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A Framework for Testing the Equality Between the Health Concentration Curve and the 45-Degree Line

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Abstract

The health concentration curve is the standard graphical tool to depict socioeconomic health inequality in the literature on health inequality. This paper shows that testing for the absence of socioeconomic health inequality is equivalent to testing if the regression function of health on income is a constant function that is equal to average health status. In consequence, any test for parametric specification of a regression function can be used to test for the absence of socioeconomic health inequality (subject to regularity conditions). Furthermore, this paper illustrates how to test for this equality using the Härdle and Mammen (1993) test for correct parametric regression functional form, and applies it to the National Health Survey 2014.

JEL Classification: D63, I10

Keywords: health concentration curves, socioeconomic health inequality, inference

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1 Introduction

The health concentration curve provides a profile of how health varies across the full distribution of living standards. It is one of the most widely accepted analytical tools in the health inequality literature and is used to portray the evolution of socioeconomic health inequalities from a global health perspective as well as a national perspective.[1] Empirical researchers often emphasize that it is crucial to test whether (or not), the health concentration curve is statistically different from the 45-degree line (O'Donnell, van Doorslaer, Wagstaff and Lindelow, 2008).[2] Allowing researchers to address this question will provide them with some guidance as to whether health inequality is an issue that needs further consideration. Building on the literature on inference for Lorenz and concentration curves' dominance, O'Donnell et *al.*(2008) propose testing for the equality of these two curves, but on a predesignated grid of points. Hence, their testing procedure does not test for the equality of the two curves over their domain of definition. An undesirable consequence of their test is that it has no power against alternatives whose curves are equal at points in the predesignated grid and differ at other points in their domain. This renders the test inconsistent. A more desirable approach would be to compare the curves at all points in the domain. This paper shows that testing for the equality of the health concentration curve and the 45-degree line at all points between 0 and 1 is equivalent to testing that the regression of health on income is equal to a constant function of income, where the constant is the population mean health status. The consequence of this equivalence is that tests for parametric specification of a regression function can be used to test for the equality of health concentration curve and the 45-degree line, against a nonparametric alternative.

[1]To mention a few, Lindelow, 2006, van Doorslaer, Clarke, Savage, and Hall, 2008, Wagstaff, 2010, Mills et *al.*, 2012, Powell-Jackson and Hanson, 2012, Randive, San Sebastian, De Costa and Lindholm, 2014, Elgar et *al.*, 2015, Mosquera et *al.*, 2016.

[2]In the theoretical case where everyone has the same level of health (i.e., the same value for the health variable), the health concentration curve is a 45-degree line.

Tests of this sort are ubiquitous in econometrics (e.g., Azzalini, Bowman and Härdle, 1989, Wooldridge, 1992, Yatchew, 1992, Härdle and Mammen, 1993, Liu, Stengos and Li, 2000, and Tripathi and Kitamura, 2003), which provides a lot of choice for the practitioner. This paper does not support the use of any particular test because such a choice would naturally depend on which regularity conditions are appropriately satisfied in practice. To provide an example of settings in which the result can be applied, we discuss in detail the implementation of the Härdle and Mammen (1993) test, and apply it to a data set. The rest of this paper is organized as follows. In the next section, we present the measurement framework. Section 3 presents our main result and the Härdle and Mammen (1993) test adapted to our framework. Section 4 gives a brief empirical illustration using data from the National Health Interview Survey 2014. Finally, Section 5 concludes the paper.

2 Measurement framework

The health concentration curve is a functional of the joint distribution of health, H, and income Y.[3] Without loss of generality, the random vector (H, Y) has a joint density $f_{H,Y}$ that is supported on $\mathcal{H} \times \mathcal{Y}$, with marginal densities given by f_H and f_Y, and marginal cumulative distribution functions F_H and F_Y.[4] Let $z(p)$ be the conditional expectation of health, H, with respect to Y equal to its p-quantile. Formally,

$$z(p) = E[H|Y = F_Y^{-1}(p)]. \tag{1}$$

The health concentration curve, $C(p)$, measures the socioeconomic health inequality. It is the plot of the cumulative proportion of total health in the population against the cumulative proportion of individuals ranked by socioeconomic statuses. It is defined on the interval $[0, 1]$ as

$$C(p) = \frac{1}{\mu_z} \int_0^p z(u)\mathrm{d}u \quad p \in [0, 1]. \tag{2}$$

[3] In this paper, we assume that this health measure is a ratio-scale variable.

