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Technology, team production and incentives

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Abstract

Incentive reversal (*IR*) is when higher rewards induce some agents to reduce their effort (Winter, 2009). We show that *IR* can hold for all agents when: there is an improvement in production technology; and rewards are based on team output. Whilst *IR* requires at least one worker's marginal return to be decreasing in team productivity when agents invest simultaneously, this is not necessary with sequential investments. Rather, *IR* can occur with sequential investment when the marginal return of effort for all agents is increasing with improvements in technology.

Key words: moral hazard in teams, technology, productivity, incentive reversal.
JEL classifications: D21, L23

1 Introduction

For people working in a team, the incentive to work hard and put in effort depends on the environment they find themselves in.[1] At first glance, it might appear that the adoption of a productivity-enhancing technology would help elicit greater effort from agents; this is, in fact, not always the case. In this paper we study how an improvement in a team's productivity influences individuals' investment incentives.

Adopting the model of Mai et al. (2014), we examine individual effort incentives when payoffs (rewards) depend on team output, *à la* Holmstrom (1982). Specifically, we show that agents might have an incentive to reduce their effort as the team itself becomes more productive. This result draws on the model of Winter (2009), who shows that incentive

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[1] Other researchers have noted this in different contexts, for example: Segal (1999, 2003), Bernstein and Winter (2012) and Winter (2004, 2006) consider the optimal design of incentive contracts when there are externalities between agents.

reversal (hereafter *IR*) – the situation when greater rewards induce lower effort from some agents – can occur when a larger payoff induces one agent to always invest, which in turn creates an incentive for other agents to free ride.[2] Here, we extend his analysis by allowing for both simultaneous and sequential investment. In this way, our model illustrates that *IR* can occur in a broader range of contexts than previously considered. Moreover, in our set-up *IR* can hold for all the team members, not just some of the agents as in Winter (2009).

2 The model and results

Following Mai et al. (2014), consider an incomplete-contracting model with two agents, A_1 and A_2. Each agent can expend some specific effort $e_i \in [0, \overline{e}_i)$[3], for $i = 1, 2$, with a cost of $C_i(e_i)$, as summarized in Assumption 1.

Assumption 1. $C_i(e_i) \geq 0$ and $C_i(0) = 0$. $C_i(e_i)$ is twice differentiable. $C_i'(e_i) > 0$ and $C_i''(e_i) \geq 0$ for $e_i \in [0, \overline{e}_i)$. In addition, $\lim_{e_i \to \overline{e}_i} C_i'(e_i) = \infty$ for $i = 1, 2$.

Working as a team or partnership, the agents produce joint surplus $v(e, s)$, where $e = (e_1, e_2)$. s is an index of team productivity; without loss of generality, $s \in [0, 1]$. There are two alternative investment timing regimes – simultaneous and sequential. With simultaneous investment, at date 0 the two agents choose their level of relationship-specific non-contractible effort. At date 1, the agents bargain over their share of gross surplus. With sequential investment the ex ante agent invests at date 0. Having observed this, the ex post agent makes her investment at date 1. At date 2, the agents bargain and the game ends.

Without loss of generality, each party receives a share $B_i(e, s)$, $i = 1, 2$, of the gross surplus generated, given that $B_1(e, s) + B_2(e, s) = v(e, s)$ and the conditions outlined in Assumption 2.

Assumption 2. $B_i(e, s)$ is twice differentiable with respect to all variables. $B_i(e, s) \geq 0$, $\frac{\partial B_i(e,s)}{\partial e_i} \geq 0$, $\frac{\partial B_i(e,s)}{\partial s} > 0$, $\frac{\partial^2 B_i(e,s)}{\partial e_i^2} < 0$, $\frac{\partial^2 B_i(e,s)}{\partial e_1^2} \frac{\partial^2 B_i(e,s)}{\partial e_2^2} - \left(\frac{\partial^2 B_i(e,s)}{\partial e_1 \partial e_2} \right)^2 > 0$ for $i = 1, 2$ and $s \in [0, 1]$.

As noted in the Introduction, Winter (2009) illustrates the possibility of *IR* when an increase in the payoffs (incentives) causes some agents to reduce their effort. In the framework presented here, *IR* arises if workers exert less effort as a result of being part of a more productive team.

