPC to A or B, whichever firm takes the role of the second entrant. The strength of these constraints can be represented by C's incentive to preempt the second entrant firm i, which is

$$D_C^{iC} = \pi(2)\int_t^{T_i^*(3)} e^{-rs}ds + \pi(3)\int_{T_i^*(3)}^{T_C^*(3)} e^{-rs}ds - ke^{-(\alpha+r)(t+\Delta_C)} + ke^{-(\alpha+r)(T_C^*(3)+\Delta_C)}. \quad (20)$$

Let T_C^{iC} be the earliest time when D_C^{iC} is zero. Observing that D_C^{iC} increases with $\pi(2)$ and decreases with Δ_i (because $T_i^*(3)$ decreases with Δ_i), firm C will introduce a tighter Follower Preemption Constraint in the markets with higher duopoly profits and less efficient rivals. In other words, earlier T_C^{iC} will be generated if $\pi(2)$ is higher or Δ_i is lower. This indicates that when competing in the same market, firm C introduces a tigher FPC to B than to A, resulting in $T_C^{BC} < T_C^{AC}$. Following the analysis of the duopoly game, the leader in this subgame has to enter earlier than the time when C starts to have positive incentive for preemption, otherwise C will always profitably deviate. Therefore, if firm B enters the market first, firm A will follow at $T_2^A = \min\{T_A^*(2), T_C^{AC}\}$. In the case where firm A acts as the market leader, B will follow and enter at $T_2^B = \min\{T_B^*(2), T_C^{BC}\}$. Note that there is no clear relationship between T_2^A and T_2^B due to the fact that $T_A^*(2) < T_B^*(2)$ and $T_C^{AC} > T_C^{BC}$.

4.3 LPC and the first entry time

Now we move to the subgame that decides whether firm A or firm B enters first. As in the asymmetric duopoly preemption game, we need to evaluate each firm's preemption incentive to derive the equilibrium entry order and entry time.

In this subgame, if j takes the role of the first entrant and i follows, i will have an incentive function for preempting the leader j of

$$D_i^{AB}(t) = \pi(1)\int_t^{T_2^j} e^{-rs}ds + \pi(2)\int_{T_2^j}^{T_2^i} e^{-rs}ds + ke^{-(\alpha+r)(T_2^i+\Delta_i)} - ke^{-(\alpha+r)(t+\Delta_i)}. \quad (21)$$

Let T_i^{AB} be the earliest time when $D_i^{AB}(t)$ is zero. If we could observe that $D_A^{AB}(t)$ is always bigger than $D_B^{AB}(t)$, firm A will always have more incentive to enter as the market leader, resulting in the efficient market entry in equilibrium. In contrast, if B starts to have positive preemption incentive earlier than A, the entry in equilibrium will be inefficient with the order $B - A - C$.

In the duopoly case above, we showed that the efficient entry order follows from the fact that if the most efficient firm A enters as a follower it would bring forward the second entrant's entry time. With a shorter period to earn monopoly profits, the less efficient firm B will always have less incentive for preempting. However, in the triopoly case, the relationship between T_2^A and T_2^B is unclear, providing the possibility of inefficient entry order in equilibrium. In other words, the strength of the LPC of each firm is influenced by the primitive parameter values. As indicated in the previous section, B is facing more threat from C, possibly resulting in $T_2^B < T_2^A$. In such cases, the more efficient firm may find it optimal to wait due to the shorter period of being monopoly. As a consequence, entry order will be inefficient with firm B entering at $min\{T_B^*(1), T_A^{AB}\}$ and A following at T_2^A. While in the case where $T_2^A < T_2^B$, the entry order is efficient and firm A will enter at $min\{T_A^*(1), T_B^{AB}\}$ with B following at T_2^B.

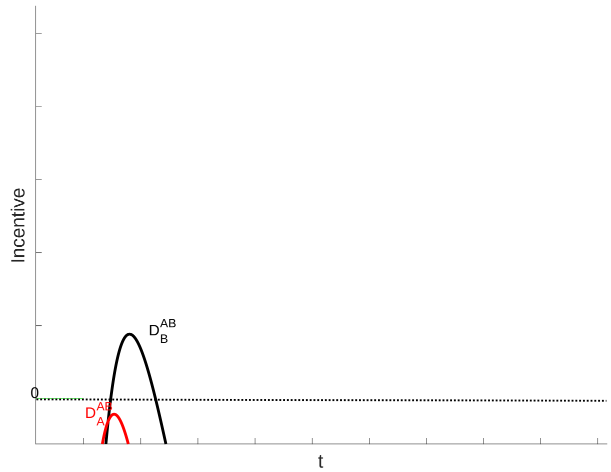
4.4 Order of entry

After illustrating the mechanism of equilibrium formation in the triopoly game, we now examine how parameters impact the outcome in equilibrium. Following from the previous analysis the parameters specifying the profit structures and properties of firms influence the equilibrium entry order and time through impacting firms' preemption incentives. To explore each factor's effect on the equilibrium, we analyze the following scenarios where all factors except for one are fixed.

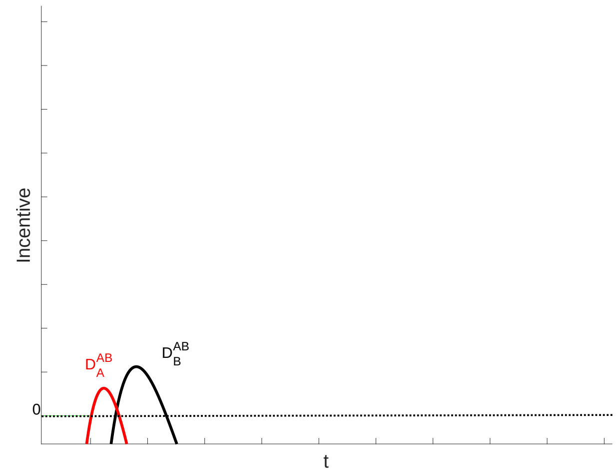
4.4.1 In a market with certain duopoly effect

If profit flows are fixed, changes in the relative efficiency among the three firms will translate into changes in the order and time of entry in equilibrium. To investigate how the relative efficiency levels impact the equilibrium outcome, we construct the scenario in which the efficiency levels of two firms are fixed whereas the efficiency level changes for the remaining potential entrant. Consider the case where firm A and C have fixed efficiency. As firm B's efficiency decreases, the equation D_C^{BC} shifts up and the earliest intersection with the horizontal axis T_C^{BC} is brought forward. This indicates that the follower preemption race between firm B and C becomes more intense. The tighter FPC on B brings forward the second entry time, making the first entry less attractive which in turn reduces A's incentive for being the leader. To some extent, when A's incentive decreases to become smaller than B's, B would act as the market pioneer instead. In particular, we could always observe that if A is a strong leader, it will always enter first no matter how intense the follower preemption race is. In the opposite case, where Δ_B increases, the FPC equation D_C^{BC} will shifts down. In other words, firm B is facing less threat from the follower preemption race. When firm B becomes sufficiently strong, its entry decision would be unperturbed by firm C. In such a scenario, the preemption game between firm A and B is just analogous to the duopoly game. As a consequence, the entry will always in the order of efficiency even if firm A is a weak leader. The following graphs depict possible outcomes as discussed above.

