Economics Working Paper Series

2017 – 02

A regime-switching stochastic volatility model for forecasting electricity prices

Peter Exterkate and Oskar Knapik

January 2017

DEPARTMENT OF ECONOMICS AND BUSINESS ECONOMICS
AARHUS UNIVERSITY

CREATES
Center for Research in Econometric Analysis of Time Series

A regime-switching stochastic volatility model for forecasting electricity prices

Peter Exterkate and Oskar Knapik

CREATES Research Paper 2017-3

Department of Economics and Business Economics
Aarhus University
Fuglesangs Allé 4
DK-8210 Aarhus V
Denmark

Email: oekonomi@au.dk
Tel: +45 8716 5515

A regime-switching stochastic volatility model for forecasting electricity prices☆

O. Knapik[a,*], P. Exterkate[a,b]

[a] *Aarhus University, Department of Economics and Business Economics, CREATES - Center for Research in Econometric Analysis of Time Series, Fuglesangs Alle 4, 8210 Aarhus V, Denmark*
[b] *School of Economics, The University of Sydney, NSW 2006, Sydney, Australia*

Abstract

In a recent review paper, Weron (2014) pinpoints several crucial challenges outstanding in the area of electricity price forecasting. This research attempts to address all of them by i) showing the importance of considering fundamental price drivers in modeling, ii) developing new techniques for probabilistic (i.e. interval or density) forecasting of electricity prices, iii) introducing an universal technique for model comparison. We propose new regime-switching stochastic volatility model with three regimes (negative jump, normal price, positive jump (spike)) where the transition matrix depends on explanatory variables. Bayesian inference is explored in order to obtain predictive densities. The main focus of the paper is on short-time density forecasting in Nord Pool intraday market. We show that the proposed model outperforms several benchmark models at this task.

Keywords: Electricity prices, density forecasting, Markov switching, stochastic volatility, fundamental price drivers, ordered probit model, Bayesian inference, seasonality, Nord Pool power market, electricity prices forecasting, probabilistic forecasting
JEL: C22, C24, Q41, Q47

☆Acknowledgment: The authors appreciate support from Center for Research in Econometric Analysis of Time Series, CREATES funded by the Danish National Research Foundation with grant number DNRF78.

*Corresponding author
Email address: `oknapik@econ.au.dk, oknapik@gmail.com` (O. Knapik)

1. Introduction

Electricity is a unique commodity, characterized by a high variability. It cannot be stored and requires immediate delivery. The electricity end-user demand shows high variability and strong weather and business cycle dependence. Moreover events like power plant outages or transmission grid (un)reliability and complexity reduce predictability. The resulting electricity price series are characterized by strong seasonality at different levels (annual, weekly, daily and hourly). However, the most distinct feature of electricity prices is very high volatility and abrupt, short-lived and generally unanticipated extreme price changes known as spikes, or jumps (see Serati et al., 2008; Janczura et al., 2013, among others). Electricity prices from Nord Pool power market are also nonstationary (see Lisi and Nan, 2014; Weron, 2014) and often have the long-memory property (see Haldrup and Nielsen, 2006a,b).

There is a large body of literature on the topic (see Weron (2014) for recent review) showing that the need for realistic models of electricity price dynamics capturing its unique characteristics and adequate derivatives pricing techniques has not been fully satisfied. It is the aim of this paper address some of the crucial challenges pointed out in (Weron, 2014) specifically the use of fundamental price drivers in modeling and developing new tools for probabilistic forecasting of electricity prices.

When building a model for electricity prices, one of the crucial steps is to find an appropriate description of seasonal pattern. Moreover, electricity prices present various forms of nonlinear dynamics, the crucial one being the strong dependence of the variability of the series on its own past. Some nonlinearities of these series are a nonconstant variance, and generally they are characterized by the clustering of large shocks, or heteroskedasticity. It is well documented that electricity prices exhibit volatility clustering (see Karakatsani and Bunn (2008) among the others). The "spiky" character of electricity prices suggests that there exists a nonlinear switching mechanism between normal and low/high states, or regimes. The requirement of stochastic jump arrival probabilities directly leads to regime switching models. Markov regime-switching (MS) models seem to be a natural candidate for modeling such nonlinear and complex structure (see Andreasen and Dahlgren, 2006; Geman and Roncoroni, 2006; Handika et al., 2014; Heydari and Siddiqui, 2010; Huisman and Mahieu, 2003; Kanamura and Ohashi, 2008; Kosater and Mosler, 2006; Mount et al., 2006, among others). MS models are successfully applied by many researchers for electricity prices modeling (see Janczura and Weron, 2010a; Haldrup and Nielsen, 2006b, among others).

This paper introduces a new regime-switching stochastic volatility model, with a time-varying transition matrix that depends on explanatory variables. The core of the model is an autoregressive process with stochastic volatility error term. The main focus of this research is on short-time density forecasting of electricity prices. Although important, this topic is however barely touched in the electricity prices forecasting literature (see Weron, 2014). Serinaldi (2011) forecasts the distribution of electricity prices using a GAMLSS approach, but computes and discusses only predictive intervals. Huurman et al. (2012) consider GARCH-type time-varying volatility models and find that models that augmented with weather forecasts statistically outperform the ones without this information. They utilize the probability integral transform scores of the realization of the variables with respect to the forecast densities. Jónsson et al. (2014) develop a semi-parametric methodology for generating densities of day-ahead electricity prices in Western Denmark (Nord Pool).

The actuality and importance of electricity price forecasting is further evidenced by a special issue of the International Journal of Forecasting (Volume 32, Issue 3, Pages 585-1102, July–September 2016) dedicated to that topic, and by the energy forecasting competition organized by this journal. The competition and the current status of the probabilistic energy forecasting research is summarized in the paper of Hong et al. (2016). One of the presented approaches by Maciejowska et al. (2016) introduces new methodology involving quantile regression to average large numbers of point forecasts and principal component analysis (PCA) to extract the major factors driving the individual forecasts. Their approach outperforms both of benchmarks autoregressive exogenous (ARX) model and the quantile regression averaging (QRA) without PCA based on comprehensive evaluation with the unconditional coverage, the conditional coverage and the Winkler score.

The paper explores Bayesian inference in order to construct predictive densities for future electricity prices. We introduce also the electricity price modeling and forecasting literature a natural, universal method for model comparisons via predictive Bayes factors. Bayesian approaches have been used in the context of electricity prices modelling by several authors. Panagiotelis and Smith (2008) use a first order vector autoregressive model with exogenous effects and a skew t distributed disturbance for hourly Australian electricity spot prices. They use a Bayesian Markov Chain Monte Carlo approach in order to construct the predictive distribution of future spot prices. Smith (2010) proposes using Bayesian inference for a Gaussian stochastic volatility model with periodic autoregressions (PAR) in both the level and log-volatility process. They include demand and day types as exogenous explanatory variables in both the mean and log-volatility equations. They confirm that there is a nonlinear relationship between demand and mean prices and construct the predictive density of prices evaluated over a horizon of one week. Our work can be seen as extending this study in two ways, with a Markov-switching structure to flexibly accommodate such nonlinearities, and by allowing for many more predictors.

The paper is divided into five sections. Section 1 introduces. Section 2 describes the data and the main electricity price drivers. Section 3 presents the model and Bayesian inference. Section 4 shows the empirical and forecasting results. Section 5 concludes.

2. Data

The data set comes from Nordic power exchange, Nord Pool owned by the Nordic and Baltic transmission system operators, one of the leading power markets in Europe. We consider the information from two different markets within Nord Pool - day-ahead auction market Elspot and intraday market Elbas.