[4] It is important to note that the assumption of (H, Y) having a density can be relaxed.

where, $\mu_z = \int_0^1 z(p)\mathrm{d}p$. Note that the concentration curve in equation (2) can be rewritten as

$$C(p) = \frac{\int_0^p \mathrm{E}\left[H \mid Y = F_Y^{-1}(u)\right] \,\mathrm{d}u}{\int_0^1 \mathrm{E}\left[H \mid Y = F_Y^{-1}(u)\right] \,\mathrm{d}u} \quad p \in [0, 1]. \tag{3}$$

When $C(p)$ lies above the 45-degree line, health inequality is referred to as pro-poor (i.e. the poor have better health than the rich). When it lies under the diagonal, health inequality is considered pro-rich (Wagstaff, Paci and van Doorslaer,1991).[5] If the health concentration curve lies on the 45-degree line, then there is no socioeconomic health inequality. In addition to its usual role in measuring the degree of health inequality associated with socioeconomic status, the health concentration curves can be used to identify robust rankings of socioeconomic health inequality when comparing two distributions (see Makdissi and Yazbeck, 2014).

3 Result

This section presents the result of the paper, which yields a framework for testing the equality of the concentration curve with the 45-degree line (i.e. the line of perfect socioeconomic health equality). The null hypothesis of interest is

$$H_0 : C(p) = p \quad \forall p \in (0, 1) \quad \text{versus} \quad H_1 : C(p) \neq p \quad \text{for some} \quad p \in (0, 1).$$

The following result is the basis for testing H_0.

Proposition 1 *Suppose that the joint density $f_{H,Y}(\cdot, \cdot)$ is continuous on $\mathcal{H} \times \mathcal{Y}$ and bounded away from zero such that $E[H] \neq 0$. Furthermore, let the function $C(\cdot)$ on $[0, 1]$ be defined as in (2). Then $C(p) = p \quad \forall p \in (0, 1) \iff E[H \mid Y = y] = E[H] \quad \forall y \in \mathcal{Y}$.*

Proof. See Appendix A. ■

[5] An opposite conclusion may be reached if the analysis is based on a ill-health variable.

Proposition 1 implies that the null hypothesis for testing equality between the health concentration curve and the 45-degree line is equivalent to testing whether the regression function of health on income is a constant function that is equal to $\mathrm{E}[H]$. This is a test for correct specification of a parametric regression function (i.e. a test of the parametric hypothesis) $\mathrm{E}[H \mid Y=y] = \mathrm{E}[H] \ \forall y \in \mathcal{Y}$, against a nonparametric alternative, where the (unknown) parameter is $\mathrm{E}[H] \in \mathcal{H}$.

Many tests for correct parametric regression functional form were proposed in the literature (e.g., Azzalini, Bowman and Härdle, 1989, Wooldridge, 1992, Yatchew, 1992, Härdle and Mammen, 1993, Liu, Stengos and Li, 2000, Tripathi and Kitamura, 2003 and Li and Racine, 2007). The choice of a suitable test naturally hinges upon which regularity conditions are appropriately satisfied. To provide an example of settings in which the result can be applied, we discuss in detail the implementation of the Härdle and Mammen (1993) test.