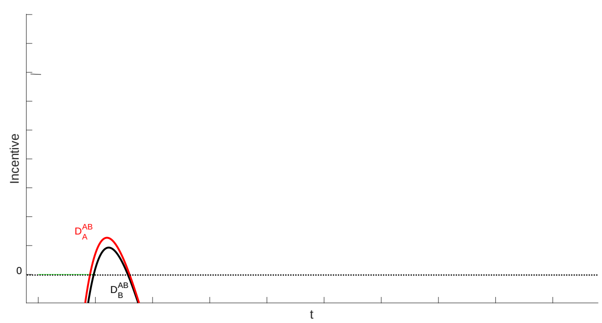
Comparing the above graphs (a) and (b), in the presence of tight FPC, the order of entry is inefficient in the game with a weak leader A and efficient with a strong leader A. Graph (c) and (d) indicate that when B is a sufficiently strong efficient firm, the entry order is always in the order of efficiency. These observations support our analysis above.



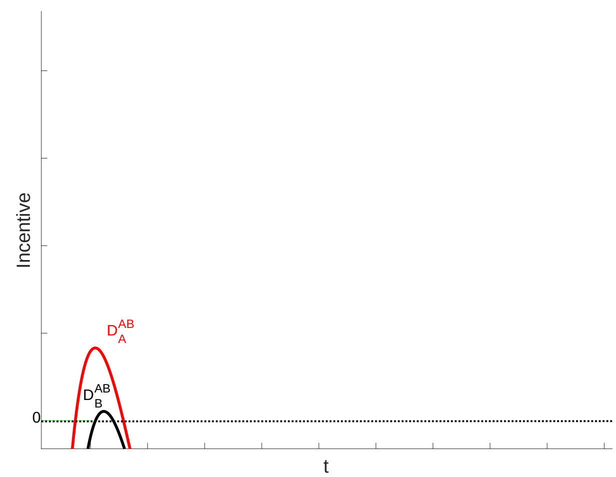
(a) A is a weak leader, $\Delta_A = 0.1$



(b) A is a strong leader, $\Delta_A = 0.2$



(c) A is a weak leader, $\Delta_A = 0.21$



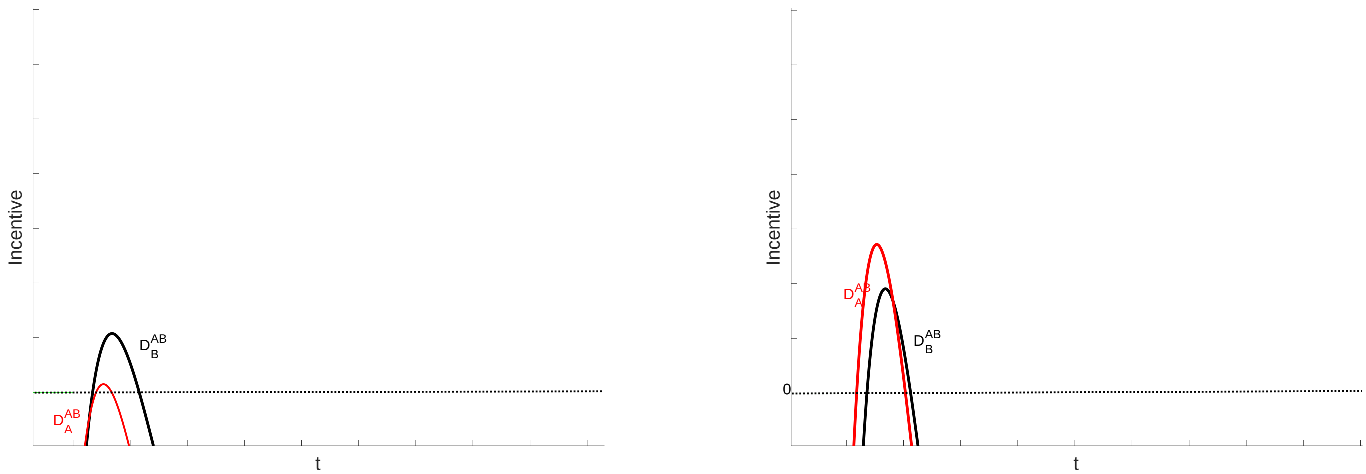
(d) A is a strong leader, $\Delta_A = 0.25$

Figure 5: Triopoly asymmetric model with ($\pi(1)$, $\pi(2)$, $\pi(3)$)=(300, 200, 100);
$(a)(b)$: (Δ_B, Δ_C)=(0.01, 0), weak B and tight FPC;
$(c)(d)$: (Δ_B, Δ_C)=(0.2, 0), strong B and weak FPC

4.4.2 The duopoly effect and efficiency

Now we fix the firms' efficiency levels to explore the relationship between the profit structure in the market and the entry order in equilibrium. Given that C will enter as the last entrant in all cases, we need to evaluate how attractive the monopoly and duopoly profits are to A and B and how willing these firms are to preempt each other under certain monopoly and duopoly profits. Therefore, we could simplify our analysis by exploring the impact of duopoly effect s, which captures the relative value in monopoly and duopoly profits on each firms' preemption incentives. In the market with a large s, the monopoly profit is less tempting to both firms. In this case, both firms would be more willing to wait as compared to the case with lower s. Given that benefits from cost reduction is higher for firm A, firm A might have a greater incentive to wait and enter later in market with a higher s. Therefore, we could expect that in the market with higher duopoly effect, entry order in equilibrium might be inefficient.

The two graphs in Figure 6 present the relative leader preemption incentive between firm A and B in market with fixed $\pi(2)$ and $\pi(3)$ but with different s. In case (a) where s is higher, entry order in equilibrium is inefficient, while in case (b) with lower s, entry order is efficient. These observations are consistent with the analysis above.



