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Abstract

We develop a panel unobserved components model of household income and consumption that can be estimated using full information methods. Maximum likelihood estimates for a simple version of this model suggests similar income risk, but higher consumption insurance relative to the partial information moments-based estimates of Blundell, Pistaferri, and Preston (2008) when using the same panel dataset. Bayesian model comparison supports this simple version of the model that only allows a spillover from permanent income to permanent consumption, but assumes no cointegration and no persistence in transitory components. At the same time, consumption insurance and income risk estimates are highly robust across different specifications.

Keywords: panel unobserved components; Bayesian model comparison; permanent income; household consumption behavior

JEL codes: E32; E22; C32

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1 Introduction

For panel data analysis, generalized method of moments is often used to estimate parameters of interest. For time series analysis, by contrast, a full information likelihood approach is typically employed. Our objective here is to apply a full information likelihood approach to a panel dataset of household income and consumption to answer two related fundamental questions in macroeconomics: What is the degree of household income risk and how much consumption insurance is there against this risk?[1]

A large literature has examined the amount of income risk faced by households and the fraction that gets transmitted to consumption. Deaton (1997) proposed measuring consumption insurance using panel data on household income and (imputed) nondurable consumption. However, Blundell, Pistaferri and Preston (2008) (BPP hereafter) were the first to implement this suggestion. In particular, BPP construct a novel panel dataset of household income and consumption and employ partial information moments-based estimation of income risk and consumption insurance without imposing a particular structural model for households' behavior and decisions. They find a relatively low degree of consumption insurance in response to idiosyncratic shocks to permanent income. However, their results have been challenged by Kaplan and Violante (2010) (KV hereafter), who argue that the BPP estimation strategy leads to a downward bias in consumption insurance, with the bias being more pronounced for borrowing-constrained households.

In this paper, we propose an alternative estimation strategy based on full information methods for estimation of a general panel unobserved components (UC) model of household income and consumption. Maximum likelihood estimates (MLE) for a simple version of the model suggest similar income risk, but higher consumption insurance relative to BPP when using the same panel dataset. Strikingly, our consumption insur-

[1]See Kaplan and Violante (2010) on the importance of accurately measuring consumption insurance to compare predictions of different incomplete-markets models and to deliver reliable predictions for policy experiments.

ance estimate is 17 percentage points higher than what was found by BPP using partial information moments-based estimation. Thus, a full information approach helps address KV's concern regarding downward bias in BPP's estimation of the consumption insurance parameter.

In the full information environment, Bayesian methods allow us to easily compare different specifications of our panel UC model. In particular, we calculate marginal likelihoods to consider spillovers across different components, persistence in transitory income and consumption, and possible cointegration between income and consumption in driving our findings. This Bayesian model comparison supports the simple version of our model used in our MLE analysis that only has a spillover from permanent income to permanent consumption, but no cointegration and no persistence in transitory components. At the same time, consumption insurance and income risk estimates are highly robust across the various specifications under consideration. However, prior sensitivity analysis makes it clear that the degree of consumption insurance is not particularly well identified in the data, although it would take a highly informative and distorted prior to obtain the lower estimates previously found by BPP with partial information inference.

Subgroup estimation reveals a highly intuitive pattern of heterogeneity in consumption insurance estimates. In particular, we find that consumption insurance is higher for older or more educated households. Furthermore, comparing our estimates with the corresponding BPP estimates, we find that the difference in estimates is larger for the subgroups that are likely to be more borrowing constrained, for example the subgroup without college education, which is completely consistent with the arguments in KV about bias.

Obtaining accurate estimates of household income risk and consumption insurance is of paramount importance in macroeconomics both to inform the incomplete-markets literature and to prescribe policies after understanding the extent of market incompleteness. In particular, the literature has keenly focused on questions about the extent to which there are formal markets or informal arrangements that help insure households against idiosyncratic and unexpected movements in their income or wealth, as well as how this insurance varies across different types of households and over a given household's life-cycle as borrowing and lending opportunities change.

The literature has employed a variety of empirical approaches and made important advances that enhance our understanding of consumption insurance. Early studies that provide a simple test of consumption insurance are Cochrane (1991) and Townsend (1994)'s consumption insurance in village India. The study by BPP employs a quasi-structural approach to estimating consumption insurance and permanent risk from their panel data of income and consumption. More recently, Heathcote, Storesletten, and Violante (2014) estimate a fully structural model. Following these studies, it has clearly been established that, for the U.S. economy at least, there is no evidence for either of the two extremes of full insurance or zero insurance. However, quantitative estimates differ significantly, as do the methods with respect to identification of the permanent and transitory shocks to income. For example, Kruger and Perri (2011) simply compute the change in nondurable consumption related to a change in income, while others, see, for example, Souleles (1999), have proxied permanent and transitory income shocks. Primiceri and van Rens (2009) use Consumer Expenditure Survey (CEX) repeated cross-section data on consumption and income to decompose idiosyncratic changes in income into predictable life-cycle changes, transitory and permanent shocks and estimate the contribution of each to total inequality.

Our contribution to this large and extensive literature is threefold: First, we are the first to our knowledge to employ full information likelihood-based estimation for a panel unobserved components (UC) model of household income and consumption data. Second, the full information approach allows us to use Bayesian model comparison to determine the best specification for our panel UC model. Third, our estimates for a simple version of the model confirm KV's concern of a downward bias in BPP's estimate of consumption insurance. In particular, we find a similar income risk, but higher consumption insurance relative to BPP when using the same panel dataset. Furthermore, using our flexible empirical framework, we are able to show that income risk and consumption insurance estimates are highly robust across various specifications.

The rest of this paper is organized as follows: Section 2 presents the general panel UC model proposed in this paper. Section 3 describes the data. Section 4 reports empirical results. Section 5 concludes.

2 A Panel Unobserved Components Model

In this section we present the details of our panel UC model of household income and consumption. We also write the BPP model in a similar form to better understand how it compares with our model.

