THE UNIVERSITY OF SYDNEY

Economics Working Paper Series

2015 - 14

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Denny Lie & Anirudh S. Yadav

May 2015

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Denny Lie† — University of Sydney

Anirudh S. Yadav — Reserve Bank of Australia

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Abstract

This paper investigates whether the persistence and the time-varying nature of trend inflation can explain the persistence of inflation in Australia — that is, whether it can explain the apparent need for the backward-looking inflation term in the New Keynesian Phillips curve (NKPC) estimated using Australian data. We derive and estimate an extended open-economy NKPC equation, accounting explicitly for time-varying trend inflation. The paper finds that although the estimated role for backward-looking indexation is near zero in some difference-equation specifications, when one considers the closed-form specifications of the NKPC, the parameter estimate increases dramatically, implying a high degree of indexation to past inflation. Thus, in contrast to Cogley and Sbordone's (2008) result for the US economy, our estimates suggest that accounting for time variation in trend inflation in the NKPC cannot explain away the inertia in the Australian inflation data. Our preferred estimates suggest that lagged inflation and future expectations of inflation enter the NKPC with almost equal weights. Finally, notwithstanding the previous results, we find a marked decline in the role of the backward-looking inflation terms since the adoption of an inflation targeting regime by the Reserve Bank in 1993.

JEL Classification: E12, E31, E52

Keywords: time-varying trend inflation, inflation persistence, New Keynesian Phillips Curve, Australia, backward-looking indexation.

*This paper is a revised version of and is based on Anirudh Yadav's thesis in the undergraduate honours program at the University of Sydney's School of Economics. We thank Tim Robinson for generously sharing his data set. This paper has benefited from comments, suggestions, and advice from Yunjong Eo, Stella Huangfu, Diane Hutchinson, Adrian Pagan, Russell Ross, and Aarti Singh. Views expressed in this paper are those of the authors and not necessarily those of the Reserve Bank of Australia. All errors are ours.

†Corresponding author | Email: denny.lie@sydney.edu.au | Address: School of Economics, The University of Sydney, Rm 370 Merewether H04, NSW 2006.

1 Introduction

The true nature of inflation dynamics is an ongoing matter of debate and investigation in macroeconomics. That such attention is devoted to dynamics of inflation is due to its importance, not only for understanding the nature of business cycles, but also for determining the appropriate path for monetary policy. Modern micro-founded models of inflation dynamics are typically derived from the assumption that the price-adjusting firms are forward-looking when setting their new prices, e.g. as in the infrequent-adjustment model of Calvo (1983) or the staggered-contract model of Taylor (1980). This assumption leads to a so-called purely forward-looking New Keynesian Phillips curve (NKPC) where inflation depends only on its future expectation and a measure of real activity, typically either real marginal cost or an output gap measure. Despite its theoretical elegance, the purely forward-looking incarnation of the NKPC has been shown to perform poorly against the data. The empirical shortcomings of the purely forward-looking NKPC are generally attributed to its inability to replicate the innate persistence present in actual inflation (see e.g. Fuhrer and Moore, 1995 and Fuhrer, 2010).[1] In order to enhance the degree of persistence within the model several authors have proposed somewhat ad-hoc rationales for the inclusion of lagged inflation terms in the NKPC, e.g. the assumption of ad-hoc backward-looking firms in Gali and Gertler (1999) or the past indexation in Christiano, Eichenbaum and Evans (2005). While the resulting 'hybrid Phillips curves' do indeed improve the fit of the model, their questionable microfoundations are an obvious source of criticism.

The unsound structural interpretation of the nature of the backward-looking term in the hybrid NKPC begs the question of whether some other source of persistence is missing from the standard NKPC formulation. In a recent seminal contribution, Cogley and Sbordone (2008) propose that the apparent need for a backward-looking inflation component in the NKPC in order to fit the data can be attributed to the persistence and the time-varying nature of trend inflation. They stress that to understand inflation persistence it is paramount to model variation in this slow-moving inflation trend, which can be interpreted as the central bank's implicit medium- to long-run inflation target. Given that shifts in trend inflation are attributable to movements in the policy target, it follows that this source of persistence is independent of any intrinsic persistence derived from the price-setting process. Failing to take these shifts into account would thus lead to biased estimates of the

[1] Fuhrer and Moore (1995) and Fuhrer (2010) attribute the inadequate persistence to the fact that the purely forward-looking NKPC does not have its own intrinsic persistence mechanism — that is, inflation only has inherited persistence, coming from its driving process (e.g. the output gap).