(a) large duopoly effect, $s = 100$

(b) small duopoly effect, $s = 50$

Figure 6: Triopoly asymmetric model with ($\pi(2)$, $\pi(3)$)=(200, 100); (Δ_A, Δ_B, Δ_C)=(0.1, 0.05, 0)

4.5 Formal investigation of the timing of entry and efficiency

In the previous section, we provided a general explanation for the relationships between the entry order and parameter values and supportive observations. In this section, we formally investigate how the efficiency in entry order relates to the model primitives. Following the above analysis, changes in parameter values can have complicated influences in firms' strategic interaction, which results in an unclear relationship between T_2^B and T_2^A. Entry is in order of efficiency if $T_2^B > T_2^A$. This provides the sufficient condition for the efficient entry order. This condition can be satisfied by imposing restrictions on the efficiency level of firms given certain pair of $\pi(2)$ and $\pi(3)$, which generates the following proposition.

Proposition 1. *In the triopoly market entry order is always efficient if at least one of the following two conditions is satisfied:*

1. *the most efficient firm A is sufficiently more efficient than the least efficient one (C), such that $T_A^*(2) < T_C^{BC}$;*

2. *firm B with middle efficiency level is sufficiently more efficient than the least efficient firm C such that* $T_B^*(2) < T_C^{AC}$*.*

Proof. See Appendix A.

However, if in the market of interest, $T_2^B < T_2^A$ is observed, there is ambiguity in the efficiency of entry order. Note that T_2^i depends on all efficiency parameters Δ_i and market structure π. Let us fix all of them except for $\pi(1)$ or s. The duopoly effect parameter s is a variable that determines the entry order in this case. Formally, we find that there always exists a s_0 above which entry order is inefficient when we have $T_2^B < T_2^A$. This generates the following Proposition 2.

Proposition 2. *There exists a value* s_0 *for the duopoly effect* $s = 2\pi(2) - \pi(1)$ *above which entry order in equilibrium is always inefficient in the market where* $T_2^B < T_2^A$ *is observed.*

Proof. See Appendix A.

To capture the intuition for Proposition 1, we consider how the relative efficiency level among firms influences the second entry time T_2^i. Observe that the first condition in Proposition 1, which specifies that A is a strong leader with $T_A^*(2) < T_C^{BC}$ would result in $T_2^A < T_2^B$. Revisiting the preemption incentive function of firm A and B, we can obtain the result by comparing term by term in the two firms' incentive functions. By restricting $T_2^A < T_2^B$, if firm A enters first in the market, it earns monopoly profits longer than B would as a leader. Therefore, the first term in $D_A^{AB}(t)$ is larger. As for the third term, bringing entry forward from T_2^A to t is cheaper for the more efficient firm A than from $T_B^*(2)$ to t for B. Hence, the third term in $D_A^{AB}(t)$ is always larger. Next, we compare the second terms in $D_A^{AB}(t)$ and $D_B^{AB}(t)$. In particular it has to be the case that monopoly profits for A earned from T_2^A to T_2^B and the duopoly profits not earned by A over the same period do not offset the previous two effects to support the efficient entry order. The intuition is as follows. Preempting B would result in a delay in the time from which A earns duopoly profits from T_2^A to T_2^B and in this interval duopoly profits are replaced by the higher monopoly profits. Hence, the total effect for firm A is positive. Considering firm B, by preempting A, the time from which it earns duopoly profits is brought forward. In contrast to A, extra duopoly profits are more than offset by the increase in the entry cost since T_2^B is further away from the duopoly stand-alone time where B has maximum payoff. Given that $D_A^{AB}(t)$ is bigger than $D_B^{AB}(t)$, we can conclude that in equilibrium B cannot enter first.

This condition implies that if we observe efficient entry order in the market with similar less productive or high-cost firms, the first entrant is expected to be a strong leader in that market. Alternatively, in empirical study of market preemption, if the most efficient firm involved in the game is sufficiently stronger than the least efficient one. It is always reasonable to expect an efficient entry order.

The second condition in Proposition 1 restricts B to be a strong leader over C such that $T_B^*(2) < T_C^{BC}$. In this case, B's entry decision is not influenced by the presence of C, thereby simplifying the triopoly game to a duopoly game of A and B. Therefore, entry order will always be efficient. By this necessary condition for efficient entry order, we could conclude with a remark that if we observe efficient entry order in market with two similar efficient firms, we could expect the second entrant to be significantly stronger than the last entrant. Alternatively, in the study of the preemption game with set of firms satisfying this condition, efficient entry order could be reasonably assumed.

As for Proposition 2, the intuition could be captured by looking at how the instantaneous post-entry monopoly benefits impact the preemption incentives. In the market with larger s, the monopoly profit is less attractive, dampening firm A and B's preemption incentives. Given earlier second entry by firm B, A's preemption incentive is more dampened by the shorter period of monopoly. When s is large enough, the benefit for being the market leader could be less than the benefits of the cost-saving for A via waiting. As a result, the efficient firm A will be more willing to wait to enter later.

By using matlab, we could obtain some graphic illustration for Propositions 1 and 2.