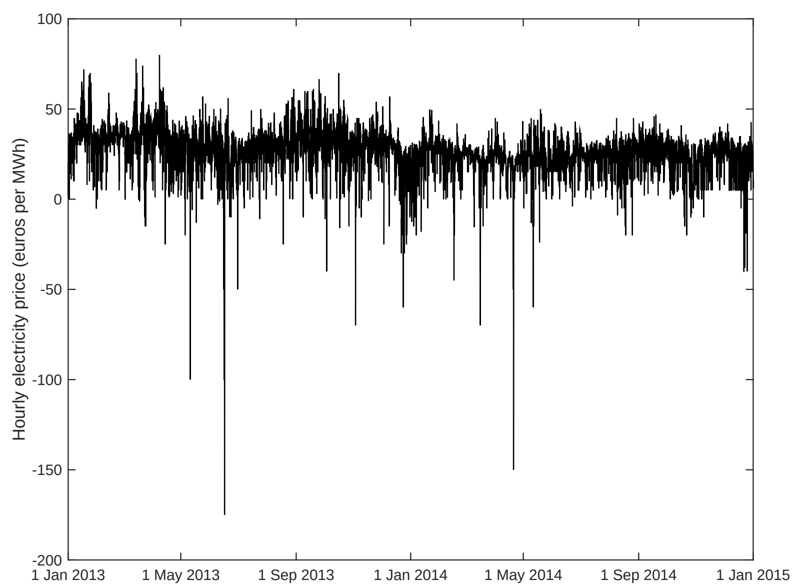
There is about 380 companies from 20 countries that trade on the Nord Pool Spot (Elspot) market including both producers and large consumers, for a trading volume of approximately 500 terawatt hours in 2015. Within the Nord Pool Spot, Elspot is the auction market for day-ahead electricity delivery. The Nord Pool Spot web-based trading system enables participants to submit bids and offers for each individual hour of the next day. Orders can be made between 08:00 and 12:00 a.m. Central European Time (CET). The aggregated buy and sell orders form demand and supply curves for each delivery hour of the next day. The intersection of the curves constitutes the system price for each hour (quoted per megawatt hour, MWh). The hourly prices are announced to the market at 12:42 CET and contracts are invoiced between buyers and sellers between 13:00 and 15:00. All 24 prices on day $t+1$ are determined on a given day t and released simultaneously. A detailed review of the operation of the market is given in (Nord Pool, 2016).

The intraday market, Elbas, supplements the day-ahead market and helps secure the necessary balance between supply and demand in the power market for Northern Europe. The majority of the volume handled by Nord Pool is traded on the day-ahead market. For the most part, the balance between supply and demand is secured here. However, incidents may take place between the closing of the day-ahead market at noon CET and delivery the next day. On the intraday market, buyers and sellers can trade volumes close to real time to bring the market back in balance. Elbas is a continuous market, and trading takes place every day around the clock until one hour before delivery. Prices are set based on a first-come, first-served principle, where best prices come first – highest buy price and lowest sell price. The Elbas market is becoming increasingly important as the amount of wind power entering the grid rises. Wind power is unpredictable by nature, and imbalances between day-ahead contracts and produced volume often need to be offset.

The data for the estimation period consist of a series of hourly observations of electricity prices in two Nord Pool markets: Elspot and Elbas. The study is conducted using hourly electricity prices for the whole area (system prices) from Elspot market $spot_t$ and corresponding hourly volume-weighted average prices from Elbas market y_t. The data covers the period from 1 January 2013 to 31 December 2014 (17520 observations). The focus of this research is on modeling and forecasting of electricity prices from the Elbas market. We will present and evaluate out-of-sample forecasts for 2 January 2015 (24 observations), the first working day after the in-sample period.

Figure 1 presents the hourly time series of volume-weighted electricity prices from Elbas market in the period 1/1/2013 till 31/12/2014. The prices exhibit typical characteristics, including seasonality and spikes.

Figure 1: Hourly electricity prices from Elbas power market, y_t



In order to model the hourly volume-weighted average prices from Elbas market y_t, we consider the following explanatory variables considered to be the main price drivers in the Nord Pool power market:

- hourly Elspot electricity system price, $spot_t$,
- turnover at system price from Elspot market, $load_t$,
- water reservoir level, res_t,
- heating degrees days, hdd_t,
- wind power production, $wind_t$,
- seasonal component, $seas_t$[1].

In the Nord Pool electricity market, about 53% of power production is generated from hydropower reservoirs. The influence of water reservoir levels in electricity prices at Nord Pool has been studied by (Gjolberg and Johnsen, 2001), (Botterud et al., 2002), (Førsund and Hoel, 2004) and (von der Fehr et al., 2005). The researchers conclude that hydropower reservoir levels are an important factor that explains futures and spot prices. The ability of the Nordic power system to store energy in hydro reservoirs causes less variation in the Nordic price structure, than that of for example Germany. Inflow during summer and in periods with low demand can be used in the winter. The data on hydropower reservoir levels are collected from the first week of 2013 to the end of the last week of 2014. Reservoirs are taken as a percentage of the total hydropower capacity available in the Nord Pool area. The reservoir levels and capacity data are from Norwegian Water Resources and Energy Directorate (NVE), Svensk Energi (Swedenergy AB), and the Finnish Environment Institute (SYKE). Reservoirs taken into account from Sweden and Finland are those after their integration in the Nord Pool market. The data is published on the weekly frequency[2]. The seasonality of reservoir levels has a highly important influence on electricity spot prices.

Temperature is the main price driver in the Nordic countries. Cold temperatures increase heat demand, since electricity is very much used for heating in the Nordic countries. Colder temperatures usually increase prices because of higher

[1] The seasonality component in the electricity price series is captured by sine and cosine terms taken with hourly, daily and weekly frequencies.

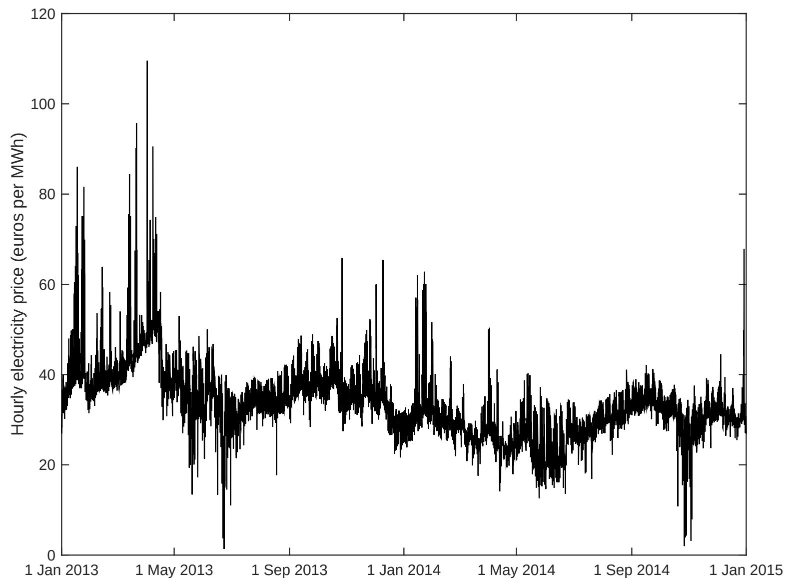
[2] The series of water reservoir levels is very regular. Therefore, the transformation from the weekly to hourly frequency is done by simple linear interpolation.

power demand. However, in special cases, combined heat and power plants where heat is the primary product, the demand for the heat could trigger secondary electricity production and causes the prices to decrease. The behavior of weather variables can also produce some predictable seasonal pattern in electricity prices. The relationship between weather variables and electricity load and price has been studied by many researchers. Li and Sailor (1995) and Sailor et al. (1998) show in a few US states that temperature is the most significant weather variable explaining electricity and gas demand. The influence of air temperature has also been described by other authors, who obtained a significant explicative power in their modeling; see, for example, (Peirson and Henley, 1994), (Peirson and Henley, 1998).Heating degree day (HDD) is a variable that shows the demand for energy needed for heating. It is taken from measurements of outside air temperature. The heating requirements for a specific structure at a specific place tend to be directly proportional to the number of HDD at that location. In this study we will consider average temperature measured on a daily basis in 13 Nordic cities (Oslo, Bergen, Trondheim, Tromsø, Helsinki, Sodankyla, Vaasa, Tampere, Stockholm, Göteborg, Östersund, Luleå and Copenhagen).