2.1 General model specification

Following Friedman and Kuznet (1945), household income is typically assumed to have a random walk permanent component, a transitory component that dies away, and zero correlation between movements in the two components – see, for example, Moffitt and Gottschalk (2002), Storesletten, Telmer, and Yaron (2004), Guvenen (2007), Blundell, Pistaferri, and Preston (2008), Primiceri and van Rens (2009), Low, Meghir, and Pistaferri (2010), and Heathcote, Perri, and Violante (2010), among many others. However, it is straightforward to show that, if the zero correlation assumption is incorrect, the model misspecification will bias the estimate of permanent risk, a key ingredient in heterogeneous agent quantitative macro models.[2] In the time series literature using aggregate U.S. quarterly real GDP data, Morley, Nelson, and Zivot (2003) clearly establish that the assumption of zero correlation between permanent and transitory movements can be rejected in the univariate case, while Morley (2007) finds evidence in favor

[2] For example, see Ejrnaes and Browning (2014) for more details.

of correlated movements using U.S. quarterly real GDP and consumption data in a multivariate unobserved components model.[3] Motivated by these results, the general model presented below allows for correlated movements in unobserved components of income and consumption, with random walk permanent components and possibly persistent dynamics for the transitory components.

Our panel unobserved components model decomposes idiosyncratic income and consumption for household i (measured as residuals from regressions of household income and consumption on common observed factors) into permanent components and transitory deviations from the permanent components:[4]

$$y_{i,t} = \tau_{i,t} + (y_{i,t} - \tau_{i,t}), \tag{1}$$

$$c_{i,t} = \gamma_\eta \tau_{i,t} + \kappa_{i,t} + (c_{i,t} - \gamma_\eta \tau_{i,t} - \kappa_{i,t}). \tag{2}$$

The permanent components are specified as random walks with possible drift:

$$\tau_{i,t} = \mu_{\tau,i} + \tau_{i,t-1} + \eta_{i,t}, \qquad \eta_{i,t} \sim i.i.d.N(0, \sigma_\eta), \tag{3}$$

$$\kappa_{i,t} = \mu_{\kappa,i} + \kappa_{i,t-1} + u_{i,t}, \qquad u_{i,t} \sim i.i.d.N(0, \sigma_u), \tag{4}$$

where for household i, the common stochastic trend of income and consumption is $\tau_{i,t}$ and $\kappa_{i,t}$ is the additional trend of consumption. In our specification, γ_η captures the impact of permanent income shocks on permanent consumption.

The transitory components are specified as ARMA(p,q) processes:

$$\phi_y(L)(y_{i,t} - \tau_{i,t}) = \lambda_{y\eta}\eta_{i,t} + \theta_y(L)\epsilon_{i,t}, \tag{5}$$

[3]Note, however, Morley (2007) considers total income for the aggregate data, not just labor income, as is considered for household data in this paper.

[4]In particular, following BPP, we compute idiosyncratic income and consumption for households by removing the impact of observables such as education, race, family size, number of children, region, employment status, year and cohort effects, residence in large city, and income recipients other than husband and wife from total household disposable labor income.

$$\phi_c(L)(c_{i,t} - \gamma_\eta \tau_{i,t} - \kappa_{i,t}) = \lambda_{c\eta}\eta_{i,t} + \lambda_{c\epsilon}\epsilon_{i,t} + \theta_c(L)v_{i,t}, \tag{6}$$

where $\phi_j(L) = (1 - \phi_{j,1}L - \phi_{j,2}L^2 - ... - \phi_{j,p}L^p)^{-1}$ and $\theta_j(L) = (1 - \theta_{j,1}L - \theta_{j,2}L^2 - ... - \theta_{j,q}L^q)^{-1}$ for $j = \{y, c\}$ are lag polynomials that satisfy stationarity and invertibility constraints, respectively.[5]

The permanent income shock, $\eta_{i,t}$, can be interpreted as reflecting shocks to health, promotion, or other idiosyncratic factors that result in an idiosyncratic change in permanent income. Other permanent shocks to consumption, $u_{i,t}$, beyond permanent shocks to income could be taste and preference shocks or other shocks to non-labor income, such as wealth shocks. The transitory income shock is $\epsilon_{i,t} \sim i.i.d.N(0, \sigma_\epsilon)$ while the transitory consumption shock is $v_{i,t} \sim i.i.d.N(0, \sigma_v)$, where the latter could capture measurement error which could be due to the imputation of non-durable consumption. We note that the model assumes time-invariant volatilities of shocks, although it is relatively easy to test for and allow structural breaks in these parameters.[6]

Instead of directly specifying shocks to be correlated across equations, as in Morley, Nelson, and Zivot (2003) and Morley (2007), we assume that shocks are orthogonal, but permanent shocks can affect the transitory components according to impact coefficients $\lambda_{y\eta}$ and $\lambda_{c\eta}$. Thus, permanent and transitory movements will be correlated as in Morley, Nelson, and Zivot (2003) and Morley (2007). However, following Morley and Singh (2016), we explicitly model the basis of this correlation as being due to the effects of permanent shocks on transitory components. Meanwhile, $\lambda_{c\epsilon}$ captures the response of consumption to transitory income shocks. For

[5]In this paper we assume that income and consumption shocks are drawn from a normal distribution. Recent papers, for example Madera (2016), study the joint distribution of income and durable and non-durable consumption by examining tails of the distributions.

[6]For example, BPP look at changes in income and consumption inequality over time using the same panel dataset. We leave such analysis for future research and focus on estimating average levels of income risk and degree of consumption insurance over the full sample period. However, we note that preliminary subsample analysis suggests that the full sample estimates are generally very close to averages of the subsample estimates. These results are available from the authors upon request.

simplicity, we assume no corresponding effect of transitory consumption shocks on income.