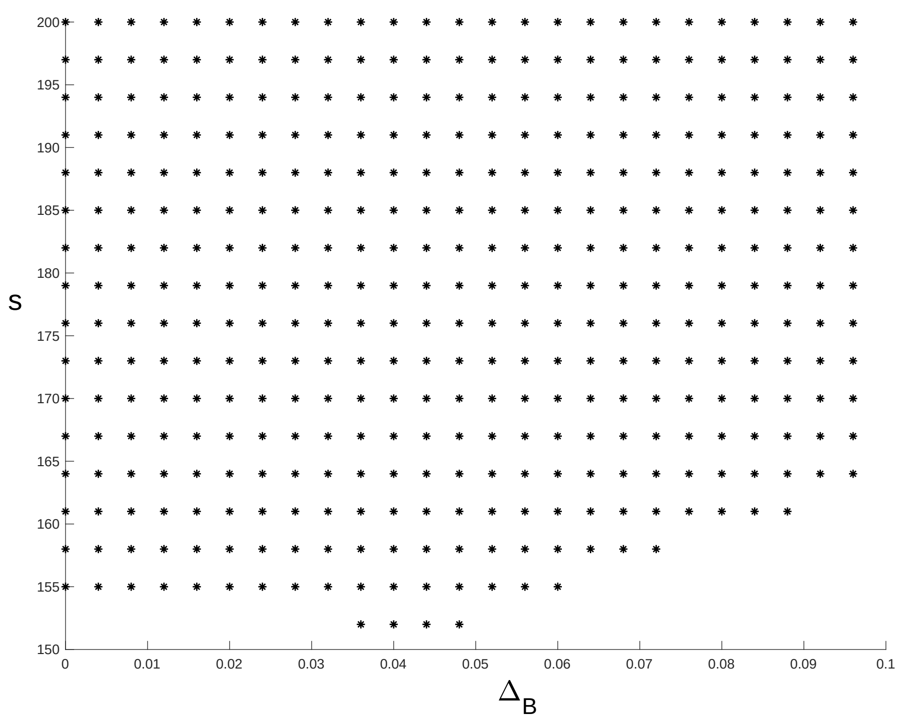


Figure 7: Parameters and inefficient entry order

The figure depicts that in the case where $\Delta_C = 0$, $\Delta_A = 0.1$, $\pi(2) = 200$ and $\pi(3) = 100$, when s is smaller than 150, entry order will always be in efficiency. In addition, this figure also indicates that as Δ_B increases, inefficiency in entry order is less like to occur. This result confirms Propositions 1 and 2 above.

4.6 Dynamics of initial entry in triopoly market

As in the duopoly preemption model, we attempt to explore the dynamics of the initial entry in the market with changes in the relevant factors, namely, duopoly effect s and relative efficiency among firms. In this section, we restrict our attention to the simpler scenario where entry in equilibrium is always in order of efficiency to obtain the welfare implication. To ensure the efficient entry order, we pretest the parameter used in the model with matlab. The basic facts about the market preemption game we are studying for the dynamics are as follows.

- Δ_A and Δ_C are fixed at 0.2 and 0 and Δ_B alternates in $(-0.1,0.3)$
- $\pi(3) = 100$, $\pi(2) = 200$ and $s_1 = 100$, $s_2 = 150$

In this scenario, the ranking among the three firms alters with Δ_B. When Δ_B is smaller than 0, the efficiency ranking is $A > C > B$. While in the case where $\Delta_B \in (0,0.2)$, the ranking is $A > B > C$ and when Δ_B is above 0.2, the ranking becomes $B > A > C$.

The figure below presents different dynamics in the initial entry time in market with different duopoly effects.

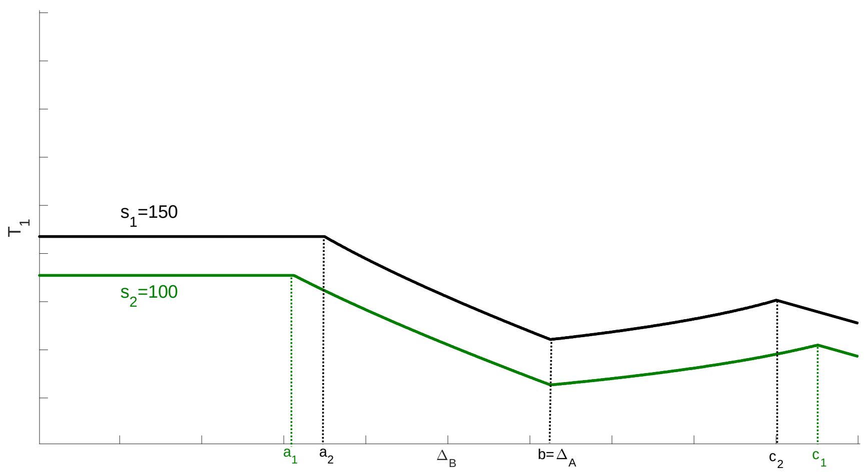


Figure 8: Dynamics of first entry time

As shown in Figure 8, the general pattern observed in triopoly model is similar to the one in duopoly. t_1 is constant when Δ_B is small ($\Delta_B < a$) after which it decreases until $\Delta_B = b$. Following an increase from b to c, t_1 linearly decreases with Δ_B.

To capture the intuition of the observed patterns, we take a further look at the changes in the preemption incentive of firms competing in the preemption race for the role of initial entrant. When firm B is very inefficient, the competition between A and C is not impacted by the existence of B. In other words, the first entry time is just the duopoly initial entry time, which is purely determined by the duopoly game between A and C. As a result, t_1 remains unchanged at $\min\{T_A^*(1), T_C^{AC}\}$. By the same logic, when firm B is really efficient, it takes the role of a strong leader whose entry decision is unperturbed by the preemption threat from the other inefficient rivals and it enters at its monopoly stand-alone time $T_B^*(1)$, which is a decreasing function with Δ_B. During the descending period from a to b, firm A

acts as a weak leader whose entry decision is impacted by the follower preemption game between B and C. As a result, $t_1 = T_C^{AC}$ or $t_1 = T_B^{AB}$, depending on the rank between B and C. As Δ_B increases but remains smaller than Δ_C, firm C is facing more threat from B in that the follower preemption race is intensified with larger Δ_B. Therefore, T_2^C is brought forward, shifting up C's leader preemption incentive function D_C^{AC} thereby decreasing $t_1 = T_C^{AC}$. When firm B becomes more efficient than C while A remains to be the weak leader in the game($\Delta_B < b$), t_1 is determined by B's preemption incentive. In this case, a stronger firm B has a growing preemption incentive with the increase in its efficiency level, bringing forward $t_1 = T_B^{AB}$. As Δ_B surpasses b, firm B becomes the weak leader in the market and $t_1 = T_A^{AB}$. Note that D_A^{AB} is a decreasing function of Δ_B since firm A becomes less willing to preempt a stronger rival, leading to an increase in t_1. The figure above also reveals the impact of duopoly effect on the initial entry decision in the triopoly market. As we enlarge s, the pattern generally shifts up, with the descending and ascending periods from a to c becoming shorter. The intuition for the observed shift-up is that all firms would be more willing to wait for benefits brought by the duopoly competition and this results in a general delay in the first entry time. In terms of the shorter interval from a to c, this intuitively tells us that the weak leader period is shortened because the inefficient rivals have less incentive for preemption given a higher relative duopoly benefits.

In the following, we further explore the impact of relative difference in efficiency levels between A and C on the initial entry. We consider the dynamic pattern with different Δ_A.