Wind power production is also an important electricity price driver. Due to the fact that there is no fuel cost for production and unpredictability, additional wind energy can lead to price decrease. This type of energy may in some cases cause even negative prices in hours with low demand and additional supply. On the other hand, when wind production falls short of expected values, it can trigger high prices, both in the Day-Ahead and Intra-day markets.

Finally, Elspot electricity market is another significant source of information about Elbas electricity prices. The same situation is in futures markets, where the basis is the difference between futures price and the underlying spot price. Figure 2 presents hourly system prices from Elspot power market.

Figure 2: Hourly electricity prices from Elspot market, $spot_t$



3. Model

The Markov regime-switching (MS) model allows for temporal dependence within the regimes, and in particular, for mean reversion. As the latter is a characteristic feature of electricity prices, it is important to have a model that captures this phenomenon. However, several modeling questions have to be addressed to build a proper MS model (see Janczura and Weron, 2010a).

First, the number of regimes has to be chosen. The important advantage of the MS models over the alternatives is no need for explicitly specifying threshold variable and level for the regimes, and therefore they are preferred for modeling risk purposes. There is no fundamental reason for considering specific number of regimes for electricity prices modeling (see Janczura and Weron (2010a)). However, almost all published papers consider only 2-regime models due to the computational convenience. Additional to the base regime, a spike (or excited) regime was introduced to capture extreme price behavior. Karakatsani and Bunn (2008) introduced third regime for capturing the most extreme prices. Also, the existence of an additional "down-spike" or "drop" regime can by justified for many of the very low prices. This research considers three regimes (down-spike (drop), normal/base and spike) Markov regime-switching stochastic-volatility model for electricity prices. We also introduce a data-driven mechanism of switching between different regimes in the form of an ordered probit model. Our approach enables to extend the number of regimes easily.

Secondly, the model defining the price dynamics in each of the regimes has to be selected. The base regime is usually modeled by a mean-reverting AR (see Ethier and Mount (1998), Deng (1999) among others) or diffusion model (for reviews see Huisman, 2009; Janczura and Weron, 2010b), which is sometimes heteroscedastic (Janczura and Weron, 2009). For the spike regime(s), on the other hand, a number of different specifications have been suggested in the literature, ranging from mean-reverting diffusions (Karakatsani and Bunn, 2008), to Gaussian (Huisman and de Jong, 2003; Liebl, 2013), lognormal (Weron et al., 2004), (Bierbrauer et al., 2004), exponential (Bierbrauer et al., 2007), heavy tailed (Weron, 2009) and non-parametric (Eichler and Türk, 2013) random variables, to mean-reverting diffusions with Poisson jumps (Arvesen et al., 2013; de Jong, 2006; Keles et al., 2012; Mari, 2008). One of the advantages of the regime-switching framework is that we can explicitly model the short-lived characteristics of power spike. We consider ARX-SV models for each of the regimes.

Finally, the dependence between the regimes has to be decided. Dependent regimes with the same random noise process in all regimes (but different parameters) are computationally less demanding that independent ones. However, independent regimes model enables greater flexibility and seems to be a natural choice for a process which significantly changes its dynamics. We follow the second mentioned approach. An empirical comparison in the paper (Janczura and Weron, 2010a) shows that the best structure is that of an independent spike three-regime model with time-varying transition probabilities, heteroscedastic diffusion-type base regime dynamics and shifted spike regime distributions.

We propose a model which capture more accurately each of the characteristics of the best structure model considered in Janczura and Weron (2010a) . In this section we provide the details on Markov-switching stochastic volatility model for electricity prices. We first introduce the stochastic volatility model that exists within each of the three regimes (negative jump, regular, positive jump), followed by a description of the transition dynamics between the regimes. Finally, we derive a Gibbs sampler that we use for parameter estimation and forecasting.

3.1. Stochastic volatility model

Denote the (scalar) price y_t. Any exogenous observables will be dated t for convenience, but it is assumed that they are known at time $t-1$, when the forecast is being made. To avoid any potential scaling issues, all variables including the regressand are studentized over the estimation window. The latent regime is denoted $R_t \in \{1, 2, 3\}$, which correspond to the negative jump, regular, and positive jump regimes, respectively. All parameters $(\alpha, \beta, \gamma, \delta, \tau)$, which are introduced below, are collected in θ.

We specify our stochastic volatility model as

$$\begin{aligned} y_t \,|\theta, y_{t-1}, \ldots, y_{t-p}, \sigma_t, R_t &\sim \mathcal{N}\left(x_t' \beta_{R_t}, \sigma_t^2\right), \\ \log \sigma_t \,|\theta, \sigma_{t-1}, \ldots, \sigma_{t-q}, R_t &\sim \mathcal{N}\left(z_t' \gamma_{R_t}, \tau_{R_t}^{-1}\right). \end{aligned}$$

The regressors in the mean equation are $x_t' = (1, y_{t-1}, \ldots, y_{t-p}, spot_t, seas_t')$, and in the volatility equation, $z_t' = (1, \log \sigma_{t-1}, \ldots, \log \sigma_{t-q}, spot_t, seas_t')$. We set the lag lengths to $p = q = 48$, so our price process has a two-day memory.

For notational simplicity, introduce $T \times 1$ vectors y, σ, and $\log \sigma$, the $T \times N$ matrix X, and the $T \times M$ matrix Z. It will also be convenient to collect all T_r observations that belong to regime r in separate vectors and matrices, for $r = 1, 2, 3$. The $T_r \times 1$ vectors y_r, σ_r, and $\log \sigma_r$, the $T_r \times N$ matrices X_r, and the $T_r \times M$ matrices Z_r contain only those rows of the original vectors and matrices with $R_t = r$. Finally, let $\Sigma = \text{diag}\left(\sigma_t^2\right)$ be a $T \times T$ matrix, and create diagonal $T_r \times T_r$ matrices Σ_r similarly. We may then write our stochastic volatility model as

$$y_r \,|\theta, \sigma_r, R \sim \mathcal{N}\left(X_r \beta_r, \Sigma_r\right), \qquad \log \sigma_r \,|\theta, R \sim \mathcal{N}\left(Z_r \gamma_r, \tau_r^{-1} I_{T_r}\right), \qquad \text{for } r = 1, 2, 3.$$

We use a standard set of priors, which are independent across regimes. For each regime r, we specify $p\left(\beta_r\right)$ as $\mathcal{N}\left(0, \lambda^{-1} I_N\right)$, $p\left(\gamma_r\right)$ as $\mathcal{N}\left(0, \mu^{-1} I_M\right)$, and the uninformative $p\left(\tau_r\right) \propto \tau_r^{-1}$. Preliminary experiments suggest that our results are largely insensitive to the choice of hyperparameters λ and μ; we use $\lambda = \mu = 1$ in our application. Finally, a prior needs to be specified for the pre-sample volatilities $\sigma_{1-q}, \ldots, \sigma_0$. The standard approach of using the unconditional distribution implied by τ and the autoregressive parameters in γ, as advocated by Jacquier et al. (2002), is not feasible in our setup, since we are not imposing stationarity on the volatility process. Instead, we follow de Jong and Shephard (1995) and set $\log \sigma_t \,|R_t \sim \mathcal{N}\left(0, \tau_{R_t}^{-1}\right)$ independently for $t = 1 - q, \ldots, 0$.