Based on our panel UC model, we can solve for consumption growth for household i as follows:

$$\Delta c_{i,t} = \gamma_\eta \eta_{i,t} + u_t + (1-L)\phi_c(L)^{-1}(\lambda_{c\eta}\eta_{i,t} + \lambda_{c\epsilon}\epsilon_{i,t} + \theta_c(L)v_{i,t}), \qquad (7)$$

which suggests that changes in consumption depend on the full history of permanent shocks to income.[7]

To calculate the implied consumption insurance based on our model, a change in consumption at date t due to the permanent income shock η_t is $\gamma_\eta + \lambda_{c\eta}$. Therefore, the consumption insurance coefficient is

$$\vartheta_c = 1 - (\gamma_\eta + \lambda_{c\eta}). \qquad (8)$$

Note that KV define the insurance coefficient with respect to permanent income shock as the share of the variance of the shock that does not translate into consumption growth such that

$$\vartheta_c = 1 - \frac{cov(\Delta c_t, \eta_t)}{var(\eta_t)}, \qquad (9)$$

which can be shown to be $1 - (\gamma_\eta + \lambda_{c\eta})$ for our panel UC model. See Appendices A and B for the state-space representation of the panel UC model and the implied variances that allow us to see that the analytical expression for equation (9) is the same as equation (8).

2.2 BPP model

We re-write the BPP model in a similar form to our model for the purpose of comparison (the state-space representation and implied variances for the BPP model are also given in Appendices). In particular, the BPP model has an implicit UC representation for income:

[7] In KV's terminology, this means that there is no "short memory" in our model.

$$y_{i,t} = \tau_{i,t} + (y_{i,t} - \tau_{i,t}). \tag{10}$$

The permanent component of income is specified as follows:

$$\tau_{i,t} = \mu_i + \tau_{i,t-1} + \eta_{i,t}, \qquad \eta_{i,t} \sim i.i.d.N(0, \sigma_\eta). \tag{11}$$

The transitory component has a moving average, in particular, an $MA(1)$, specification as follows:

$$(y_{i,t} - \tau_{i,t}) = \epsilon_{i,t} + \theta\epsilon_{i,t-1}, \qquad \epsilon_{i,t} \sim i.i.d.N(0, \sigma_\epsilon). \tag{12}$$

Meanwhile, consumption growth is given by the following process:

$$\Delta c_{i,t} = \gamma_\eta \eta_{i,t} + \gamma_\epsilon \epsilon_{i,t} + u_{i,t} + \Delta u^{\star}_{i,t}, \qquad u_{i,t} \sim i.i.d.N(0, \sigma_u), \tag{13}$$

where η_t and ϵ_t are the permanent and transitory income shocks, u_t is the permanent shock to consumption, and $u^{\star}_{i,t} \sim i.i.d.N(0, \sigma_{u^\star})$ is measurement error for consumption. As discussed in BPP, the permanent shock to consumption captures taste and preference shocks, while the measurement error reflects errors of imputing nondurable consumption for the PSID.

Our panel UC model differs from the BPP specification in one key way. In particular, a transitory income shock can only impact transitory consumption in our model, while in the BPP model, transitory income shocks are assumed to have a completely permanent impact on consumption. To see this, we can rewrite the level of consumption, after suppressing the individual specific subscript for simplicity, as

$$c_t = \gamma_\eta \tau_t + \gamma_\epsilon Z_{\epsilon,t} + Z_{u,t} + u^{\star}_{i,t}, \tag{14}$$

$$Z_{\epsilon,t} = Z_{\epsilon,t-1} + \epsilon_t, \tag{15}$$

$$Z_{u,t} = Z_{u,t-1} + u_t, \tag{16}$$

In section 4, we examine which specification has more support in the data.

3 Data

In this section we briefly describe the novel dataset constructed by BPP and look at sample autocorrelations in income and consumption growth to help motivate our model specification in the next section. For full details of the dataset, we refer the reader to the BPP paper.

3.1 BPP dataset

BPP use the Panel Study of Income Dynamics (PSID) sample from 1978-1992 of continuously married couples headed by a male (with or without children) age 30 to 65. The income variable is family disposable income which includes transfers. They adopt a similar sample selection in the Consumer Expenditure Survey (CEX). Since CEX has detailed nondurable consumption data, unlike PSID which primarily has food expenditure data, they impute nondurable consumption for each household per year by using the estimates of the food demand from CEX. The constructed dataset is a panel of income and imputed nondurable consumption. To get idiosyncratic (residual) income and consumption, BPP regress income and consumption for households on a vector of regressors including demographic and ethnic factors and other income characteristics observable/known by consumers. It is this residual idiosyncratic income and consumption that is modeled in section 2.

3.2 Sample autocorrelations

To help motivate our model specification in the next section, we compute the sample autocorrelation function (ACF) and partial autocorrelation function (PACF) for idiosyncratic income and consumption growth from the BPP dataset by pooling individuals of all ages and over all years. Table 1 reports the results.

Based on the sample ACFs and PACFs, we can see that the ACFs completely die off after 1 lag, but the PACFs die off more gradually for both

TABLE 1. SAMPLE ACF AND PACF

a_1	a_2	a_3	p_1	p_2	p_3
		Δy			
−0.29	−0.03	−0.01	−0.29	−0.13	−0.07
		Δc			
−0.34	−0.01	−0.02	−0.34	−0.14	−0.04

Notes: Autocorrelations and partial autocorrelations are calculated using 12041 observations.

income growth and consumption growth. This pattern is consistent with an MA(1) process, not an MA(2) process, as would be implied for income growth by the BPP model. Moreover, this pattern is suggestive of a simple specification for the general panel UC model in section 2. In particular, it is consistent with a simple model in which both income and consumption follow random walk permanent components plus noise for their respective transitory components. We start with this specification, but also conduct formal model comparison to determine the preferred specification.

4 Empirical Results

Instead of relying on only certain moments to estimate key parameters of the permanent-transitory model of income and consumption, we make use the entire likelihood for our estimation. A clear benefit of this full information approach is that it addresses possible extreme sensitivity of inferences to particular moments. For example, in the idiosyncratic income/wage risk literature Heathcote, Perri and Violante (2010) find that the estimates of the variance of the wage shocks are different whether one uses moment conditions based on log residual wage growth or moment conditions based on log residual wage level.[8] Using the panel dataset of residual income and consumption, we report our empirical results using a full information approach in this section.