[2]These theoretical predictions are supported by experimental evidence of *IR* in Klor et al. (2014). See Winter (2009) for a discussion of real-world examples of *IR*.

[3]Note that $\overline{e}_i > 0$ and it is possible that $\overline{e}_i = \infty$.

2.1 Simultaneous case

The objective function for each agent in the simultaneous case is

$$A_i(e,s) = B_i(e,s) - C_i(e_i), \;\; \forall \;\; i = 1,2. \tag{1}$$

The equilibrium levels of effort $e(s)$ solve the following first-order conditions:

$$\frac{\partial B_i(e,s)}{\partial e_i} = \frac{\partial C_i(e_i)}{\partial e_i}, \;\; \forall \;\; i = 1,2. \tag{2}$$

The solution for each agent's investment choice exists and is unique due to Assumptions 1 and 2. From this we derive our benchmark result – the conditions for *IR* with simultaneous investment.

Proposition 1. *With simultaneous investment, for* IR *to hold:*

1. $\frac{\partial^2 B_i(e,s)}{\partial e_i \partial s} < 0$, *for at least one* i *and some* $s \in [0,1]$ *is necessary; and*
2. $\frac{\partial^2 B_i(e,s)}{\partial e_i \partial s} < 0$ *and* $\frac{\partial^2 B_i(e,s)}{\partial e_1 \partial e_2} > 0 \; \forall \; i = 1,2$ *and* $\forall \; s \in [0,1]$ *is sufficient.*

If the marginal return of effort for both agents is always increasing in productivity (s), there will never be *IR* – hence, a necessary condition for *IR* is that, for some s and at least one agent, their effort and the productivity level are not complementary. From Part 2, if efforts are complementary with one another $\left(\frac{\partial^2 B_i(e,s)}{\partial e_1 \partial e_2} > 0\right)$, the effort choices of team members move together. Consequently, if the marginal productivity of effort for both agents is decreasing in s $\left(\frac{\partial^2 B_i(e,s)}{\partial e_i \partial s} < 0\right)$, it must be the case that in equilibrium both efforts fall with s.[4]

While there are various specific cases around these conditions, we will highlight one when $\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial s} < 0$ and $\frac{\partial^2 B_2(e,s)}{\partial e_2 \partial s} \geq 0$. In this case, the marginal productivity of only effort e_1 is decreasing in s. If there is to be *IR*, the negative impact of s (directly via $\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial s} < 0$ for e_1 and through $\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial e_2}$ for e_2) in equilibrium needs to dominate its positive impact (directly via $\frac{\partial^2 B_2(e,s)}{\partial e_2 \partial s} \geq 0$ for e_2 and through $\frac{\partial^2 B_2(e,s)}{\partial e_1 \partial e_2}$ for e_1).[5]

Example 1 illustrates the possibility of *IR* with simultaneous investment.

Example 1

Assume that the joint surplus is $v(e_1, e_2, s) = 3s(1 - e_1 - e_2) + 21(e_1 + e_2) - 80(e_1^3 + e_2^3)$. Each agent's effort cost is $C_i(e_i) = 9e_i$ for $i = 1,2$, and $s \in [0,1]$. For simplicity, we assume

[4] An equivalent scenario, when a change that improves overall productivity also being associated with a decrease in marginal returns, has been analyzed previously in the literature, for example see Bel (2013) and Bel et al. (2015).

[5] See the proof of Proposition 1 in the Appendix.

that the shares for the two parties are $B_1(e,s) = B_2(e,s) = v(e,s)/2$. Using (2), the optimal efforts are

$$e_1^{sim} = e_2^{sim} = \sqrt{\frac{1-s}{80}}. \tag{3}$$

The efforts are decreasing in the productivity level s – there is an *IR*.[6] □

It is worth noting several additional points. First, in our model *IR* can result in all agents decreasing their effort. This is not possible in Winter (2009) where *IR* can only hold for a subset of agents. Second, whilst *IR* in Winter (2009) relies on increasing returns with respect to agents' efforts, we make the usual assumption of diminishing returns to effort. Third, the mechanism underlying *IR* here is different from in Winter (2009). *IR* in Winter (2009) arises from an agent's best-response choice to shirk in response to another agent's dominant strategy to invest in a discrete-choice setting; *IR* occurs here because the marginal return of effort for at least one agent is not aligned with overall team output. Taken together, our model shows that *IR* can occur in a broader range of circumstances than initially considered by Winter (2009).