The test proposed by Härdle and Mammen (1993) employs the weighted L_2-distance between the nonparametric and parametric fits as a measure of discrepancy.[6] Given a random sample $\{(H_i, Y_i)\}_{i=1}^n$, the general form of their test statistic in our framework is

$$T_n = n\sqrt{h} \int_{\mathcal{Y}} \left[\hat{m}(y) - \overline{H}\right]^2 \psi(y) dy, \tag{4}$$

where $\hat{m}(y)$ is the Nadaraya-Watson estimator of $m(y) = \mathrm{E}[H \mid Y=y]$ with bandwidth h and kernel K,

$$\hat{m}(y) = \frac{\sum_{i=1}^n K_h(y - Y_i) H_i}{\sum_{i=1}^n K_h(y - Y_i)}, \quad K_h(y - Y_i) = h^{-1} K\left(\frac{y - Y_i}{h}\right), \tag{5}$$

$\overline{H} = \frac{1}{n}\sum_{i=1}^n H_i$, and $\psi(y)$ is a weight function on $\mathcal{Y}$, its purpose is to control for areas where there are relatively few observations.

[6] It should be noted that the code for Härdle and Mammen (1993) test is readily available in Stata. For more details see Verardi and Debarsy (2012).

In their paper, Härdle and Mammen propose to use a wild bootstrap testing procedure. They employ regularity conditions on $h, K(\cdot), \psi(\cdot)$, and $f_{H,Y}(\cdot,\cdot)$ that yields the validity of their statistical test. In this paper, the regularity conditions required are the same as those in Härdle and Mammen (1993). For ease of exposition, we state these conditions explicitly in Appendix B.

As for the bootstrap procedure, it is as follows:

1. Using the data, estimate the nonparametric regression model $\hat{m}(y)$, compute $\overline{H}$ and the test statistic T_n.

2. Compute the residuals $\hat{\varepsilon}_i = H_i - \overline{H}$.

3. Generate a bootstrap sample from the following data-generating process: $H_i^* = \overline{H} + \hat{\varepsilon}_i \eta_i^*$, which satisfies H_0, and where η_i^* is a random draw from a probability distribution with mean 0 and variance of 1.

4. Use this bootstrap sample to calculate a bootstrap test statistic T_n^*, in the same way as the statistic T_n is computed.

5. Repeat the two preceding steps a large number of times, say, $B = 999$ times[7], and then construct the bootstrap P-value, $p_{bs} = \frac{1}{B}\sum_{j=1}^{B} 1\left[T_{n,j}^* > T_n\right]$.

6. "Reject H_0" if $p_{bs} < \alpha$, where α is the nominal level.

4 Empirical illustration

To illustrate the proposed methodology empirically, we use data from the National Health Interview Survey 2014. We focus on comparisons of three health related behaviors that have been of great interest in the health economics literature: cigarettes consumption,

[7]For work on the appropriate number of bootstrap replications, see, for example, Davidson and Mackinnon (2000).

overweightness, and sleeping habits. We measure sleep using reported daily sleep hours, we also measure cigarette consumption by the reported number of cigarettes consumed per day, finally, following Bilger, Kruger and Finkelstein (2016) we measure overweightness, Ow, by taking the $(\max[0, BMI - 25])$.[8]

The NHIS monitors health outcomes of Americans since 1957. It is a cross-sectional household interview survey representative of American households and non-institutionalized individuals. It contains data on a broad range of health topics that are collected via personal household interviews. We use the 2014 public-use data and restrict our attention to adult population for whom we have information about their socioeconomic status and at least one of the three health-related behaviors. After applying all these restrictions to the data, we end up with a sample size of 35,408 for sleep habit, 36,363 for cigarettes consumption and 35,408 for overweightness. We use the sample adult file to extract information on health-related behaviour and use family income adjusted for family size to infer the socioeconomic rank of individuals.[9]

Figure 1 illustrates the health concentration curves for our three health related behaviors. The concentration curve of sleep habit almost lies on the 45-degree line. The concentration curve of overweightness is slightly above and the one for cigarettes consumption seems further away above this 45-degree line. These three curves give us a perfect setting to illustrate the proposed methodology.