3.2. State dynamics

We model the regime R_t according to a hidden Markov process, where each state transition is governed by an ordered probit model. Our specification is an extension of the two-regime model in Filardo and Gordon (1998). Specifically, the transition probabilities $P\left[R_{t+1}\,|R_t = r, \theta\right]$ are given implicitly by

$$R_{t+1} = \begin{cases} 1 & \text{if} & & & w_t'\delta_r + \varepsilon_t & < & 0, \\ 2 & \text{if} & 0 & \leq & w_t'\delta_r + \varepsilon_t & \leq & \alpha_r, \\ 3 & \text{if} & \alpha_r & < & w_t'\delta_r + \varepsilon_t & & \end{cases}$$

where ε_t is i.i.d. $\mathcal{N}(0,1)$, $w_t' = (spot_t, res_t, load_t, hdd_t, wind_t)$, and the parameters δ_r and α_r may again be different for each regime r. Note that no generality is lost by fixing the mean and variance of ε_t, as well as the threshold between the first two regimes; these restrictions serve to identify the model.

To simplify notation, we write $R^*_{t+1} = w_t'\delta_{R_t} + \varepsilon_t$. We create regime-specific $T_r \times K$ matrices W_r and $T_r \times 1$ vectors R^*_r as above, so that $R^*_r\,|\theta \sim \mathcal{N}(W_r\delta_r, I_{T_r})$, for $r = 1, 2, 3$. In particular, R^*_r includes R^*_{t+1} if $R_t = r$; the r-th ordered probit model describes all transitions from regime r to any of the three regimes.

As in the mean and volatility equations, the regression coefficients in these probit models are also given independent priors $\delta_r \sim \mathcal{N}(0, \nu^{-1}I_K)$, where we set $\nu = 1$ after finding that the results are not very sensitive to this choice. As in Albert and Chib (1993), the regime thresholds α_r have uninformative priors, uniform over $(0, \infty)$. Finally, pre-sample states $R_{1-q}, \ldots, R_0$ are required in the prior specification for the volatilities. We specify $P\left[R_{1-q} = 1\right] = P\left[R_{1-q} = 3\right] = 0.05$ and $P\left[R_{1-q} = 2\right] = 0.90$, and the ordered probit model then automatically implies a prior for $R_{2-q}, \ldots, R_0$.

3.3. Gibbs sampler

Because of its modular nature, our model lends itself well to estimation using a Gibbs sampler with data augmentation. We can obtain draws from all required conditional posteriors analytically. Standard results (Koop, 2003) apply for all regression coefficients; for $r = 1, 2, 3$, we may draw

- β_r from $\mathcal{N}\left(\left(X_r'\Sigma_r^{-1}X_r + \lambda I_N\right)^{-1} X_r'\Sigma_r^{-1}y_r, \left(X_r'\Sigma_r^{-1}X_r + \lambda I_N\right)^{-1}\right)$,
- γ_r from $\mathcal{N}\left(\left(\tau_r Z_r'Z_r + \mu I_M\right)^{-1}\left(\tau_r Z_r' \log\sigma_r\right), \left(\tau_r Z_r'Z_r + \mu I_M\right)^{-1}\right)$, and
- δ_r from $\mathcal{N}\left(\left(W_r'W_r + \nu I_K\right)^{-1} W_r'R^*_r, \left(W_r'W_r + \nu I_K\right)^{-1}\right)$.

The conditional posterior for each τ_r is the usual gamma distribution, with shape parameter $T_r/2$ and scale parameter $2\left/\left[\left(\log\sigma_r - Z_r\gamma_r\right)'\left(\log\sigma_r - Z_r\gamma_r\right)\right]\right.$.

The conditional posterior for the latent volatilities is nonstandard, but an auxiliary variable v_t can be introduced to obtain draws analytically. For $t = 1, \ldots, T$, define v_t to be $\left(y_t - x_t'\beta_{R_t}\right)^2/(2\sigma_t^2)$ plus a draw from the exponential distribution with mean one, and then draw $\log\sigma_t$ from $\mathcal{N}\left(z_t'\gamma_{R_t} - \tau_{R_t}^{-1}, \tau_{R_t}^{-1}\right)$, truncated to the interval $\left(\frac{1}{2}\log\left(\left(y_t - x_t'\beta_{R_t}\right)^2/(2v_t)\right), \infty\right)$; for further details see Damien et al. (1999).

Sampling the state dynamics R_t is done along the same lines as in Filardo and Gordon (1998). For $t = 1-q, 2-q, \ldots, T$, the conditional posterior distribution of R_t has support $\{1, 2, 3\}$, with probability for state r proportional to the product $p\left(R_t = r \,|R_{t-1}, \theta\right) \cdot p\left(R_{t+1} \,|R_t = r, \theta\right) \cdot p\left(\log\sigma_t \,|\log\sigma_{t-1}, \ldots, \log\sigma_{t-q}, \theta, R_t = r\right) \cdot p\left(y_t \,|\sigma_t, y_{t-1}, \ldots, y_{t-p}, \theta, R_t = r\right)$. The factors in this expression can be explicitly computed as

$$p\left(R_t = r \,|R_{t-1}, \theta\right) = \begin{cases} \Phi\left(-w_{t-1}'\delta_{R_{t-1}}\right) & \text{if } r = 1, \\ \Phi\left(\alpha_{R_{t-1}} - w_{t-1}'\delta_{R_{t-1}}\right) - \Phi\left(-w_{t-1}'\delta_{R_{t-1}}\right) & \text{if } r = 2, \\ 1 - \Phi\left(\alpha_{R_{t-1}} - w_{t-1}'\delta_{R_{t-1}}\right) & \text{if } r = 3, \end{cases}$$

$$p\left(R_{t+1} \,|R_t = r, \theta\right) = \begin{cases} \Phi\left(-w_t'\delta_r\right) & \text{if } R_{t+1} = 1, \\ \Phi\left(\alpha_r - w_t'\delta_r\right) - \Phi\left(-w_t'\delta_r\right) & \text{if } R_{t+1} = 2, \\ 1 - \Phi\left(\alpha_r - w_t'\delta_r\right) & \text{if } R_{t+1} = 3, \end{cases}$$

$$p\left(\log\sigma_t \,|\log\sigma_{t-1}, \ldots, \log\sigma_{t-q}, \theta, R_t = r\right) = \phi\left(\log\sigma_t; z_t'\gamma_r, \tau_r^{-1}\right),$$

$$p\left(y_t \,|\sigma_t, y_{t-1}, \ldots, y_{t-p}, \theta, R_t = r\right) = \phi\left(y_t; x_t'\beta_r, \sigma_t^2\right),$$

where Φ is the standard normal CDF, and ϕ is the normal PDF with specified mean and variance.

Finally, the sampling distributions for the latent R_{t+1}^* as well as the thresholds α_r were obtained by Albert and Chib (1993). For $t = 1, 2, \ldots, T$, the latent R_{t+1}^* can be drawn from $\mathcal{N}\left(w_t'\delta_{R_t}, 1\right)$, truncated to the correct interval, which is $(-\infty, 0)$ if $R_{t+1} = 1$, $(0, \alpha_{R_t})$ if $R_{t+1} = 2$, and (α_{R_t}, ∞) if $R_{t+1} = 3$. The conditional posterior for α_r is uniform with lower bound $\max\left\{\max\left\{R_{t+1}^* : R_t = r \text{ and } R_{t+1} = 2\right\}, 0\right\}$ and upper bound $\min\left\{R_{t+1}^* : R_t = r \text{ and } R_{t+1} = 3\right\}$, for $r = 1, 2, 3$.