2.2 Sequential case

Let us now consider the case when efforts are made sequentially by the two agents, which parallels the timing of investment in Winter (2009). The objective function for ex ante agent in this case is

$$A_1(e_1,s) = B_1(e_1, e_2(e_1), s) - C_1(e_1), \tag{4}$$

whilst the objective function of ex post agent is given by

$$A_2(e,s) = B_2(e,s) - C_2(e_2), \tag{5}$$

where $e_2(e_1)$ is the ex post agent's best response to the choice of the ex ante agent at date 0. The equilibrium efforts $e(s)$ solve the first-order conditions

$$\begin{cases} \frac{\partial B_1(e,s)}{\partial e_1} + Y(e,s) = \frac{\partial C_1(e_1)}{\partial e_1}, \text{ and} \\ \frac{\partial B_2(e,s)}{\partial e_2} = \frac{\partial C_2(e_2)}{\partial e_2}, \end{cases} \tag{6}$$

where

$$Y(e,s) = \frac{\partial B_1(e,s)}{\partial e_2}\frac{\partial e_2}{\partial e_1} = \frac{\partial B_1(e,s)}{\partial e_2}\left(\frac{\frac{\partial^2 B_{12}(e,s)}{\partial e_1 \partial e_2}}{\frac{\partial^2 C_2(e_2)}{\partial e_2^2} - \frac{\partial^2 B_{22}(e,s)}{\partial e_2^2}}\right)$$

[6]Note Assumptions 1-2 are satisfied for effort levels $e_i \le e_i^{sim}$. For higher effort levels we assume that alternative specific functional forms apply to ensure that the assumptions of the model are satisfied.

is the *internalization effect* introduced in Mai et al. (2014). This effect captures how the ex ante agent strategically manipulates her effort, accounting for the ex post agent's response. To ensure uniqueness, we make the following additional assumption.[7]

Assumption 3. The objective function of an ex ante agent (4) is strictly concave in e_1.

The solution to (6) exists and is unique due to Assumptions 1-3.

Proposition 2. *With sequential investment, for* IR *to hold:*

1. *either* $\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial s} + \frac{\partial Y(e,s)}{\partial s} < 0$ *or* $\frac{\partial^2 B_2(e,s)}{\partial e_2 \partial s} < 0$ *for some* $s \in [0,1]$ *is necessary;*
2. $\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial s} + \frac{\partial Y(e,s)}{\partial s} < 0$, $\frac{\partial^2 B_2(e,s)}{\partial e_2 \partial s} < 0$, $\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial e_2} + \frac{\partial Y(e,s)}{\partial e_2} > 0$ *and* $\frac{\partial^2 B_2(e,s)}{\partial e_1 \partial e_2} > 0 \ \forall \ s \in [0,1]$ *is sufficient.*

The intuition here is similar to the simultaneous case, with the additional complication that we need to take into account the *internalization effect*. If an increase in s enhances the net marginal return of effort for both the ex ante and ex post agent, there will never be *IR*; consequently, *IR* requires that the net impact of s on marginal return for at least one effort is negative, as outlined in Part 1. From Part 2, there will be *IR* if the net marginal return of effort for both agents is reduced by an increase in s when efforts are complementary $\left(\frac{\partial^2 B_2(e,s)}{\partial e_1 \partial e_2} > 0\right)$.

Again, it is possible to consider other cases. For example, suppose that the net marginal return of ex ante and ex post effort is declining and increasing in the productivity level s, respectively: that is, $\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial s} + \frac{\partial Y(e,s)}{\partial s} < 0$ and $\frac{\partial^2 B_2(e,s)}{\partial e_2 \partial s} \geq 0$. In this instance, *IR* requires the negative influence of s, directly via $\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial s} + \frac{\partial Y(e,s)}{\partial s} < 0$ for e_1 and through $\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial e_2}$ for e_2, in equilibrium dominates its positive impact on marginal productivity, directly via $\frac{\partial^2 B_2(e,s)}{\partial e_2 \partial s} \geq 0$ for e_2 and through $\frac{\partial^2 B_2(e,s)}{\partial e_1 \partial e_2}$ for e_1.[8]

One difference with the simultaneous case is worth mentioning: *IR* is possible with sequential investment even when the marginal return of effort for all agents is increasing with improvements in technology $\left(\frac{\partial^2 B_i(e,s)}{\partial e_i \partial s} \geq 0 \text{ for } i = 1,2\right)$ provided that the ex ante agent's net marginal return is negative $\left(\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial s} + \frac{\partial Y(e,s)}{\partial s} < 0\right)$. It is the presence of the *internalization effect*, captured here by $\frac{\partial Y(e,s)}{\partial s}$, that makes this possible.