From Proposition 1 we know that testing for equality between a health concentration curve and the 45-degree line is equivalent to testing whether the regression function of health on income is a constant function that is equal to the average value of the health variable. To do so, we first compute Nadaraya-Watson nonparametric regressions of (three) health related behaviours on income. Figure 2 displays these regressions where, on each

[8]Note that we use 25 as a benchmark as it is the upper threshold for normal weight.

[9]We compute equivalent income by dividing family income by the square root of household size.

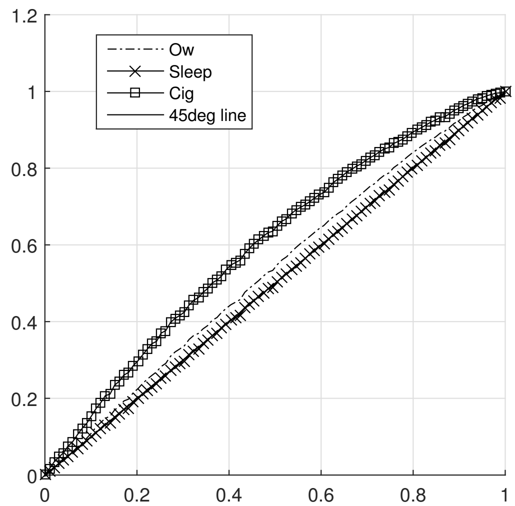

Figure 1: Concentration curves

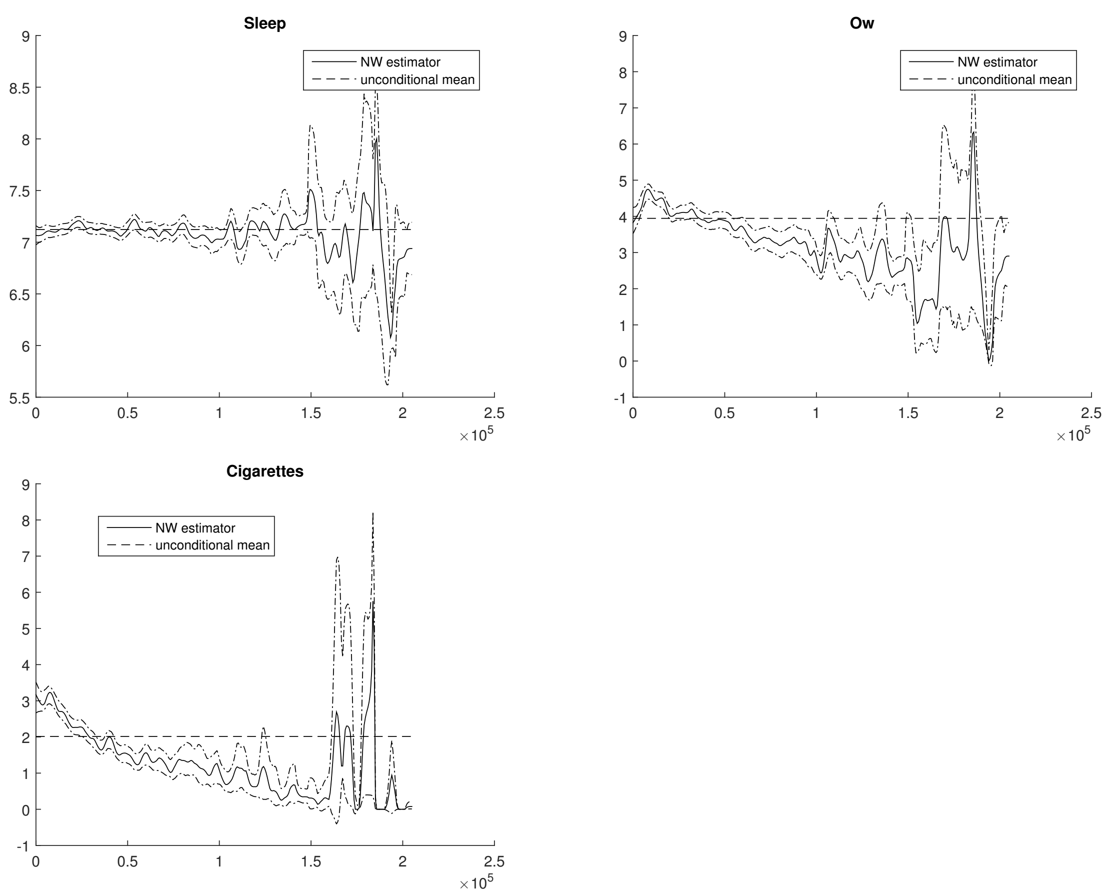

Figure 2: Nonparametric regressions

Table 1: Härdle and Mammen tests

	p-value
$H_0 : \mathrm{E}[Sleep \mid Y = y] = \mathrm{E}[Sleep] \quad \forall y \in \mathcal{Y}$ $H_1 : \mathrm{E}[Sleep \mid Y = y] \neq \mathrm{E}[Sleep]$ for some $y \in \mathcal{Y}$	0.2910
$H_0 : \mathrm{E}[Ow \mid Y = y] = \mathrm{E}[Ow] \quad \forall y \in \mathcal{Y}$ $H_1 : \mathrm{E}[Ow \mid Y = y] \neq \mathrm{E}[Ow]$ for some $y \in \mathcal{Y}$	0.0000
$H_0 : \mathrm{E}[Cig \mid Y = y] = \mathrm{E}[Cig] \quad \forall y \in \mathcal{Y}$ $H_1 : \mathrm{E}[Cig \mid Y = y] \neq \mathrm{E}[Cig]$ for some $y \in \mathcal{Y}$	0.0000

graph, the horizontal doted line represents the average value for the health related behavior variable. We then perform a 999 replications wild bootstrap test to get the p-values. Table 1 displays the p-values of the Härdle and Mammen test. Given these tests, we cannot reject this equality for hours of sleep (p-value = 0.2910), in other words, we cannot reject the null hypothesis that sleep hours are equally distributed across socioeconomic statuses. However, this equality can be rejected for overweightness and cigarettes consumption.