3.4. Density forecasting

We can obtain draws from the one-step-ahead predictive density within the Gibbs sampler. At each step $d = 1, 2, \ldots, D$, we draw $R_{T+1}^{*(d)} \sim \mathcal{N}\left(w_T'\delta_{R_T^{(d)}}^{(d)}, 1\right)$ to find $R_{T+1}^{(d)}$, which is 1 if $R_{T+1}^{*(d)} < 0$, 2 if $0 \leq R_{T+1}^{*(d)} \leq \alpha_{R_T^{(d)}}^{(d)}$, and 3 otherwise.

Finally, we draw $\log\sigma_{T+1}^{(d)} \sim \mathcal{N}\left(z'_{T+1}\gamma^{(d)}_{R^{(d)}_{T+1}}, \tau^{(d)\,-1}_{R^{(d)}_{t+1}}\right)$, and use it to draw $y^{(d)}_{T+1} \sim \mathcal{N}\left(x'_{T+1}\beta^{(d)}_{R^{(d)}_{T+1}}, \sigma^{(d)\,2}_{T+1}\right)$. The empirical distribution formed by these draws after discarding D_0 initial burn-in draws, $F(c) = \frac{1}{D-D_0}\sum_{d=D_0+1}^{D} \mathbf{1}\left\{y^{(d)}_{T+1} \leq c\right\}$, approximates the CDF of $y_{T+1}\,|y$. A kernel density estimate of the corresponding PDF is used to visualize this distribution in the Results section below.

In our empirical application we will also be interested in h-step-ahead forecasts for $h = 2, 3, \ldots, 24$; that is, density forecasts for every hour of the next day. We may recursively obtain draws of each y_{T+h}, using the same procedure as outlined for $h = 1$ above. Most exogenous regressors in x_t, z_t, and w_t are available one day ahead, and highly accurate forecasts are available for the others. For the endogenous R^*_t, R_t, σ_t, and y_t that are needed for $t > T$, we may simply substitute the forecasts that were made at shorter horizons. This procedure is justified by the standard decomposition

$$p\left(y_{T+1}, y_{T+2}, \ldots, y_{T+24}\,|y\right) = p\left(y_{T+1}\,|y\right)\cdot p\left(y_{T+2}\,|y_{T+1}, y\right)\cdot\cdots\cdot p\left(y_{T+24}\,|y_{T+23}, \ldots, y_{T+1}, y\right).$$

3.5. Forecast evaluation

We evaluate the quality of our density forecasts using predictive Bayes factors, as suggested by Geweke and Amisano (2010). The predictive Bayes factor comparing two competing models is given by the ratio of the predictive densities implied by these models, evaluated at the realized prices. A number greater than one indicates a preference for the model in the numerator.

For one-step-ahead forecasts, we may approximate the predictive density evaluated at the realized price y_{T+1} using

$$p\left(y_{T+1}\,|y\right) = \int\int\int p\left(y_{T+1}\,|y, \theta, \sigma_{T+1}, R_{T+1}\right) dR_{T+1} d\sigma_{T+1} d\theta \approx \frac{1}{D-D_0}\sum_{d=D_0+1}^{D} \phi\left(y_{T+1}; x'_{T+1}\beta^{(d)}_{R^{(d)}_{T+1}}, \sigma^{(d)\,2}_{T+1}\right).$$

A similar iterative procedure as outlined above for density forecasting can then be used to approximate $p\left(y_{T+2}\,|y_{T+1}, y\right), \ldots, p\left(y_{T+24}\,|y_{T+23}, \ldots, y_{T+1}, y\right)$, except that now realized rather than simulated values of y_t need to be substituted into x_t, for $t > T$. Finally, the joint predictive density is again given by

$$p\left(y_{T+1}, y_{T+2}, \ldots, y_{T+24}\,|y\right) = p\left(y_{T+1}\,|y\right)\cdot p\left(y_{T+2}\,|y_{T+1}, y\right)\cdot\cdots\cdot p\left(y_{T+24}\,|y_{T+23}, \ldots, y_{T+1}, y\right).$$

4. Results

We study the hourly volume-weighted electricity prices from Elbas power market in order to understand the data generating mechanism and examine the proposed model. For that reason we consider four model specifications: the basic autoregressive process with explanatory variables (ARX), the autoregressive model with stochastic volatility error and explanatory variables (ARX-SV), the three-state Markov regime-switching model (MS-ARX) and our proposed three-state Markov regime-switching model with stochastic volatility (MS-ARX-SV). Formal Bayesian model comparison in terms of the predictive adequacy is measured by predictive Bayes factors.

Table 1: The model specifications considered in the empirical study.

Mnemonic	Restrictions on the model introduced in Section 3
ARX	no regime switching ($R_t = 2$ for all t), no stochastic volatility ($\sigma_t^2 = \sigma^2$ for all t)
ARX-SV	no regime switching ($R_t = 2$ for all t)
MS-ARX	no stochastic volatility ($\sigma_t^2 = \sigma_{R_t}^2$ for all t)
MS-ARX-SV	no restrictions

4.1. Full-sample results

As a preliminary check, we run the classical neural network test for neglected nonlinearity introduced by Lee et al. (1993) on the ARX model in each of our 363 estimation windows. The null hypothesis of linearity is rejected in the vast majority of cases, as expected based on the literature surveyed in Section 1. Specifically, nonrejection at the 5% level occurs on only 39 days, all of which are in the last 2.5 months of the year. Moving to the 10% level, only one nonrejection remains. We conclude that the in-sample evidence is strongly in favor of nonlinear modeling.

Below we present out-of-sample results obtained within four model specifications: ARX, ARX-SV, MS-ARX and MS-ARX-SV (see Table 1), estimated for hourly electricity prices (see Figure 1) for every hour of 2015.[3] In each case posterior analysis is based on 15000 MCMC samples from the relevant joint posterior, preceded by 5000 burnin draws. Calculations have been carried out with the authors' own codes run under Matlab. MCMC convergence is deemed satisfactory, as measured using standardized CUSUM plots (see Yu and Mykland,

[3]A preliminary analysis of the results did not reveal any obvious daily, weekly, or annual patterns in model performance. For this reason, only aggregate results are reported here.

1998), which are not reported here but may be obtained from the authors upon request.

We report on the model comparison first. Relevant quantities, including the predictive density values and predictive Bayes factors, are displayed in Table 2. It is clear that all three nonlinear models provide a better out-of-sample forecasting performance than the simple linear ARX model. To quantify the performance differences, we follow the interpretation of Bayes factors suggested by Kass and Raftery (1995): a Bayes factor greater than three provides "positive" evidence of the outperformance, and the evidence is "strong" for Bayes factors greater than twenty and "very strong" beyond 150. Thus, there is very strong evidence that MS-ARX-SV outperforms the simple ARX benchmark, since $\exp(1219.1797) \approx 3 \times 10^{529}$. In fact, the same could be said for every pairwise model comparison in this table; the evidence in favor of the presence of both stochastic volatility (ARX-SV versus ARX) and especially regime switching (MS-ARX versus ARX) is very strong. We conclude that the overall forecasting performance of the MS-ARX, ARX-SV, and MS-ARX-SV models is much better than that of the ARX model.

However, our proposed MS-ARX-SV model is outperformed by each of the simpler nonlinear MS-ARX and ARX-SV models, which are special cases of it. Our interpretation of this result is that regime switching and stochastic volatility are both good ideas for modeling the electricity prices, since each of these features individually strongly enhances the performance of a pure ARX model. Joining both of these ideas, on the other hand, still requires some further investigation. Perhaps our highly parametrized specification is simply asking to much from the data; perhaps three regimes are not needed for this particular data set and two would be sufficient. We are currently in the process of assessing these issues.

Table 2: Predictive performance measures in the empirical study.