4.1 Maximum likelihood estimates for a simple version of the model

Motivated by the sample autocorrelations, we estimate a simple version of the model in section 2 using maximum likelihood. For this simple model, which we refer to as the UC-WN model hereafter, the transitory components are assumed to have no persistence (i.e., the ϕ's in the general model in section 2 are set to zero) and, for simplicity, we only consider a spillover from permanent income to permanent consumption, as captured by γ_η (i.e., the λ's in the general model are set to zero). Despite having only a short time series for each individual in the sample, with a maximum time dimension of T=14, maximum likelihood estimation is feasible for this model because each observation is effectively treated as an independent draw from the data generating process, making only the total sample size TN relevant for precision of inference rather than T mattering separately in addition to N for identifying parameters, as would be the case when there is implied dependence between observations across time.

Based on Table 2, the variance of the permanent income shock is 0.02 (0.13^2). This is similar to the estimate in BPP and is also close to what one finds in the related idiosyncratic income risk literature. However, what is striking is that, using the same dataset as BPP, but taking a full information approach, our estimate of consumption insurance, $1 - \gamma_\eta$ is 0.58 while the corresponding estimate in BPP is 0.36. This result provides some support

[8]This inconsistency between the estimates has been reported by Brzozowski, Gervais,Klien and Suzuki (2010) for Canada; Fuchs-Schundeln, Krueger and Sommer (2010) for Germany, Domeij and Floden (2010) for Sweden and Chatterjee, Singh and Stone (2016) for Australia. However, due to differences in the dataset, sample selection and the estimated equation, our results may not be directly address the level versus growth puzzle. See Daly, Hryshko and Manovskiis (2016) who study this puzzle much more closely.

TABLE 2. MAXIMUM LIKELIHOOD ESTIMATES

	INCOME
σ_η	0.13 (0.002)
σ_ϵ	0.20 (0.002)
	CONSUMPTION
σ_u	0.09 (0.002)
σ_v	0.28 (0.002)
γ_η	0.42 (0.017)

Notes: The table reports maximum likelihood estimates with standard deviations reported in parentheses for UC-WN model.

for KV, who argue that BPP estimate of consumption insurance is biased downward.

It is not clear based on these results alone whether the differences in estimates are because of differences in model specification or because of the full information approach versus a partial information moments-based approach. Moreover, despite the suggestion from the autocorrelations, it is not clear that the UC-WN model is really the best specification for the BPP dataset. We address these issues in the subsequent subsections and employ Bayesian methods to do so. We take a Bayesian approach because the general panel UC model in section 2 can suffer from weak identification or maximum likelihood estimation may even be infeasible given a small time dimension T when the transitory components of income and consumption are persistent, thus inducing strong dependence in observations for each individual household across time. By imposing reasonable priors for parameters based on past studies and *a priori* reasoning, we are able to estimate specifications of the panel UC model that imply time dependence, as well as compute marginal likelihoods in order to determine the preferred model specification. Another advantage of the Bayesian approach is that we can easily make statistical inferences about the implied variances of idiosyncratic income and consumption growth, which are complicated functions of the model parameters, and compare these inferences directly with the corresponding sample moments.

4.2 Bayesian model comparison

Some of the literature on earnings has moved away from a simple model in which the permanent component is a random walk and the transitory component is white noise (i.e., the UC-WN model) in recent years. It is often believed that the earnings dynamics are more complicated. For example, MaCurdy (1982) and Abowd and Card (1982) find that the covariance matrix of earnings differences fits an MA(2), Gottschalk and Moffitt (1994) fit random walk plus ARMA(1,1) in levels which is an ARMA(1,2) in first differences, and Heathcote, Storesletten and Violante (2010) employ a very persistent "permanent" component and a white noise transitory component. We focus on two main specifications of the general model discussed in section 2.1, a UC-AR(2) model and the more traditional UC-WN model to encompass the main differences in views held in the literature.[9] Motivated by the findings for persistent autoregressive dynamics in the aggregate data found in the time series literature, we also investigate whether these dynamics play an important role in the household data and if income and consumption share a common trend, as they appear to do in the aggregate data.

We estimate our panel UC models using Bayesian posterior simulation based on Markov-chain Monte Carlo (MCMC) methods. We use multi-block random-walk chain version of the Metropolis-Hastings (MH) algorithm with 20,000 draws after a burn-in of 20,000 draws. To check the robustness of our posterior moments, we use different starting values. Our prior distributions are loosely motivated by the vast empirical literature on modeling income and consumption dynamics. First, the priors for the precisions (inverse variances) are $\Gamma(2.5, 2.5)$. Meanwhile, because there is no

[9]Following Morley, Nelson, Zivot (2003), the UC-AR(2) model is identified because $p = q + 2$ for the implied ARMA(p,q) process in first differences.

consensus in the literature regarding the estimate of the impact of permanent income shock on consumption, we choose an uninformative $U(0,1)$ prior for γ_η. The priors for the impact coefficients, $\lambda_{y\eta}$, $\lambda_{c\eta}$, and $\lambda_{c\epsilon}$ are $TN_{[-1,1]}(0, 0.5^2)$ – i.e., they are truncated to ensure that they lie between -1 and 1. The priors for autoregressive and moving-average coefficients are $TN_{|z|>1,\phi(z)=0}(0, 0.5^2)$ and $TN_{|z|>1,\theta(z)=0}(0, 0.5^2)$ – i.e., they are truncated to ensure stationarity or invertibility.