NKPC coefficients. Cogley and Sbordone find that once we incorporate this time-varying trend inflation in the estimation of the NKPC the coefficient on the backward-looking inflation term is zero and the NKPC is thus purely forward looking. Put another way, it is the persistence of the trend inflation, rather than an inherent feature of firms' price adjustment behavior, that gives rise to the persistence of inflation observed in the data.[2]

This paper investigates whether the persistence and the time-varying nature of trend inflation can also explain the persistence of inflation in Australia — that is, whether it can also explain the apparent need for the backward-looking inflation term in NKPC estimated using Australian data. Similar to Cogley and Sbordone (2008) we incorporate time-varying trend inflation into an otherwise-standard Calvo model with backward-looking indexation and derive the NKPC equation by log-linearising the equilibrium conditions of the model around a shifting steady-state associated with time-varying trend inflation. We extend the model by incorporating small open-economy aspects relevant to the Australian economy, resulting in an extended open-economy NKPC in which the coefficients are functions not only of the model's structural parameters, but also trend inflation. Trend inflation is model as a driftless random walk, as in Cogley and Sargent (2005) and Cogley, Primiceri, and Sargent (2010).

In terms of the econometric method, we utilize a two-stage estimation procedure, also used in Cogley and Sbordone (2008), to estimate the structural parameters of the NKPC. The first stage involves estimating a Bayesian vector autoregression (VAR) with time-varying trends, subject to reflecting barriers, described in Cogley and Sargent (2005). As in the derivation of the NKPC, the Bayesian VAR is also adapted to capture the small open-economy aspects of the Australian economy. The second stage involves a minimum distance estimator, which exploits the cross-equation restrictions implied by the NKPC and the first-stage VAR estimates. We also investigate the sensitivity of the two-stage estimation procedure to the imposition of different degrees of model-consistent restrictions on expectations during the second stage. As shown in Barnes, Gumbau-Brisa, Lie, and Olivei (2011) and Gumbau-Brisa, Lie, and Olivei (2015), this partial-information method could yield drastically different estimates of the structural parameters, depending on whether the *difference-equation* specification or the *closed-form* specification of the NKPC is imposed in the second estimation stage. One can interpret the difference-equation and the closed-form specifications of the NKPC as imposing minimum and maximum degree of model-consistent restrictions on expectations, respectively. Further to this, we discuss evidence that seems to suggest preference

[2] Cogley, Primiceri and Sargent (2010) term this persistence as "inflation-gap persistence."

towards the closed-form estimates.[3]

Our results point to three main conclusions. First, although the estimated role for backward-looking indexation is near zero in some difference-equation specifications, when one considers the closed-form specifications of the NKPC, the parameter estimate increases dramatically, implying a high degree of indexation to past inflation. Thus, the estimates suggest that accounting for time-varying trend inflation in the NKPC cannot explain away the inertia in Australian inflation. Second, despite the time variation in trend inflation, which alters the relative magnitude of the NKPC coefficients, the NKPC assigns near equal weights to lagged inflation and to expected future inflation. The enhanced magnitude of the backward-looking coefficient is in contrast to conventional GMM estimates of the Australian NKPC, which suggest a predominant role for forward-looking behaviour. Finally, and notwithstanding the aforementioned conclusions, the structural analysis reveals a marked decline in the persistence of inflation since the Reserve Bank's implementation of an inflation targeting regime in 1993.

The remainder of the paper proceeds as follows. Section 2 describes the standard NKPC equation for Australia under the assumption of zero, constant trend inflation. We also introduce the two-stage estimation procedure and present both the difference-equation and the closed-form specifications of the NKPC. Section 3 contains the primary structural analysis and results, incorporating the time-varying trend inflation. A robustness analysis is conducted in Section 4. Section 5 concludes.

1.1 Review of past NKPC and inflation-persistence studies based on Australian data

Here, we briefly survey the existing relevant NKPC and inflation-persistence studies in the literature based on Australian data.