		Log predictive Bayes factor against...			
Model	Predictive log density	MS-ARX-SV	MS-ARX	ARX-SV	ARX
MS-ARX-SV	-4571.9895		2242.4402	1565.7482	-1219.1797
MS-ARX	-2329.5493			-676.6921	-3461.6199
ARX-SV	-3006.2413				-2784.9278
ARX	-5791.1691				

4.2. Subsample results

In order to better understand the differences in performance between the four models under consideration, as well as to highlight the ease of formally obtaining predictive densities within our framework, we repeat the analysis that we just performed restricted to four specific days in 2015. These days were selected to be the ones where each of the four models performed best relative to its competitors: Monday 12 January (ARX performs best on this day), Sunday 18 January (MS-ARX-SV), Sunday 10 May (MS-ARX), and Tuesday 25 August (ARX-SV).

Tables 3–6 below are analogous to the full-sample Table 2. We observe that, except in the special case where ARX performs best (Table 3), this simple linear benchmark performs far worse than all nonlinear competitors (Tables 4–6). When our full MS-ARX-SV model performs best (Table 4), it does so by a very wide margin; note that the smallest Bayes factor is $\exp(12.3544) \approx 2 \times 10^5$ already.

This leaves us with the intermediate cases to analyze, where either Markov switching turned out to be useful for forecasting but stochastic volatility did not, or vice versa. Table 5 presents a case in which MS-ARX strongly outperforms all other models, and we observe that the MS-ARX-SV model is still "the best of the rest". That is, the full, highly-parametric model is preferred over the ARX-SV model, which gets the nature of the nonlinearity wrong in this instance. In the opposite case (Table 6), where ARX-SV is the preferred model, the other nonlinear models MS-ARX-SV and MS-ARX are virtually indistinguishable, with a predictive Bayes factor of $\exp(0.8272) \approx 2$. These results confirm our intuition based on the full-sample results: leaving out Markov switching when we need it has a larger negative impact on forecast accuracy than leaving out stochastic volatility when we need it.

Table 3: Predictive performance measures in the empirical study, Monday 12 January 2015.

		Log predictive Bayes factor against...			
Model	Predictive log density	MS-ARX-SV	MS-ARX	ARX-SV	ARX
MS-ARX-SV	-62.7767		33.2098	27.9920	38.0507
MS-ARX	-29.5669			-5.2178	4.8409
ARX-SV	-34.7847				10.0587
ARX	-24.7260				

Table 4: Predictive performance measures in the empirical study, Sunday 18 January 2015.

		Log predictive Bayes factor against…			
Model	Predictive log density	MS-ARX-SV	MS-ARX	ARX-SV	ARX
MS-ARX-SV	-1.2465		-12.3544	-15.7705	-20.9411
MS-ARX	-13.6009			-3.4161	-8.5867
ARX-SV	-17.0169				-5.1706
ARX	-22.1875				

Table 5: Predictive performance measures in the empirical study, Sunday 10 May 2015.

		Log predictive Bayes factor against…			
Model	Predictive log density	MS-ARX-SV	MS-ARX	ARX-SV	ARX
MS-ARX-SV	-83.2826		39.4483	-7.5094	-84.2646
MS-ARX	-43.8343			-46.9577	-123.7129
ARX-SV	-90.7920				-76.7552
ARX	-167.5473				

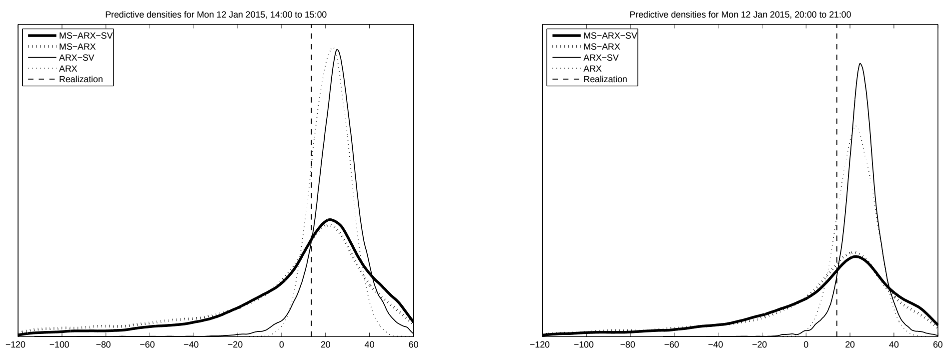
Table 6: Predictive performance measures in the empirical study, Tuesday 25 August 2015.

		Log predictive Bayes factor against…			
Model	Predictive log density	MS-ARX-SV	MS-ARX	ARX-SV	ARX
MS-ARX-SV	-11.0719		0.8272	22.3026	-2.8123
MS-ARX	-10.2447			21.4755	-3.6395
ARX-SV	11.2308				-25.1150
ARX	-13.8842				

To further investigate what drives these results, Figures 3–6 show the predictive densities obtained for two selected hours on these four days, one in the afternoon ($h = 15$) and one in the evening ($h = 21$). The ex-post realized price is also included in each of these figures. 12 January (Figure 3) was a relatively uneventful day, for which the linear ARX model was "good enough" and its nonlinear extensions turned out to be needless complications.

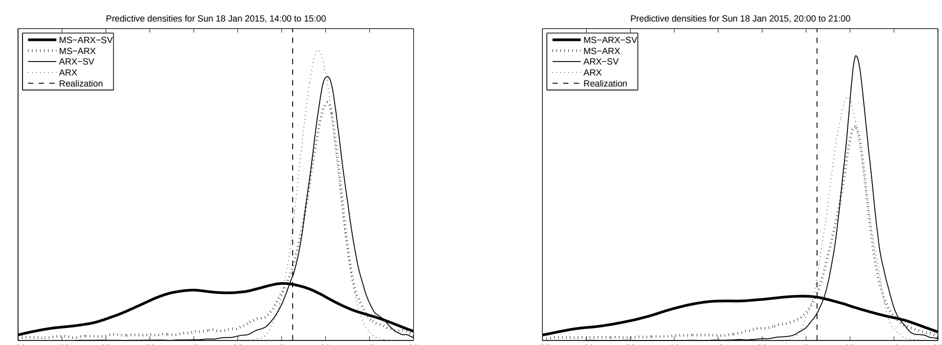
The MS-ARX-SV model performed best on 18 January (Figure 4). It appears that the models without a Markov switching component provide forecasts that are centered at the wrong location on this day. Clearly, allowing for multiple regimes provides a safeguard against such problems. The price fluctuated considerably on this day, a fact that is picked up by the relatively flat predictive densities produced by the stochastic volatility models.

A large negative jump occurred in the afternoon of 10 May (Figure 5), which explains the good performance of Markov switching models on this date. Finally, 25 August (Figure 6) saw large price fluctuations but no jumps, so that stochastic volatility was an important model component on that day.



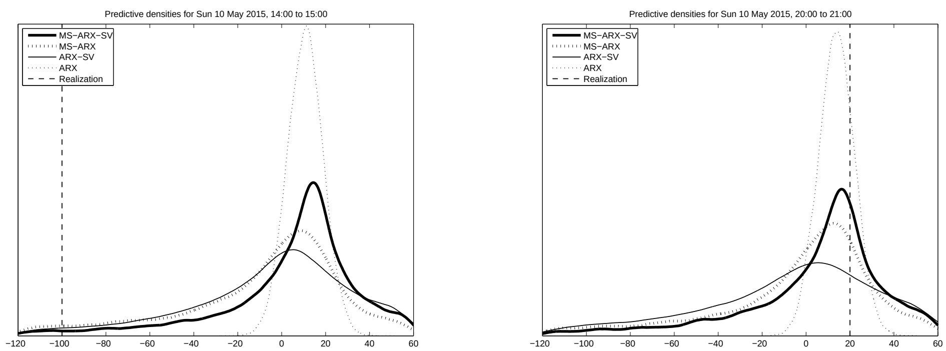
(a) Predictive density for $h = 15$ (b) Predictive density for $h = 21$

Figure 3: Predictive densities for two selected hours on Monday 12 January 2015.