Table 3 reports results for our different model specifications. First, estimates for the full UC-AR(2) model in column 2 suggest no persistent transitory dynamics, but permanent income shocks have an immediate positive impact on transitory income. This stands in contrast to some other studies, such as Hyrshko (2010) and Belzil and Bognanno (2008), which find a negative correlation between the permanent and transitory shocks. For both the income and consumption processes, transitory shocks are more volatile compared to permanent shocks. The implied variances of income and consumption closely match their corresponding counterparts in the data, 0.09 and 0.16. Second, when we shut down the additional permanent shocks to consumption beyond permanent income shocks in column 3, the implied variance of consumption is much lower than the variance of consumption in the data. Third, when we set the impact coefficients to zero in column 4, we find relatively similar estimates for the other parameters, as for the full UC-AR(2) model in column 2. Fourth, when we shut down all dynamics and set all impact coefficients to zero in column 5, which corresponds to the UC-WN model, we find, again, that the estimates of the remaining parameters remain similar to the full UC-AR(2) model. In particular, the variance of income shocks and transitory shocks to consumption, as well as the pass-through of the permanent income shock to consumption, are quite similar across specifications.

Using Bayesian methods, we can easily compare different models to determine which model is supported by the data. We do so by computing the marginal likelihood following the method in Chib and Jeliazkov (2001). The last row in Table 3 clearly shows that the UC-WN model is

TABLE 3. ESTIMATES OF PANEL UC MODELS

	UC-AR(2)			UC-WN
	full	$\sigma_u = 0$	$\lambda = 0$	
		INCOME		
$\phi_{y,1}$	-0.02 (0.01)	-0.10 (0.02)	-0.08 (0.01)	
$\phi_{y,2}$	-0.05 (0.01)	-0.10 (0.02)	-0.11 (0.01)	
σ_η	0.14 (0.01)	0.14 (0.01)	0.14 (0.01)	0.14 (0.01)
σ_ϵ	0.16 (0.02)	0.16 (0.02)	0.16 (0.02)	0.17 (0.01)
$\lambda_{y\eta}$	0.11 (0.01)	0.10 (0.01)		
		CONSUMPTION		
$\phi_{c,1}$	-0.12 (0.01)	-0.37 (0.01)	-0.30 (0.01)	
$\phi_{c,2}$	-0.07 (0.01)	-0.32 (0.01)	-0.20 (0.01)	
σ_u	0.13 (0.01)		0.14 (0.01)	0.13 (0.01)
σ_v	0.20 (0.02)	0.29 (0.02)	0.19 (0.02)	0.21 (0.02)
$\lambda_{c\eta}$	0.05 (0.01)	0.02 (0.01)		
$\lambda_{c\epsilon}$	-0.03 (0.01)	-0.02 (0.01)		
γ_η	0.47 (0.02)	0.43 (0.02)	0.47 (0.02)	0.47 (0.02)
		IMPLIED VARIANCE		
Δy	0.08 (0.00)	0.08 (0.00)	0.08 (0.00)	0.08 (0.00)
Δc	0.16 (0.00)	0.12 (0.00)	0.17 (0.00)	0.15 (0.00)
		MARGINAL LIKELIHOOD (IN LOGS)		
	-89595	-110416	-90295	-89041

Notes: The table reports posterior means of panel UC model parameters with posterior standard deviations reported in parentheses. The third panel reports the variance of residual income and residual consumption growth implied by the model and the marginal likelihood is in the bottom panel. The total number of households are 1765.

preferred.[10] Note that the estimates of the key parameters of interest in the last column of Table 3, the permanent shock to income and the consumption insurance, are similar to the maximum likelihood estimates.

Based on our preferred model the variance of the permanent income shock is 0.02. Most importantly, we find that using an uninformative prior, our estimate of consumption insurance is 17 percentage points higher than what was previously estimated by BPP. This fact that our estimate of consumption insurance is higher than in BPP is consistent with KV, who argue that the BPP estimate of consumption insurance is biased downward. Moreover, our results of higher consumption insurance are also in line with Heathcote, Storesletten, and Violante (2014), who take a more structural approach. Finally, the implied volatilities of income and consumption growth for our preferred model are 0.08 and 0.15. The corresponding counterparts in the BPP dataset are 0.09 and 0.16.

Our analysis so far suggests that the UC-WN model is the preferred specification for the BPP data. In the next subsection we examine whether the estimates that we find are different due to differences in model specification or due to differences in approach – i.e., a full information approach versus a partial information approach.

4.3 Bayesian estimates for the BPP model and prior sensitivity analysis

In this subsection, we employ the same full information approach as in subsection 4.2 to estimate the BPP model. The priors for the additional parameters, θ and γ_ϵ are $TN_{[-1,1]}(0, 0.5^2)$, i.e., they are truncated to lie between -1 and 1.[11] In addition, we conduct prior sensitivity analysis by

[10] These results stand in contrast to those for the aggregate data in Morley (2007), although this is perhaps not surprising given that common shocks have been removed from the data and idiosyncratic shocks are likely due to very different factors with different behaviors than the common shocks that drive the aggregate data. Also, we are using annual data, while Morley (2007) considers quarterly data.

[11] All the other priors are the same as for our panel UC model.

TABLE 4. BPP ESTIMATES USING BAYESIAN APPROACH

Prior γ_η	$TN_{[-1,1]}(0.65, 0.25^2)$	$TN_{[-1,1]}(0.65, 1^2)$	$U(0,1)$	BPP estimate/data
		INCOME		
θ	0.01(0.01)	0.00(0.01)	−0.04(0.02)	0.11
σ_η	0.14(0.01)	0.14(0.01)	0.15(0.01)	0.14
σ_ϵ	0.17(0.01)	0.17(0.01)	0.17(0.02)	0.17
		CONSUMPTION		
γ_η	0.64(0.01)	0.55(0.02)	0.46(0.02)	0.64
γ_ϵ	0.002(0.00)	−0.02(0.01)	−0.01(0.01)	0.05
σ_u	0.13(0.01)	0.13(0.01)	0.13(0.01)	0.11
$\sigma_{u^\star}$	0.21(0.02)	0.21(0.02)	0.21(0.02)	NA
		IMPLIED VERSUS ACTUAL VARIANCE		
Δy	0.08(0.00)	0.08(0.00)	0.08(0.00)	0.09
Δc	0.11(0.00)	0.11(0.00)	0.11(0.00)	0.16

Notes: The table reports posterior means of model parameters with posterior standard deviations reported in parentheses. The bottom panel reports the variance of residual income and residual consumption growth implied by the model versus the corresponding averages in the BPP data.

varying the prior on γ_η in particular to investigate its role in determining the posterior estimate.