[7] Gal-Or (1985) and Mai et al. (2014) make an equivalent assumption to ensure unique solutions in their Stackelberg-style games examining first- and second-mover advantages. Moreover, as noted by Gal-Or, uniqueness is guaranteed here if the third and higher-order derivatives of all value and cost functions are zero.

[8] The specific conditions are outlined in the Appendix.

Example 2

Assume that the joint surplus is $v(e_1, e_2, s) = (5.2 + 0.8s)(e_1 + e_2) + 0.8(1 - s)e_1 e_2 - (e_1^2 + e_2^2)$. Each agent's effort cost is $C_i(e_i) = 1.2e_i$ for $i = 1, 2$. In addition, we assume that $s \in [0, 1]$ and the shares for the two parties are $B_1(e, s) = B_2(e, s) = v(e, s)/2$. Note, Assumptions 1-3 are satisfied for $e_i \leq 1$.[9]

From the simultaneous case FOC, investments are $e_1^{sim}(s) = e_2^{sim}(s) = \frac{7+2s}{8+2s}$.

Now consider the sequential investment. The following system of equations describes this case

$$\begin{cases} 2.6 + 0.4s + 0.4(1 - s)e_2 - 2e_1 + 0.24(1 - s) = 1.2; \\ 2.6 + 0.4s + 0.4(1 - s)e_1 - 2e_2 = 1.2. \end{cases} \tag{7}$$

The solutions are $e_1^{seq}(s) = \frac{2.8+0.8s+0.4(1-s)(2.6+0.4s)}{4-0.16(1-s)^2}$ and $e_2^{seq}(s) = 0.7 + 0.2s + 0.2(1 - s)e_1^{seq}(s)$. Both efforts are higher with the sequential regime rather than the simultaneous regime due to the positive *internalization effect* $Y(e, s) = 0.24(1 - s)$, see Mai et al. (2014). As $e_i^{sim}(s)$ increases with productivity s, while the *internalization effect* decreases with productivity s, the combined effect on sequential investment could go either way. As we can see from Figure 1, for $s \leq 0.5$ both investments decrease with s because the decrease in the *internalization effect* dominates the increase in the simultaneous investment. On the other hand, for $s > 0.5$ only e_1^{seq} decreases with s, while in the case of ex post investment the increase in the simultaneous investment dominates the decrease due to the *internalization effect*. Note that when $s = 1$ the *internalization effect* is absent and all four investments are the same $e_i^{sim} = e_1^{seq} = e_2^{seq}$. $\square$

3 Appendix

Proof of Proposition 1

Applying the Implicit Function Theorem to the system of equations in (2) yields that equilibrium efforts exhibit *IR* iff:

$$\frac{\frac{\partial^2 B_i(e,s)}{\partial e_i \partial s}\left(\frac{\partial^2 C_j(e_j)}{\partial e_j^2} - \frac{\partial^2 B_j(e,s)}{\partial e_j^2}\right) + \frac{\partial^2 B_j(e,s)}{\partial e_j \partial s}\frac{\partial^2 B_i(e,s)}{\partial e_i \partial e_j}}{\left(\frac{\partial^2 C_1(e_1)}{\partial e_1^2} - \frac{\partial^2 B_1(e,s)}{\partial e_1^2}\right)\left(\frac{\partial^2 C_2(e_2)}{\partial e_2^2} - \frac{\partial^2 B_2(e,s)}{\partial e_2^2}\right) - \left(\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial e_2}\right)\left(\frac{\partial^2 B_2(e,s)}{\partial e_1 \partial e_2}\right)} < 0, \tag{8}$$

where $i, j = 1, 2$ and $i \neq j$. Combining (8) with Assumptions 1 and 2 proves the proposition. $\square$

[9] As earlier, for higher effort levels we assume that alternative specific functional forms apply to ensure that these assumptions are satisfied.