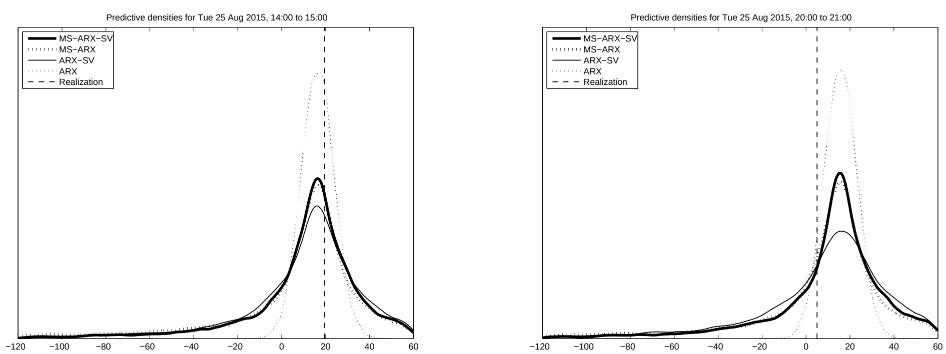
(a) Predictive density for $h = 15$ (b) Predictive density for $h = 21$

Figure 4: Predictive densities for two selected hours on Sunday 18 January 2015.



(a) Predictive density for $h = 15$ (b) Predictive density for $h = 21$

Figure 5: Predictive densities for two selected hours on Sunday 10 May 2015.



(a) Predictive density for $h = 15$ (b) Predictive density for $h = 21$

Figure 6: Predictive densities for two selected hours on Tuesday 25 August 2015.

5. Conclusions

The goal of this research was to address fundamental questions within the area of forecasting electricity prices. We proposed a new regime-switching stochastic volatility model with three regimes which takes into account fundamental price drivers. We show how the predictive densities of future electricity prices can be formally constructed via Bayesian inference. Moreover, we introduced a universal method (within the Bayesian framework) for model comparisons, predictive Bayes factors, to the electricity price forecasting literature. Based on this measure, we showed that the Markov switching structure and the stochastic volatility component of our model both contribute to its improved forecasting performance in terms of short-time density forecasting in the Nord Pool intraday market, relative to a model that lacks such features.

Both Markov switching models and stochastic volatility models provide very good forecasts on some occasions but poor ones on some others, and our subsample analysis suggests that there may be a complementarity between these two features. Since our MS-ARX-SV model nests both types of models, it strikes us as a useful contribution to the literature. However, more research is needed in order to obtain a desirable empirical performance from this rich model. One possible way to reduce the dimensionality of its parameter space would be to simplify the volatility dynamics, e.g. $\gamma_{R_t} = \gamma$ for each regime.

Another avenue, which appears more promising in our view, is to go back to models with two rather than three regimes. Most evidence in favor of the existence of a third regime (Karakatsani and Bunn, 2008; Janczura and Weron, 2010a) is several years old by now. As integrated energy markets have matured, a "spike" regime may no longer be necessary to describe the dynamics in electricity prices. Tentatively, visual inspection of Figure 1 confirms this intuition, but a more thorough investigation is required.

J.H. Albert and S. Chib. Bayesian analysis of binary and polychotomous response data. *Journal of the American Statistical Association*, 88:669–679, 1993.

J. Andreasen and M. Dahlgren. At the flick of a switch. *Energy Risk*, 10:71–75, 2006.

T. Arvesen, V. Medbø, S.-E. Fleten, A. Tomasgard, and S. Westgaard. Linepack storage valuation under price uncertainty. *Energy*, 52:155–164, 2013.

M. Bierbrauer, S. Trück, and R. Weron. Modeling electricity prices with regime switching models. In M. Bubak, G.D. van Albada, P. Sloot, and J. Dongarra, editors, *Proceedings of the Fourth International Conference on Computational Science*, pages 859–867. Springer, Berlin/Heidelberg, 2004.

M. Bierbrauer, C. Menn, S.T. Rachev, and S. Trück. Spot and derivative pricing in the EEX power market. *Journal of Banking and Finance*, 31:3462–3485, 2007.

A. Botterud, A.K. Bhattacharyya, and M. Ilic. Futures and spot prices: An analysis of the Scandinavian electricity market. In *Proceedings of the North American Power Symposium*. Tempe, 2002.

P. Damien, J. Wakefield, and S. Walker. Gibbs sampling for Bayesian non-conjugate and hierarchical models by using auxiliary variables. *Journal of the Royal Statistical Society: Series B*, 61:331–344, 1999.

C. de Jong. The nature of power spikes: A regime-switch approach. *Studies in Nonlinear Dynamics and Econometrics*, 10:27–46, 2006.

P. de Jong and N. Shephard. The simulation smoother for time series models. *Biometrika*, 82:339–350, 1995.

S.J. Deng. Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes. *POWER working paper*, University of California Energy Institute, Berkeley, 1999.

M. Eichler and D. Türk. Fitting semiparametric Markov regime-switching models to electricity spot prices. *Energy Economics*, 36:614–624, 2013.

R. Ethier and T. Mount. Estimating the volatility of spot prices in restructured electricity markets and the implications for option values. *Working paper*, Cornell University, Ithaca, 1998.

A.J. Filardo and S.F. Gordon. Business cycle durations. *Journal of Econometrics*, 85:99–123, 1998.

F.R. Førsund and M. Hoel. Properties of a non-competitive electricity market dominated by hydroelectric power. *Working paper*, University of Oslo, 2004.

H. Geman and A. Roncoroni. Understanding the fine structure of electricity prices. *Journal of Business*, 79:1225–1261, 2006.

J. Geweke and G. Amisano. Comparing and evaluating Bayesian predictive distributions of asset returns. *International Journal of Forecasting*, 26:216–230, 2010.

O. Gjolberg and T. Johnsen. Electricity futures: Inventories and price relationships at Nord Pool. *Working paper*, Norwegian School of Economics and Business Administration, Bergen, 2001.

N. Haldrup and Ø.M. Nielsen. Directional congestion and regime switching in a long memory model for electricity prices. *Studies in Nonlinear Dynamics & Econometrics*, 10:1–24, 2006a.

N. Haldrup and Ø.M. Nielsen. A regime switching long memory model for electricity prices. *Journal of Econometrics*, 135:349–376, 2006b.

R. Handika, C. Truong, S. Trück, and R. Weron. Modelling price spikes in electricity markets: The impact of load, weather and capacity. *Working paper*, Hugo Steinhaus Center, Wrocław, 2014.

S. Heydari and A. Siddiqui. Valuing a gas-fired power plant: A comparison of ordinary linear models, regime-switching approaches, and models with stochastic volatility. *Energy Economics*, 32:709–725, 2010.

T. Hong, P. Pinson, S. Fan, H. Zareipour, A. Troccoli, and R.J. Hyndman. Probabilistic energy forecasting: Global energy forecasting competition 2014 and beyond. *International Journal of Forecasting*, 32(3):896 – 913, 2016.

R. Huisman. *An introduction to models for the energy markets*. Risk Books, London, 2009.

R. Huisman and C. de Jong. Option pricing for power prices with spikes. *Energy Power Risk Management*, 7:12–16, 2003.

R. Huisman and R. Mahieu. Regime jumps in electricity prices. *Energy Economics*, 25:425–434, 2003.

C. Huurman, F. Ravazzolo, and C. Zhou. The power of weather. *Computational Statistics and Data Analysis*, 56:3793–3807, 2012.

E. Jacquier, N.G. Polson, and P.E. Rossi. Bayesian analysis of stochastic volatility models. *Journal of Business and Economic Statistics*, 20:69–87, 2002.

J. Janczura and R. Weron. Regime-switching models for electricity spot prices: Introducing heteroskedastic base regime dynamics and shifted spike distributions. In *IEEE Proceedings of the International Conference on the European Energy Market*, Leuven, 2009.