Table 4 reports the estimates of the BPP model using Bayesian methods. It can be seen that our estimation method can only recover the volatility of income shocks and the consumption insurance parameter from BPP when the prior on γ_η is tight around the BPP estimate.[12] However, when the prior is less informative in the case of $TN_{[-1,1]}(0.65, 1^2)$, the estimate moves towards higher consumption insurance and similar to what we find with a uniform prior for our preferred UC-WN model. Note that, in the last row of Table 4, the implied variance of residual consumption growth is 0.11, while the variance of residual consumption growth in the BPP sample is 0.16. This seems plausible, as Figure 5 in BPP suggests that the process of consumption growth implied by their baseline model does not match

[12]Note that the estimate of the standard deviation of the measurement error is not reported in Table 6 of BPP.

the data all that well in the latter part of the sample. Finally, the marginal likelihood from the BPP model where γ_η is uniformly distributed, column 4 in Table 4, is -89946. This suggests that our UC-WN model provides a better fit to the BPP dataset given a higher marginal likelihood.[13]

In this subsection, we have demonstrated that using the full information approach we can estimate the BPP model. Based on the prior sensitivity analysis, we find that consumption insurance is not particularly well identified in the data. However, it is necessary to impose a highly informative and distorted prior to obtain the lower estimates previously found with partial information methods.

5 Subgroup Estimates

In this section, we examine how estimates vary across different groups of households based on education and age. Because our results regarding model specification are robust to different types of households, we report results for UC-WN specification only in Tables 5 and 6.

From Table 5, the pass-through of permanent income shocks to consumption is 34 percent for the college educated, which is approximately half of the pass-through for the households with no college education. Qualitatively, these results are similar to BPP, although the magnitudes are different. KV find that the downward bias in consumption insurance using BPP estimator is much more pronounced for households that are borrowing constrained and our results seem consistent with this result. In particular, households without college education are more likely to be more borrowing constrained than households with college education and we find that our estimate of consumption insurance for no college group is 6 times higher than BPP (0.35 for our estimate versus 0.06 in BPP), while

[13]Using simulated data, we find that the full information approach performs reasonably well when there is model misspecification. In particular, if the DGP is UC-WN and we fit a BPP model, our full-information approach recovers the true parameters quite well. See Appendix C for more details.

Table 5. Education heterogeneity

	No college	College
	Income	
σ_η	0.14 (0.02)	0.14 (0.02)
σ_ϵ	0.18 (0.02)	0.16 (0.02)
	Consumption	
σ_u	0.13 (0.02)	0.13 (0.02)
σ_v	0.24 (0.03)	0.19 (0.02)
γ_η	0.65 (0.03)	0.34 (0.02)

Notes: The table reports posterior means of model parameters with posterior standard deviations reported in parentheses. There are 883 households in the no college group and 882 in the college group.

Table 6. Age heterogeneity

	Young (30-47)	Old (48-65)
	Income	
σ_η	0.13 (0.01)	0.14 (0.02)
σ_ϵ	0.15 (0.01)	0.19 (0.02)
	Consumption	
σ_u	0.13 (0.01)	0.13 (0.02)
σ_v	0.21 (0.02)	0.20 (0.02)
γ_η	0.55 (0.03)	0.40 (0.03)

Notes: The table reports posterior means of model parameters with posterior standard deviations reported in parentheses. There are 1413 households for the young while the number of households for the old is 708.

the apparent downward bias is not so large for households with college education (0.66 for our estimate versus 0.58 in BPP).

Estimating our model for subgroups based on age, we find that the results are again intuitive. From Table 6, we can see that γ_η for the old is 0.40, while it is 0.55 for the young. This implies that older households insure their consumption against fluctuations in income more relative to younger households. BPP mention that they find some evidence of an age profile in their estimates of the consumption insurance parameter, although their estimates are imprecise.

Because the total sample size, TN, is smaller for the subgroups, we examine the sensitivity of our results to smaller sample size via simulations (see Table C2 in Appendix C). Our full information approach does reasonably well, but the estimate of γ_η has some apparent upward bias. Therefore our estimates of consumption insurance using the actual data can be seen to provide a lower bound for the true consumption insurance. Notably, our preferred UC-WN specification, consumption insurance is 53 percent, while Heathcote, Storesletten, and Violante (2014) find consumption insurance to be close to 60 percent in a structural model of household income and consumption.

6 Conclusion

In this paper, we have followed BPP and others by taking a reduced-form approach that associates permanent income with the random walk stochastic trend in income and examining the correlation of movements in consumption with permanent income. However, unlike BPP, we consider full information estimation of the parameters for our panel UC model. By definition, this approach eliminates sensitivity of results to the choice of which moments to consider in estimation. It also allows for Bayesian model comparison to determine the most appropriate model specification in practice. As a result, we have a statistical model of household income and consumption that is consistent with the data and allows us to estimate the empirical relationship between permanent income and permanent consumption given the link between the statistical notion of permanent components and households' notions under rational expectations.

Our full information approach to estimation for a panel UC model could be useful for estimating more structural models of household income and consumption that would still be consistent with the data in a way that is analogous to Smets and Wouters' (2003) estimation of a DSGE model of aggregate data. However, our reduced-form approach has the benefit of potentially being robust across different structural assumptions, which makes it more analogous to VAR modeling of aggregate data – see, for example, Fernandez-Villaverde et al. (2007) on this point, although the general idea relates back at least to Sims (1980). Thus, we see the estimates for our reduced-form model as providing guidance for and a means of evaluating more structural models. But we leave more structural analysis using full information inference to future research.