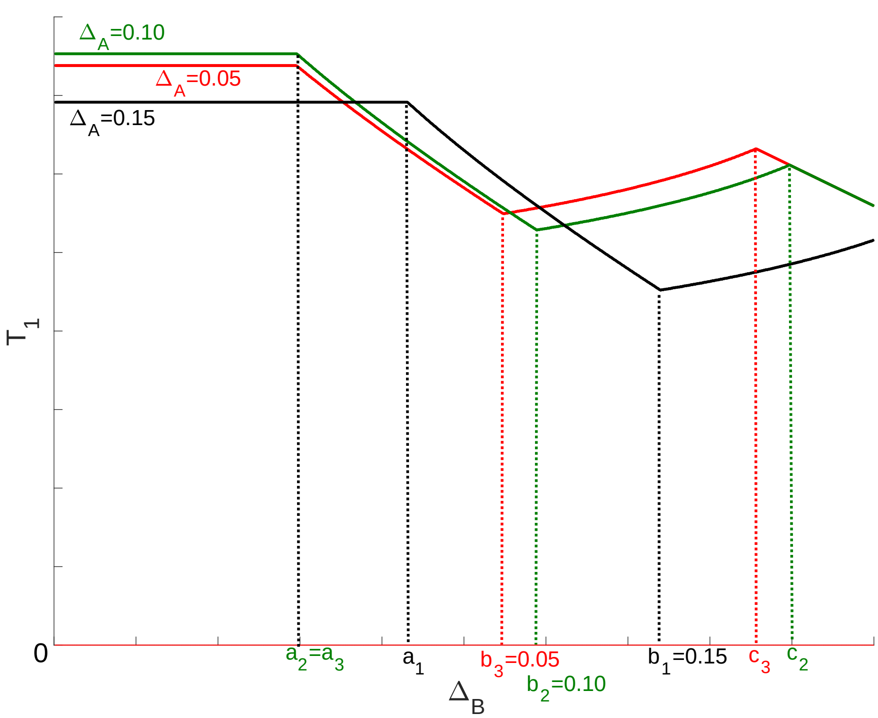
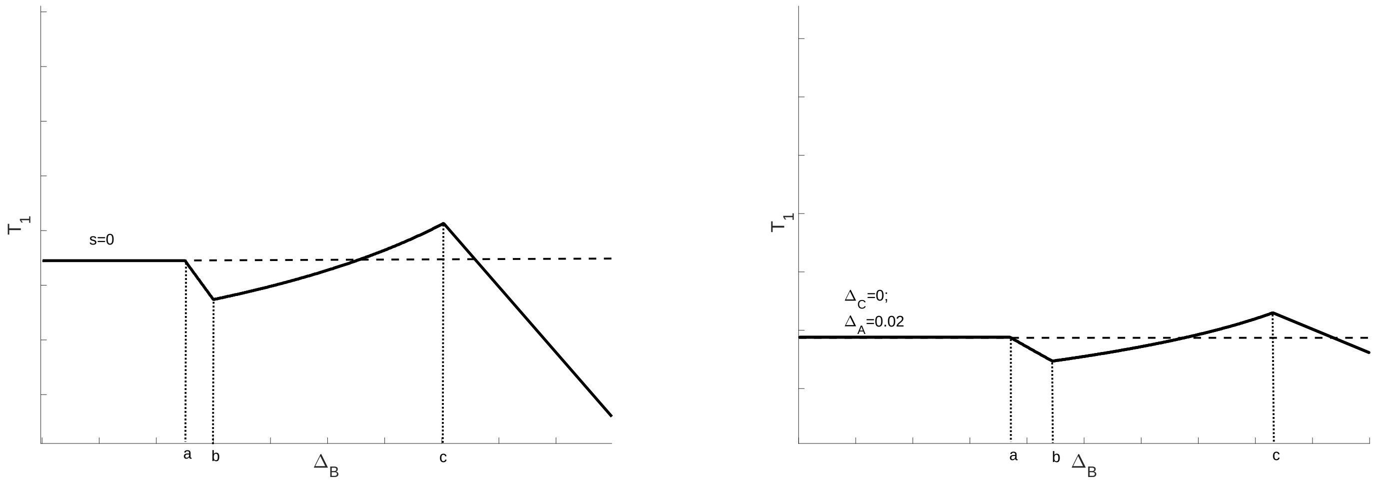


Figure 9: Dynamics of first entry time

As revealed in the above figure, as Δ_A decreases from 0.15 to 0.1, the weak-leader period becomes longer ((a_1, c_1) to (a_2, c_2)) and the dynamic pattern generally shifts up. However, as Δ_A decreases further from 0.1 to 0.05, the turning point a remains unchanged with b and c becoming smaller (b_2 to b_3 and c_2 to c_3). In addition, the pattern observed before B becomes the efficient firm in the market at point b_2 is lower, while it is higher after B takes the role of the leader in the game in the case of Δ_A being 0.05. The intuition for the first observation is that as Δ_A changes from 0.15 to 0.1, firm A transforms from a strong leader to a weak leader in the duopoly game with firm C. Therefore, with $\Delta_A = 0.1$, the duopoly competition between A and C when B is very inefficient is less intense and thus results in a higher platform. In addition, with a weak leader A, it is easier for B to influence the initial entry decision which brings the first turning point a earlier. Moreover the required efficiency level for B to be the strong leader is lower when facing a weaker A.

In terms of the results when Δ_A decreases from 0.1 to 0.05, in both cases, firm A is a weak leader in the duopoly game with C. Therefore, during the platform period $t_1 = T_C^{AC}$. As Δ_A decreases from 0.1 to 0.05, by anticipating an later entry by weaker A as a follower, C's incentive for preempting becomes bigger contributing to an earlier initial entry and thus a lower platform. The turning point a remains the same because it is the point where B starts to impact C's decision, which is not influenced by A.

As mentioned previously, the issue concerning whether there is a delay in initial entry as market structure changes has welfare implications. In the symmetric model, given the fact that initial entry is socially sub-optimal in the duopoly market, initial entry delays in the triopoly market further ruins social welfare because there would be a longer period when there is no production in the market to generate consumer surplus. However, this is not always the case in asymmetric model. The dynamic patterns above provide inference for the delay effect as market structure changes from duopoly to triopoly since the platform period reflects the initial entry time in the two-firm game between A and C. Based on the above observations, we could conclude that delay does not necessarily happen and it is likely to occur when s and Δ_A are small. With a lower s, the initial entry time would descend from a lower platform for the same length of period and increase for a longer period. With a lower Δ_A, t_1 decreases from a lower platform for a shorter period and increases for a longer period. The following graphs present the situation where delay occurs as a result of small s and Δ_A.