5 Conclusion

This paper shows that testing for the equality between the health concentration curve and the 45-degree line is equivalent to testing for constant parametric specification of the regression function of health on income, where the constant is the population mean health status. This equivalency ameliorates the testing problem of interest because now standard tests for parametric specification of a regression function can be utilized. These tests are ubiquitous in econometrics. Building on this equivalency, we describe in detail the implementation of the Härdle and Mammen (1993) testing procedure within the paper's setup. Finally, we present an empirical illustration using the 2014 public-use data from the National Health Interview Survey.

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A Proof of Proposition 1

We first prove $C(p) = p \quad \forall p \in (0,1) \implies \mathrm{E}[H \mid Y = y] = \mathrm{E}[H] \quad \forall y \in \mathcal{Y}$. The proof proceeds by the direct method. First, noting that

$$\int_0^1 \mathrm{E}\left[H \mid Y = F_Y^{-1}(u)\right] \mathrm{d}u = E[H] \tag{6}$$

is a result of applying the change of variables

$$u = F_Y(y), du = f_Y(y)dy \quad \text{and} \quad y \in \mathcal{Y}, \tag{7}$$

it follows that

$$C(p) = p \quad \forall p \in (0,1) \iff \frac{\int_0^p \mathrm{E}\left[H \mid Y = F_Y^{-1}(u)\right] \mathrm{d}u}{E[H]} = p \quad \forall p \in (0,1) \tag{8}$$

$$\iff \int_0^p \mathrm{E}\left[H \mid Y = F_Y^{-1}(u)\right] \mathrm{d}u = pE[H] \quad \forall p \in (0,1) \tag{9}$$

$$\implies \mathrm{E}\left[H \mid Y = F_Y^{-1}(p)\right] = E[H] \quad \forall p \in (0,1), \tag{10}$$

where the implication (10) is a result of differentiating both sides of the equality in (9) with respect to p. We can differentiate the left side of the equality in (9) with respect to