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A State-Space Representations

In this appendix, we present the state-space representations for our general panel UC model and for the BPP model.

The state-space representation for our panel UC model is standard. The observation equation is

$$\tilde{\mathbf{y}}_t = \mathbf{H}\,\boldsymbol{\beta}_t$$

where

$$\tilde{\mathbf{y}}_t = \begin{bmatrix} y_t \\ c_t \end{bmatrix},\ \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & \gamma_\eta & 1 \end{bmatrix} \text{ and } \boldsymbol{\beta}_t = \begin{bmatrix} y_t - \tau_t \\ y_{t-1} - \tau_{t-1} \\ \tau_t \\ c_t - \tau_t \\ c_{t-1} - \tau_{t-1} \\ \kappa_t \end{bmatrix}$$

The state equation is

$$\boldsymbol{\beta}_t = \boldsymbol{F}\boldsymbol{\beta}_{t-1} + \tilde{\boldsymbol{v}}_t$$

where

$$\mathbf{F} = \begin{bmatrix} \phi_{y,1} & \phi_{y,1} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{c,1} & \phi_{c,2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},\ \tilde{\boldsymbol{v}}_t = \begin{bmatrix} \lambda_{y\eta}\eta_t + \epsilon_t \\ 0 \\ \lambda_{c\eta}\eta_t + \lambda_{c\epsilon}\epsilon_t + v_t \\ 0 \\ \eta_t \\ u_t \end{bmatrix}$$

and the covariance matrix of $\tilde{\boldsymbol{v}}_t$, $\mathbf{Q}$, is given by

$$\mathbf{Q} = \begin{pmatrix} \lambda_{y\eta}^2\sigma_\eta^2 + \sigma_\epsilon^2 & 0 & \lambda_{y\eta}\lambda_{c\eta}\sigma_\eta^2 + \lambda_{c\epsilon}\sigma_\epsilon^2 & 0 & \lambda_{y\eta}\sigma_\eta^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{y\eta}\lambda_{c\eta}\sigma_\eta^2 + \lambda_{c\epsilon}\sigma_\epsilon^2 & 0 & \lambda_{c\eta}^2\sigma_\eta^2 + \lambda_{c\epsilon}\sigma_\epsilon^2 + \sigma_v^2 & 0 & \lambda_{c\eta}\sigma_\eta^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{y\eta}\sigma_\eta^2 & 0 & \lambda_{c\eta}\sigma_\eta^2 & 0 & \sigma_\eta^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_u^2 \end{pmatrix}$$

For the BPP model, the observation equation is

$$\tilde{\mathbf{y}}_t = \mathbf{H}\,\boldsymbol{\beta}_t$$

where

$$\tilde{\mathbf{y}}_t = \begin{bmatrix} y_t \\ c_t \end{bmatrix},\ \mathbf{H} = \begin{bmatrix} 1 & \theta & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \gamma_\epsilon & \gamma_\eta & 1 \end{bmatrix} \text{ and } \boldsymbol{\beta}_t = \begin{bmatrix} \epsilon_t \\ \epsilon_{t-1} \\ u_t^\star \\ Z_{\epsilon,t} \\ \tau_t \\ Z_{u,t} \end{bmatrix}$$

The state equation is

$$\boldsymbol{\beta}_t = \boldsymbol{F}\boldsymbol{\beta}_{t-1} + \tilde{\boldsymbol{v}}_t$$

where

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},\ \tilde{\boldsymbol{v}}_t = \begin{bmatrix} \epsilon_t \\ 0 \\ u_t^\star \\ \epsilon_t \\ \eta_t \\ u_t \end{bmatrix}$$

and the covariance matrix of $\tilde{\boldsymbol{v}}_t$, $\mathbf{Q}$, is given by

$$\mathbf{Q} = \begin{pmatrix} \sigma_\epsilon^2 & 0 & 0 & \sigma_\epsilon^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{u^\star}^2 & 0 & 0 & 0 \\ \sigma_\epsilon^2 & 0 & 0 & \sigma_\epsilon^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_\eta^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_u^2 \end{pmatrix}$$

B Implied Variances

In this appendix, we derive formulas of the variances of income and consumption growth for our general panel UC model and for the BPP model.

For our UC model, income and consumption growth are given as follows:

$$\Delta y_t = \mu + \eta_t + z_t^y, \tag{B.1}$$

where $(1 - \phi_{y,1}L - \phi_{y,2}L^2)z_t^y = (1 - L)x_t^y$ and $x_t^y = \lambda_{y\eta}\eta_t + \epsilon_t$ and

$$\Delta c_t = \mu + \gamma_c \eta_t + z_t^c, \tag{B.2}$$

where $(1 - \phi_{c,1}L - \phi_{c,2}L^2)z_t^c = (1 - L)x_t^c$ and $x_t^c = \lambda_{c\eta}\eta_t + \lambda_{c\epsilon}\epsilon_t + v_t$.

We can then write a vector representation for z_t^y and z_t^c as

$$\mathbf{z}_t = \mathbf{K}\mathbf{z}_{t-1} + \mathbf{w}_t,$$

where

$$\mathbf{z}_t = \begin{bmatrix} z_t^y \\ z_{t-1}^y \\ z_t^c \\ z_{t-1}^c \\ x_t^y \\ x_t^c \end{bmatrix}, \mathbf{K} = \begin{bmatrix} \phi_{y,1} & \phi_{y,2} & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{c,1} & \phi_{c,2} & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mathbf{w}_t = \begin{bmatrix} x_t^y \\ 0 \\ x_t^c \\ 0 \\ x_t^y \\ x_t^c \end{bmatrix}.$$

Let $\mathbf{W}$ be the covariance matrix of $\mathbf{w}_t$, with the following non-zero entries: $\mathbf{W}[1,1] = \mathbf{W}[1,5] = \mathbf{W}[5,1] = \mathbf{W}[5,5] = \lambda_{y\eta}^2\sigma_\eta^2 + \sigma_\epsilon^2$, $\mathbf{W}[1,3] = \mathbf{W}[3,1] = \mathbf{W}[1,6] = \mathbf{W}[6,1] = \mathbf{W}[3,5] = \mathbf{W}[5,3] = \mathbf{W}[5,6] = \mathbf{W}[6,5] = \lambda_{y\eta}\lambda_{c\eta}\sigma_\eta^2$, and $\mathbf{W}[3,3] = \mathbf{W}[3,6] = \mathbf{W}[6,3] = \mathbf{W}[6,6] = \lambda_{c\eta}^2\sigma_\eta^2 + \sigma_v^2$.