(a) small duopoly effect, $s = 0$

(b) $\Delta_C = 0, \Delta_A = 0.02$

Figure 10: Delay in initial entry in triopoly asymmetric model

The intuition is similar in spirit to Argenziano and Schmidt-Dengler (2013) who conclude that extra competition in triopoly market contributes to delay in first entry time. In asymmetric game, with lower duopoly effect and more symmetric firms in the market, preemption race is more intense thereby resulting in earlier second entry. This dampens firms' preemption incentive and thus first entry is more likely to be delayed. Therefore, we could conclude with a remark restricting number of firms in the market with lower duopoly effect and similar firms is a potentially effective way to enhance social welfare as delay in the first entry is likely occur with an extra firm coming into the market. However, in the markets with higher duopoly effect and more differentiated firms, encouraging extra entrants might be appropriate to alleviate distortion in time of the first entry.

5 Conclusion

We have analysed a market-entry game with asymmetric firms. We show that entry order is efficient in the duopoly game while it may be inefficient in the triopoly game. We generate necessary conditions for the efficient entry by placing restrictions in terms of triopoly and duopoly profits. We further explore the implications of monopoly/duopoly profits (captured by the duopoly effect) on the entry order, and produce essential conditions necessary for inefficient order of entry by firms. Furthermore, to investigate the welfare effect of the entry in equilibrium we explore the dynamics of initial entry time in both duopoly and triopoly markets and show that the first entry could occur earlier in the triopoly market (contrary to the literature). Based on this result, we conclude that encouraging an extra competitor in the duopoly market could help improve social welfare.

Appendix A

Proof of Lemma 2

The preemption incentive for firm A and B are:

$$D_A^{AB}(t) = \pi(1)\int_t^{T_B^*(2)} e^{-rs}ds + \pi(2)\int_{T_B^*(2)}^{T_A^*(2)} e^{-rs}ds - (ke^{-(\alpha+r)(t+\Delta_A)} - ke^{-(\alpha+r)(T_A^*(2)+\Delta_A)}),$$

and

$$D_B^{AB}(t) = \pi(1)\int_t^{T_A^*(2)} e^{-rs}ds + \pi(2)\int_{T_A^*(2)}^{T_B^*(2)} e^{-rs}ds - (ke^{-(\alpha+r)(t+\Delta_B)} - ke^{-(\alpha+r)(T_B^*(2)+\Delta_B)}).$$

For $D_A^{AB}(t) > D_B^{AB}(t)$, we need to prove that:

$$(\pi(1)-\pi(2))\int_{T_A^*(2)}^{T_B^*(2)} e^{-rs}ds - (ke^{-(\alpha+r)(t+\Delta_A)} - ke^{-(\alpha+r)(T_A^*(2)+\Delta_A)})$$
$$> \pi(2)\int_{T_A^*(2)}^{T_B^*(2)} e^{-rs}ds - (ke^{-(\alpha+r)(t+\Delta_B)} - ke^{-(\alpha+r)(T_B^*(2)+\Delta_B)}).$$

The right hand side (RHS) can be further rewritten as:

$$\pi(2)\int_{T_A^*(2)}^{\infty} e^{-rs}ds - ke^{-(\alpha+r)(T_A^*(2)+\Delta_B)} - (\pi(2)\int_{T_B^*(2)}^{\infty} e^{-rs}ds - ke^{-(\alpha+r)(T_B^*(2)+\Delta_B)})$$
$$+ ke^{-(\alpha+r)(T_A^*(2)+\Delta_B)} - ke^{-(\alpha+r)(t+\Delta_B)}.$$

Given that $T_B^*(2)$ maximizes firm B's follower profit and the fact that $T_B^*(2) > T_A^*(2)$, we know that

$$\pi(2)\int_{T_A^*(2)}^{\infty} e^{-rs}ds - ke^{-(\alpha+r)(T_A^*(2)+\Delta_B)} - (\pi(2)\int_{T_B^*(2)}^{\infty} e^{-rs}ds - ke^{-(\alpha+r)(T_B^*(2)+\Delta_B)}) < 0.$$

Therefore,

$$RHS < ke^{-(\alpha+r)(T_A^*(2)+\Delta_B)} - ke^{-(\alpha+r)(t+\Delta_B)}.$$

Next, we only need to prove that

$$(\pi(1)-\pi(2))\int_{T_A^*(2)}^{T_B^*(2)} e^{-rs}ds - (ke^{-(\alpha+r)(t+\Delta_A)} - ke^{-(\alpha+r)(T_A^*(2)+\Delta_A)}) >$$
$$ke^{-(\alpha+r)(T_A^*(2)+\Delta_B)} - ke^{-(\alpha+r)(t+\Delta_B)}.$$

Since $(\pi(1)-\pi(2))\int_{T_A^*(2)}^{T_B^*(2)} e^{-rs}ds > 0$, we have to prove that

$$ke^{-(\alpha+r)(T_A^*(2)+\Delta_A)} - ke^{-(\alpha+r)(t+\Delta_A)} > ke^{-(\alpha+r)(T_A^*(2)+\Delta_B)} - ke^{-(\alpha+r)(t+\Delta_B)}.$$

Observe that,

$$T_A^*(2) + \Delta_A - (t + \Delta_A) = T_A^*(2) + \Delta_B - (t + \Delta_B) = d > 0,$$

and

$$t + \Delta_A > t + \Delta_B.$$

We could construct a function $f(x) = ke^{-(\alpha+r)(x+d)} - ke^{-(\alpha+r)(x)}$ to aid the proof. Differentiate $f(x)$ over x, we have,

$$f'(x) = k(\alpha + r)e^{-(\alpha+r)(x)} - k(\alpha + r)e^{-(\alpha+r)(x+d)} > 0.$$

Therefore, $f(x)$ is increasing in x. Hence,

$$ke^{-(\alpha+r)(T_A^*(2)+\Delta_A)} - ke^{-(\alpha+r)(t+\Delta_A)} > ke^{-(\alpha+r)(T_A^*(2)+\Delta_B)} - ke^{-(\alpha+r)(t+\Delta_B)}.$$

This concludes the proof. □

Proof of Proposition 1

The conditions (1) and (2) in Proposition 1 result in $T_2^A < T_2^B$.