J. Janczura and R. Weron. An empirical comparison of alternate regime-switching models for electricity spot prices. *Energy Economics*, 32:1059–1073, 2010a.

J. Janczura and R. Weron. Goodness-of-fit testing for regime-switching models. *Working paper*, MPRA, München, 2010b.

J. Janczura, S. Trück, R. Weron, and R.C. Wolff. Identifying spikes and seasonal components in electricity spot price data: A guide to robust modeling. *Energy Economics*, 38:96–110, 2013.

T. Jónsson, P. Pinson, H. Madsen, and H.A. Nielsen. Predictive densities for day-ahead electricity prices using time-adaptive quantile regression. *Energies*, 7: 5523–5547, 2014.

T. Kanamura and K. Ohashi. On transition probabilities of regime switching in electricity prices. *Energy Economics*, 30:1158–1172, 2008.

N.V. Karakatsani and D.W. Bunn. Intra-day and regime-switching dynamics in electricity price formation. *Energy Economics*, 30:1776–1797, 2008.

R.E. Kass and A.E. Raftery. Bayes factors. *Journal of the American Statistical Association*, 90:773–795, 1995.

D. Keles, R. Hartel, D. Möst, and W. Fichtner. Compressed-air energy storage power plant investments under uncertain electricity prices: An evaluation of compressed-air energy storage plants in liberalized energy markets. *Journal of Energy Markets*, 5:53–84, 2012.

G. Koop. *Bayesian Econometrics*. Wiley, Chichester, 2003.

P. Kosater and K. Mosler. Can Markov regime-switching models improve power-price forecasts? Evidence from German daily power prices. *Applied Energy*, 83:943–958, 2006.

T.-H. Lee, H. White, and C.W.J. Granger. Testing for neglected nonlinearity in time series models. *Journal of Econometrics*, 56:269–290, 1993.

X. Li and D.J. Sailor. Electricity use sensitivity to climate and climate change. *World Resource Review*, 7:334–346, 1995.

D. Liebl. Modeling and forecasting electricity spot prices: A functional data perspective. *Annals of Applied Statistics*, 7:1565–1592, 2013.

F. Lisi and F. Nan. Component estimation for electricity prices: Procedures and comparisons. *Energy Economics*, 44:143–159, 2014.

K. Maciejowska, J. Nowotarski, and R. Weron. Probabilistic forecasting of electricity spot prices using factor quantile regression averaging. *International Journal of Forecasting*, 32(3):957–965, 2016.

C. Mari. Random movements of power prices in competitive markets: A hybrid model approach. *Journal of Energy Markets*, 1:87–103, 2008.

T.D. Mount, Y. Ning, and X. Cai. Predicting price spikes in electricity markets using a regime-switching model with time-varying parameters. *Energy Economics*, 28:62–80, 2006.

Nord Pool. `http://www.nordpoolspot.com/`, accessed 15 July, 2016.

A. Panagiotelis and M. Smith. Bayesian density forecasting of intraday electricity prices using multivariate skew t distributions. *International Journal of Forecasting*, 24:710–727, 2008.

J. Peirson and A. Henley. Electricity load and temperature: Issues in dynamic specification. *Energy Economics*, 16:235–243, 1994.

J. Peirson and A. Henley. Residential energy demand and the interaction of price and temperature: British experimental evidence. *Energy Economics*, 20:157–171, 1998.

D.J. Sailor, J.N. Rosen, and J.R. Muñoz. Natural gas consumption and climate: A comprehensive set of predictive state-level models for the United States. *Energy*, 23:91–103, 1998.

M. Serati, M. Manera, and M. Plotegher. Modeling electricity prices: From the state of the art to a draft of a new proposal. *Working paper*, Fondazione Eni Enrico Mattei, Milan, 2008.

F. Serinaldi. Distributional modeling and short-term forecasting of electricity prices by generalized additive models for location, scale and shape. *Energy Economics*, 33:1216–1226, 2011.

M.S. Smith. Bayesian inference for a periodic stochastic volatility model of intraday electricity prices. In T. Kneib and G. Tutz, editors, *Statistical Modelling and Regression Structures: Festschrift in Honour of Ludwig Fahrmeir*, pages 353–376. Physica-Verlag, Heidelberg, 2010.

N.-H.M. von der Fehr, E.S. Amundsen, and L. Bergman. The Nordic market: Signs of stress? *The Energy Journal*, 26:71–98, 2005.

R. Weron. Heavy-tails and regime-switching in electricity prices. *Mathematical Methods of Operations Research*, 69:457–473, 2009.

R. Weron. Electricity price forecasting: A review of the state-of-the-art with a look into the future. *International Journal of Forecasting*, 30:1030–1081, 2014.

R. Weron, M. Bierbrauer, and S. Trück. Modeling electricity prices: Jump diffusion and regime switching. *Physica A*, 336:39–48, 2004.

B. Yu and P. Mykland. Looking at Markov samplers through CUSUM path plots: A simple diagnostic idea. *Statistics and Computing*, 8:275–286, 1998.

Research Papers 2016

CREATES
Center for Research in Econometric Analysis of Time Series

2016-19: Tim Bollerslev, Jia Li and Yuan Xue: Volume, Volatility and Public News Announcements

2016-20: Andrea Barletta, Paolo Santucci de Magistris and Francesco Violante: Retrieving Risk-Neutral Densities Embedded in VIX Options: a Non-Structural Approach

2016-21: Mikkel Bennedsen: Semiparametric inference on the fractal index of Gaussian and conditionally Gaussian time series data

2016-22: Søren Johansen and Morten Ørregaard Nielsen: The cointegrated vector autoregressive model with general deterministic terms

2016-23: Yunus Emre Ergemen and Carlos Vladimir Rodríguez-Caballero: A Dynamic Multi-Level Factor Model with Long-Range Dependence

2016-24: Shin Kanaya: Convergence rates of sums of α-mixing triangular arrays: with an application to non-parametric drift function

2016-25: Gustavo Fruet Dias, Marcelo Fernandes and Cristina M. Scherrer: Improving on daily measures of price discovery

2016-26: Martin M. Andreasen, Tom Engsted, Stig V. Møller and Magnus Sander: Bond Market Asymmetries across Recessions and Expansions: New Evidence on Risk Premia

2016-27: Kim Christensen, Ulrich Hounyo and Mark Podolskij: Testing for heteroscedasticity in jumpy and noisy high-frequency data: A resampling approach

2016-28: Kim Christensen, Roel Oomen and Roberto Renò: The Drift Burst Hypothesis

2016-29: Hossein Asgharian, Charlotte Christiansen, Rangan Gupta and Ai Jun Hou: Effects of Economic Policy Uncertainty Shocks on the Long-Run US-UK Stock Market Correlation

2016-30: Morten Ørregaard Nielsen and Sergei S. Shibaev: Forecasting daily political opinion polls using the fractionally cointegrated VAR model

2016-31: Carlos Vladimir Rodríguez-Caballero: Panel Data with Cross-Sectional Dependence Characterized by a Multi-Level Factor Structure

2016-32: Lasse Bork, Stig V. Møller and Thomas Q. Pedersen: A New Index of Housing Sentiment

2016-33: Joachim Lebovits and Mark Podolskij: Estimation of the global regularity of a multifractional Brownian motion

2017-01: Nektarios Aslanidis, Charlotte Christiansen and Andrea Cipollini: Predicting Bond Betas using Macro-Finance Variables

2017-02: Giuseppe Cavaliere, Morten Ørregaard Nielsen and Robert Taylor: Quasi-Maximum Likelihood Estimation and Bootstrap Inference in Fractional Time Series Models with Heteroskedasticity of Unknown Form

2017-03: Peter Exterkate and Oskar Knapik: A regime-switching stochastic volatility model for forecasting electricity prices