Because the $vec(var(\mathbf{z}_t)) = (I - \mathbf{K} \otimes \mathbf{K})^{-1}vec(\mathbf{W})$, the unconditional variance of output growth is given by

$$\begin{aligned} var(\Delta y_t) &= var(\eta_t + z_t^y) \\ &= \sigma_\eta^2 + var(z_t^y) + 2cov(\eta_t, z_t^y) \\ &= \sigma_\eta^2 + var(z_t^y) + 2\lambda_{y,\eta}\sigma_\eta^2 \end{aligned}$$

where $var(z_t^y)$ is the $[1,1]$ element of $var(\mathbf{z}_t)$. Similarly, unconditional variance of consumption growth is given by

$$var(\Delta c_t) = var(\gamma_c \eta_t + z_t^c)$$
$$= \gamma_\eta^2 \sigma_\eta^2 + var(z_t^c) + 2cov(\eta_t, z_t^c)$$
$$= \sigma_\eta^2 + var(z_t^c) + 2\lambda_{c,\eta}\sigma_\eta^2$$

where $var(z_t^c)$ is the $[3,3]$ element of $var(\mathbf{z}_t)$.

For the BPP model, computing these variances is relatively simple. They are given as follows:

$$var(\Delta y_t) = \sigma_\eta^2 + \sigma_\epsilon^2(1 + \theta^2 - \theta) \tag{B.3}$$

since $\Delta y_t = \epsilon_t - \epsilon_{t-1} + \theta\epsilon_{t-1} - \theta\epsilon_{t-2} + \eta_t$.

Similarly,

$$var(\Delta c_t) = \gamma_\eta \sigma_\eta^2 + \gamma_\epsilon \sigma_\epsilon^2 + \sigma_u^2 + +2\sigma_{u^*}^2 \tag{B.4}$$

since $\Delta c_t = \gamma_\eta \eta_t + \gamma_\epsilon \epsilon_t + u_t + \Delta u_t^*$.

C Simulations

In this appendix, we first examine whether the full information approach can help us recover the key parameters of the income and consumption process that we are interested in such as the variance of the permanent shocks to income and consumption insurance when there is model misspecification. To consider this we simulate data from a UC-WN model and fit the BPP model where all the priors are the same as in section 4 and the prior on γ_η is uniformly distributed. We estimate the model using Bayesian methods.

Our approach does well in recovering the key parameters even when the model is misspecified. For example in the DGP, 45 percent of the permanent income shocks get transmitted to consumption, while our estimate using full information is 43 percent. Our estimate of permanent income

TABLE C1. ESTIMATES UNDER MODEL MISSPECIFICATION

	DGP	BPP model
		INCOME
θ		$-0.06(0.03)$
σ_η	0.14	0.15(0.02)
σ_ϵ	0.17	0.15(0.02)
		CONSUMPTION
γ_η	0.45	0.43(0.02)
γ_ϵ		0.005(0.00)
σ_u	0.13	0.15(0.02)
σ_{u^*}	0.21	0.17(0.02)

Notes: The table reports posterior means of model parameters, with posterior standard deviations in parentheses. For the simulation, N=700 and T=10.

risk is also not impacted by model misspecification. In the partial information moments based approach, it has been documented in the level versus growth moment conditions literature that model mis-specification can bias the estimate of permanent risk. See Domeij and Floden (2010) for more details.

Second, we examine the sensitivity of our results to small samples with N=700 and T=10. Because our preferred model is UC-WN and what matters is TN, we also report results for N=1400 and T=5. For both cases, we consider three values of γ_η. Table C2 reports our simulation results. Bayesian estimation is able to recover the true parameters quite well. In particular, the estimate of permanent income risk is close to the true parameter value and the estimate of γ_η appears to have an upward bias.

TABLE C2. IMPACT OF SAMPLE SIZE ON ESTIMATES FOR PANEL UC MODEL

			$T = 10, N = 700$			$T = 5, N = 1400$	
	DGP	$\gamma_\eta = 0.25$	$\gamma_\eta = 0.45$	$\gamma_\eta = 0.65$	$\gamma_\eta = 0.25$	$\gamma_\eta = 0.45$	$\gamma_\eta = 0.65$
				INCOME			
σ_η	0.14	0.14 (0.02)	0.14 (0.02)	0.14 (0.02)	0.14 (0.01)	0.14 (0.02)	0.14 (0.02)
σ_ϵ	0.17	0.15 (0.02)	0.15 (0.02)	0.16 (0.02)	0.15 (0.01)	0.15 (0.02)	0.15 (0.02)
				CONSUMPTION			
σ_u	0.13	0.15 (0.02)	0.15 (0.02)	0.15 (0.02)	0.15 (0.02)	0.15 (0.02)	0.15 (0.02)
σ_v	0.21	0.18 (0.02)	0.17 (0.02)	0.17 (0.02)	0.15 (0.02)	0.16 (0.02)	0.15 (0.02)
γ_η		0.28 (0.03)	0.47 (0.03)	0.72 (0.03)	0.30 (0.03)	0.50 (0.03)	0.75 (0.03)

Notes: The table reports posterior means of model parameters, with posterior standard deviations in parentheses. For the simulated data, total number of observations is always TN=7,000.