p because the continuity of $f_{H,Y}$ implies that $\mathrm{E}\left[H \mid Y = F_Y^{-1}(p)\right]$ is a continuous function of $p \in (0,1)$. Finally, because $f_{H,Y}$ is bounded away from zero on $\mathcal{H} \times \mathcal{Y}$, the marginal CDF $F_Y(\cdot)$ is strictly increasing on $\mathcal{Y}$; therefore, for each $p \in (0,1)$ $\exists! y \in \mathcal{Y}$ such that $\mathrm{E}\left[H \mid Y = F_Y^{-1}(p)\right] = \mathrm{E}\left[H \mid Y = y\right]$, which concludes this part of the proof.

Next, we prove $\mathrm{E}\left[H \mid Y = y\right] = \mathrm{E}\left[H\right] \quad \forall y \in \mathcal{Y} \implies C(p) = p \quad \forall p \in (0,1)$ using the direct method. By the strict monotonicity of the marginal CDF $F_Y(\cdot)$, for each $y \in \mathcal{Y}$ $\exists! p \in [0,1]$ such that $\mathrm{E}\left[H \mid Y = y\right] = \mathrm{E}\left[H \mid Y = F_Y^{-1}(p)\right]$. Hence, for each $p \in (0,1)$

$$C(p) = \frac{\int_0^p E\left[H \mid Y = F_Y^{-1}(u)\right]\, du}{\mathrm{E}\left[H\right]} = \frac{\int_0^p \mathrm{E}\left[H\right]\, du}{\mathrm{E}\left[H\right]} = \frac{p\mathrm{E}\left[H\right]}{\mathrm{E}\left[H\right]} = p. \tag{11}$$

This concludes the proof. ■

B Regularity Conditions

This section introduces the regularity conditions for testing $\mathrm{E}\left[H \mid Y = y\right] = \mathrm{E}\left[H\right] \; \forall y \in \mathcal{Y}$. These conditions are the same as those in Härdle and Mammen (1993), but specialized to our testing problem. Assumptions (A1) and (A2) in Härdle and Mammen (1993) when specialized to our framework are given by Conditions 1 and 2 respectively:

Condition 1 *The support of Y_i, which is $\mathcal{Y}$, is a compact subset of $\mathbb{R}$. The marginal density $f_Y(\cdot)$ of Y_i is bounded away from zero on $\mathcal{Y}$.*

Condition 2 *$m(\cdot)$ and $f_Y(\cdot)$ are twice continuously differentiable. $\psi(\cdot)$ is continuously differentiable.*

Popular choices for the weight function are $\psi(y) = f_Y(y)$ or $\psi(y) = f_Y^2(y)$ in which case one uses the kernel density estimator of $f_Y(\cdot)$.

The setup in Härdle and Mammen (1993) allows for conditional heteroskesticity; see Assumption (A4) in their paper. In our framework, this assumption is given by

Condition 3 *$\sigma^2(y) = VAR\left(H_i \mid Y_i = y\right)$ is bounded away from 0 and from ∞ on $\mathcal{Y}$.*

Assumption (A5) in Härdle and Mammen (1993) is that the moment generating function is uniformly bounded in a small enough neighborhood of zero. In our framework, this assumption is given by

Condition 4 *$E[\exp(t\varepsilon_i)]$ is bounded in i and n for $|t|$ small enough, where $\varepsilon_i = H_i - m(Y_i)$ for each $i = 1, \ldots, n$.*

For the kernel K and bandwidth h we require that they satisfy Assumptions (K1) and (K2) in Härdle and Mammen (1993). These assumptions in our framework are respectively given the following conditions.

Condition 5 *The kernel K is a symmetric, twice continuously differentiable function with compact support, furthermore $\int K(u)\,du = 1$.*

Condition 6 *The bandwidth h satisfies $h = h_n \sim n^{-1/5}$.*

An example of a kernel function that satisfies Condition 5 is the quartic kernel:

$$K(u) = \frac{15}{16}\left(1 - u^2\right)^2 1\left[|u| \leq 1\right].$$