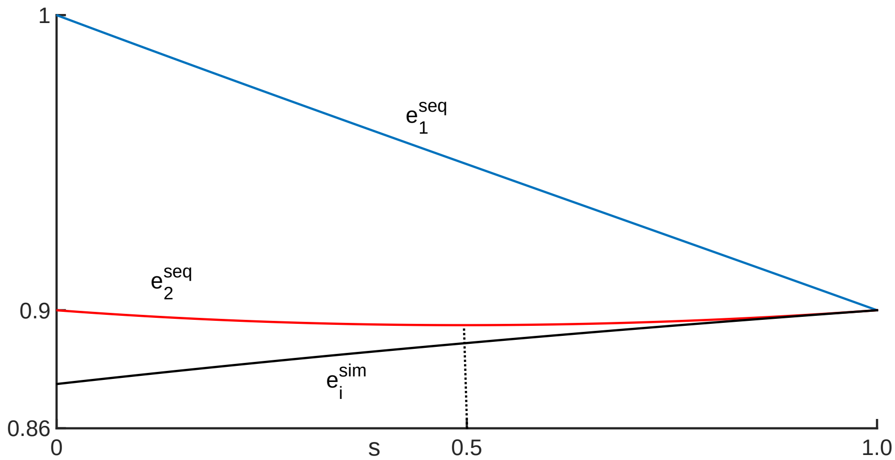

Figure 1: Effort levels e_1^{seq}, e_2^{seq} and e_i^{sim}

Proof of Proposition 2

Applying the Implicit Function Theorem to the system of equations in (6) yields that equilibrium efforts exhibit *IR* iff

$$\left(\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial s} + \frac{\partial Y}{\partial s}\right)\left(\frac{\partial^2 C_2(e_2)}{\partial e_2^2} - \frac{\partial^2 B_2(e,s)}{\partial e_2^2}\right) + \frac{\partial^2 B_2(e,s)}{\partial e_2 \partial s}\left(\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial e_2} + \frac{\partial Y}{\partial e_2}\right) < 0, \quad (9)$$

and

$$\frac{\partial^2 B_2(e,s)}{\partial e_2 \partial s}\left(\frac{\partial^2 C_1(e_1)}{\partial e_1^2} - \frac{\partial^2 B_1(e,s)}{\partial e_1^2} - \frac{\partial Y}{\partial e_1}\right) + \frac{\partial^2 B_2(e,s)}{\partial e_1 \partial e_2}\left(\frac{\partial^2 B_1(e,s)}{\partial e_1 \partial s} + \frac{\partial Y}{\partial s}\right) < 0. \quad (10)$$

Combining inequalities 9-10 with Assumptions 1 and 2 proves the proposition. □

References

R. Bel. Access, veto, and ownership in the theory of the firm. *Journal of Law, Economics and Organization,*, 29(4):871–897, 2013.

R. Bel, V. Smirnov, and A. Wait. Team composition, worker effort and welfare. *Intenational Journal of Industrial Organization*, 41:1–8, 2015.

S. Bernstein and E. Winter. Contracting with heterogeneous externalities. *American Economic Journal: Microeconomics*, 4(2):50–76, 2012.

E. Gal-Or. First mover and second mover advantages. *International Economic Review*, 26 (3):649–653, 1985.

B. Holmstrom. Moral hazard in teams. *Bell Journal of Economics*, 13(2):324–340, 1982.

E.F. Klor, S. Kub, E. Winter, and R. Zultan. Can higher rewards lead to less effort? incentive reversal in teams. *Journal of Economic Behavior and Organization*, 97:72–83, 2014.

M. Mai, V. Smirnov, and A. Wait. Ownership, access, and sequential investment. *Canadian Journal of Economics*, 47(1):203–231, 2014.

I. Segal. Complexity and renegotiation: A foundation for incomplete contracts. *Review of Economic Studies*, 66(1):57–82, 1999.

I. Segal. Collusion, exclusion, and inclusion in random-order bargaining. *Review of Economic Studies*, 70(2):439–460, 2003.

E. Winter. Incentives and discrimination. *American Economic Review*, 94(3):764–773, 2004.

E. Winter. Optimal incentives for sequential production processes. *Rand Journal of Economics*, 37(2):376–390, 2006.

E. Winter. Incentive reversal. *American Economic Journal: Microeconomics*, 1(2):133–47, 2009.