Firstly, observe that $T_A^*(2) < T_B^*(2)$. Next we compare T_C^{AC} and T_C^{BC}. Consider the subgame between A and C, the payoff for C to be the leader and follower are:

$$L_C^{AC} = \pi(2) \int_t^{T_A^*(3)} e^{-rs} ds + \pi(3) \int_{T_A^*(3)}^{\infty} e^{-rs} ds - ke^{-(\alpha+r)(t+\Delta_C)},$$

and

$$F_C^{AC} = \pi(3) \int_{T_C^*(3)}^{\infty} e^{-rs} ds - ke^{-(\alpha+r)(T_C^*(3)+\Delta_C)}.$$

Therefore, we generate C's incentive function to preempt A as

$$D_C^{AC}(t) = \pi(2) \int_t^{T_A^*(3)} e^{-rs} ds + \pi(3) \int_{T_A^*(3)}^{T_C^*(3)} e^{-rs} ds + ke^{-(\alpha+r)(T_C^*(3)+\Delta_C)} - ke^{-(\alpha+r)(t+\Delta_C)}.$$

By the same logic, consider the subgame of firm B and C, the incentive for C to preempt B is

$$D_C^{BC}(t) = \pi(2) \int_t^{T_B^*(3)} e^{-rs} ds + \pi(3) \int_{T_B^*(3)}^{T_C^*(3)} e^{-rs} ds + ke^{-(\alpha+r)(T_C^*(3)+\Delta_C)} - ke^{-(\alpha+r)(t+\Delta_C)}.$$

Next we compare C's incentive to preempt A and B by exloring the sign of

$$D_C^{AC}(t) - D_C^{BC}(t) = (\pi(3) - \pi(2)) \int_{T_B^*(3)}^{T_A^*(3)} e^{-rs} ds.$$

Since $T_B^*(3) > T_A^*(3)$ and $D_C^{AC}(t) - D_C^{BC}(t) < 0$ we know that $T_C^{AC} > T_C^{BC}$. Also notice that $T_B^*(2) > T_A^*(2)$. As a result,

- By condition (1): $T_A^*(2) < T_C^{BC}$, we have $T_2^A < T_2^B$;
- By condition (2): $T_B^*(2) < T_C^{BC}$, we have $T_2^A < T_2^B$.

The entry is in order of efficiency if at least one of the above conditions is satisfied

Firstly, note that A's preemption incentive is

$$D_A^{AB}(t) = \pi(1)\int_t^{T_2^B} e^{-rs}ds + \pi(2)\int_{T_2^B}^{T_2^A} e^{-rs}ds + ke^{-(\alpha+r)(T_2^A+\Delta_A)} - ke^{-(\alpha+r)(t+\Delta_A)},$$

and B's preemption incentive is

$$D_B^{AB}(t) = \pi(1)\int_t^{T_2^A} e^{-rs}ds + \pi(2)\int_{T_2^A}^{T_2^B} e^{-rs}ds + ke^{-(\alpha+r)(T_2^B+\Delta_B)} - ke^{-(\alpha+r)(t+\Delta_B)}.$$

For $D_A^{AB}(t) > D_B^{AB}(t)$, we need to prove:

$$(\pi(1)-\pi(2))\int_{T_2^A}^{T_2^B} e^{-rs}ds - (ke^{-(\alpha+r)(t+\Delta_A)} - ke^{-(\alpha+r)(T_2^A+\Delta_A)}) >$$
$$\pi(2)\int_{T_2^A}^{T_2^B} e^{-rs}ds - (ke^{-(\alpha+r)(t+\Delta_B)} - ke^{-(\alpha+r)(T_2^B+\Delta_B)}).$$

As in the proof of duopoly case, we rewrite the right hand side (RHS) as follows,

$$\pi(2)\int_{T_2^A}^{\infty} e^{-rs}ds - ke^{-(\alpha+r)(T_2^A+\Delta_B)}) - (\pi(2)\int_{T_2^B}^{\infty} e^{-rs}ds - ke^{-(\alpha+r)(T_2^B+\Delta_B)})$$
$$+ ke^{-(\alpha+r)(T_2^A+\Delta_B)} - ke^{-(\alpha+r)(t+\Delta_B)}.$$

In Proposition 1, we have restricted that $T_2^A = T_A^*(2) < T_2^B \leq T_B^*(2)$. Since $T_B^*(2)$ is the duopoly stand-alone entry time, when $t < T_B^*(2)$, the function $\pi(2)\int_t^{\infty} e^{-rs}ds - ke^{-(\alpha+r)(t+\Delta_B)}$ is increasing in t. Therefore we should have,

$$RHS < ke^{-(\alpha+r)(T_2^A+\Delta_B)} - ke^{-(\alpha+r)(t+\Delta_B)}.$$

Next we only need to prove that:

$$(\pi(1)-\pi(2))\int_{T_2^A}^{T_2^B} e^{-rs}ds - (ke^{-(\alpha+r)(t+\Delta_A)} - ke^{-(\alpha+r)(T_2^A+\Delta_A)}) > ke^{-(\alpha+r)(T_2^A+\Delta_B)} - ke^{-(\alpha+r)(t+\Delta_B)}.$$

Since $(\pi(1)-\pi(2))\int_{T_A^2}^{T_B^2} e^{-rs}ds > 0$, this problem can be simplified as follows,

$$ke^{-(\alpha+r)(T_2^A+\Delta_A)} - ke^{-(\alpha+r)(t+\Delta_A)} > ke^{-(\alpha+r)(T_2^A+\Delta_B)} - ke^{-(\alpha+r)(t+\Delta_B)}.$$

Note that, $T_2^A + \Delta_B - (t+\Delta_B) = T_2^A + \Delta_A - (t+\Delta_A) = d > 0$ and $t+\Delta_A > t+\Delta_B$. It has been proved before, $f(x) = ke^{-(\alpha+r)(x+d)} - ke^{-(\alpha+r)(x)}$ $(d > 0)$ is a function increasing in x. Therefore,

$$ke^{-(\alpha+r)(T_2^A+\Delta_A)} - ke^{-(\alpha+r)(t+\Delta_A)} > ke^{-(\alpha+r)(T_2^A+\Delta_B)} - ke^{-(\alpha+r)(t+\Delta_B)}.$$

Hence the proof is completed. □

Proof of Proposition 2

When $s \to \pi(2)$, $t_A^{AB} \to t_2^A$ and $t_B^{AB} \to t_2^B$

By definition of t_A^{AB}, it satisfies

$$D_A^{AB}(t_A^{AB}) = \pi(1)\int_{t_A^{AB}}^{T_2^B} e^{-rs}ds + \pi(2)\int_{T_2^B}^{T_2^A} e^{-rs}ds + ke^{-(\alpha+r)(T_2^A+\Delta_A)} - ke^{-(\alpha+r)(t_A^{AB}+\Delta_A)} = 0.$$

Since $s \to \pi(2)$, $\pi(1) \to \pi(2)$, the above equation can be simplied as

$$D_A^{AB}(t_A^{AB}) = \pi(2)\int_{t_A^{AB}}^{T_2^A} e^{-rs}ds + ke^{-(\alpha+r)(T_2^A+\Delta_A)} - ke^{-(\alpha+r)(t_A^{AB}+\Delta_A)} = 0.$$

By definition of t_2^A, $t_2^A \leq t_A^*(2)$. In addition, $\pi(1) \to \pi(2)$ indicates $t_A^*(1) \to t_A^*(2)$. Therefore, we have $t_2^A \leq t_A^*(1)$. Given that $t_A^*(1)$ maximizes the D_A^{AB} and t_2^A is the upper bound of t_A^{AB}, t_A^{AB} has to be equal to t_2^A so as to satisfy $D_A^{AB}(t_A^{AB}) = 0$.

By the same logic, t_B^{AB} has to be equal to t_2^B so as to satisfy $D_B^{AB}(t_B^{AB}) = 0$. In this proposition, we have already observe $t_2^A > t_2^B$. Therefore, $t_A^{AB} > t_B^{AB}$ or equivalently, $D_A^{AB} < D_B^{AB}$.

There always exists a range for s with inefficient entry order

We consider the relationship between t_A^{AB} and t_B^{AB} and s. With $D_B^{AB}(t_B^{AB}) = 0$, we can obtain $t_B^{AB'}$ (in terms of s) by totally differentiating $D_B^{AB}(t_B^{AB}) = 0$ in terms of s and the result is as follows:

$$(-\pi(1)e^{-rt_B^{AB}} + k(\alpha+r)e^{-(\alpha+r)(t_B^{AB}+\Delta_B)}) \times t_B^{AB'} = \frac{e^{-rt_B^{AB}} - e^{-rt_2^A}}{r}.$$

When $s \to \pi(2)$ as analysed in the above part, we should have a clear rank as follows:

- $t_B^{AB} \to t_2^B$, $t_A^{AB} \to t_2^A$, $t_B^{AB} < t_A^{AB}$
- $t_B^*(2) \to t_B^*(1)$, $t_A^*(2) \to t_A^*(1)$, $t_A^*(1) < t_B^*(1)$

In addition, by definition of $t_B^*(1)$ and concavity of $D_B^{AB}(t)$, we have,

$$-\pi(1)e^{-rt_B^*(1)} + k(\alpha+r)e^{-(\alpha+r)(t_B^*(1)+\Delta_B)} = 0,$$

and

$$-\pi(1)e^{-rt_B^{AB}} + k(\alpha+r)e^{-(\alpha+r)(t_B^{AB}+\Delta_B)} > 0.$$

Furthermore, since $t_B^{AB} < t_2^A$, the RHS: $\frac{e^{-rt_B^{AB}} - e^{-rt_2^A}}{r} > 0$.

As a consequence, when $s \to \pi(2)$, we have $t_B^{AB'} > 0$. Also notice that t_2^A and t_2^B are only determined by $\pi(2)$, $\pi(3)$ and all Δ_i, s and t_B^{AB} are thus exogenous. Therefore, $t_B^{AB'} > 0$ holds as s decreases from $\pi(2)$. As a result, we come to the conclusion that t_B^{AB} decreases from t_2^B as s decreases from $\pi(2)$.

Next, we take a further look at $D_A^{AB}(t_2^B)$ and $D_A^{AB}(t_2^A)$. It is easy to see that given $t_2^B < t_2^A \le t_A^*(2)$ ($t_A^*(2)$ is the duopoly stand-alone time)

$$D_A^{AB}(t_2^B) = \frac{2\pi(2)-s}{r}(e^{-rt_2^B} - e^{-rt_2^B}) + \frac{\pi(2)}{r}(e^{-rt_2^B} - e^{-rt_2^A}) + ke^{-(\alpha+r)(t_2^A+\Delta_A)} - ke^{-(\alpha+r)(t_2^B+\Delta_A)} < 0;$$

$$D_A^{AB}(t_2^A) = \frac{2\pi(2)-s}{r}(e^{-rt_2^A} - e^{-rt_2^B}) + \frac{\pi(2)}{r}(e^{-rt_2^A} - e^{-rt_2^A}) + ke^{-(\alpha+r)(t_2^A+\Delta_A)} - ke^{-(\alpha+r)(t_2^A+\Delta_A)} < 0,$$

for all $s = 2\pi(2) - \pi(1)$.

Given the concavity of $D_A^{AB}(t)$ and $t_B^{AB} \le t_2^B$ with equality only for $s = \pi(2)$, we know that if $D_A^{AB}(t)$'s maximizer $t_A^*(1)$ is to the right of t_2^B, $D_A^{AB}(t_B^{AB}) \le D_A^{AB}(t_2^B) < 0$.

Since $t_A^*(1)$ is the maximizer of $D_A^{AB}(t)$, we have, $D_A^{AB}(t_A^*(1))' = 0$. Therefore,

$$-\pi(1)e^{-rt_A^*(1)} + k(\alpha+r)e^{-(\alpha+r)(t_A^*(1)+\Delta_A)} = 0.$$

Solving the above equation, we have,

$$t_A^*(1) = \frac{ln\frac{k(\alpha+r)}{2\pi(2)-s_0} - (\alpha+r)\Delta_A}{\alpha}.$$

Notice that $t_A^*(1)$ decreases as s decreases and $t_A^*(1) = t_2^A > t_2^B$ when $s = \pi(2)$. We can find a $s_0 < \pi(2)$ which can result in $t_A^*(1) = t_2^B$. Based on the above anaysis, $D_A^{AB}(t_B^{AB}) < 0$ always holds for $s \ge s_0$. s_0 solves the following equation:

$$t_2^B = \frac{ln\frac{k(\alpha+r)}{2\pi(2)-s_0} - (\alpha+r)\Delta_A}{\alpha}.$$

Therefore, we have

$$s_0 = 2\pi(2) - e^{ln(k(\alpha+r)) - \alpha t_2^B - (\alpha+r)\Delta_A}.$$

Since $D_A^{AB}(t_B^{AB}) < 0$ always holds for $s \ge s_0$, $D_A^{AB}(t) < D_B^{AB}(t)$ always holds for $s \ge s_0$ in the case where we have certain profit structure which results in $t_2^B < t_2^A$. Hence the proof is completed. □

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