Documentation of inflation persistence in Australia has been limited, with the bulk of the research having been conducted for the U.S. and Europe. Benati (2008) documents empirical evidence on changes in the reduced-form persistence of inflation for a broad array of developed countries. His analysis draws evidence from long samples and focuses on differences in estimated inflation persistence across different monetary policy regimes. While the analysis does not focus specifically on Australia's inflation experience, a key finding of the paper is that inflation persistence

[3] Within a conventional NKPC with a constant (zero) trend inflation, Rudd and Whelan (2005) also find that the estimates of the structural parameters based on the closed-form specification are significantly different from those obtained from estimating the difference-equation form directly.

has declined dramatically in recent years for all countries that have adopted an inflation targeting monetary policy. In Australia the inflation targeting framework was first adopted by the Reserve Bank in 1993 as an operational interpretation of its price stability mandate.[4] Not only has inflation fallen since the adoption of an explicit target, but Benati's (2008) results also suggest that inflation in Australia exhibits a marked decline in persistence post-inflation targeting.

Discussion of structural sources of inflation persistence in Australia is also relatively sparse. Gruen, Pagan, and Thompson (1999) discuss the development of the Phillips curve in Australia from the 1950s-1990s. Although their paper does not focus on the issue of inflation persistence, the authors' preliminary results indicate that inflation expectations are predominantly backward-looking. Nimark (2009) estimates a structural model of the Australian economy and employs an ad-hoc hybrid Phillips curve, with indexation to one lag of inflation. Similarly, Jaaskela and Nimark (2011) employ a hybrid Phillips curve with an indexation parameter in their medium-scale New Keynesian model of the Australian economy. A recent study by Kuttner and Robinson (2010) discusses the flattening of the Phillips curve with particular reference to the experiences in the U.S and Australia. The authors follow a rule-of-thumb price-setting model as stipulated in Gali and Gertler (1999) and estimate the resulting hybrid NKPC for both the U.S and Australia. The authors' results for Australia indicate an economically sizeable flattening of the Phillips curve since 1960. More importantly, for our context, the estimates in all three studies above show that current inflation does have a positive and significant backward-looking component, albeit with less economic significance compared to the forward-looking component.

The empirical literature on the Australian NKPC emphasises that the open economy aspects of the inflation process demand greater attention than in structural models of the U.S economy. One approach to embedding small open-economy aspects within a New Keynesian framework was developed by Monacelli (2005).[5] In essence, Monacelli (2005) postulates that the domestic economy is populated by two types of firms: domestic producers and importers. Prices of domestically-produced goods follow Calvo dynamics and may be adjusted for backward-looking behaviour. The Australian models employed by Nimark (2009), Jaaskela and Nimark (2011) and Kuttner and Robinson (2010) simplify Monacelli's specification by assuming the law of one price holds for import prices at the docks. The two-sector generalisation yields two Phillips curves where CPI inflation is

[4] International institutions, such as the OECD and IMF, have accepted the above dating. Bernanke et. al. (2001), by contrast, date the start of inflation targeting as 1994. The regime was not formally endorsed until 1996, when a new government signed a letter of agreement with a new Governor, Ian Macfarlane, upon his appointment.

[5] And its closely related precursor, Gali and Monacelli (2005).

simply the weighted average of inflation in the two sectors. Kuttner and Robinson (2010) make a further simplifying assumption that the Calvo parameter governing the degree of price stickiness is identical across both the domestic sector and the import sector. This assumption results in a single equation aggregate Phillips curve, which is the form used in the subsequent empirical analysis.

2 A Baseline New Keynesian Phillips Curve for Australia

To fix ideas and contextualise the main structural investigation in Section 3, we first consider a conventional NKPC based on the Calvo model with an indexation mechanism in the vein of Christiano, Eichenbaum and Evans (2005). By conventional, we mean that the resulting NKPC is derived from the log-linearisation of the equilibrium conditions around a zero, constant trend inflation.

In each period t, each firm i faces a constant probability, $(1-\alpha)$, of being able to adjust its nominal price optimally. Firms that are not allowed to reoptimize their price can still update their current price according to an indexation mechanism based on lagged aggregate inflation. In line with Barnes, Gumbau-Brisa, Lie, and Olivei (2011, henceforth BGLO), the indexation mechanism is adapted to allow for two lags of inflation:

$$P_t(i) = (\Pi_{t-1}^{\tau}\Pi_{t-2}^{1-\tau})^{\rho} P_{t-1}(i).$$

Here $\Pi_{t-1} = P_t/P_{t-1}$ is the aggregate gross inflation rate. The parameter $\rho \in [0,1]$ governs the degree of indexation, with $\rho = 0$ stipulating the absence of indexation (thus, there is no mechanical updating of prices for firms who cannot reoptimize, resulting in a purely forward-looking NKPC), and $\rho = 1$ denotes full indexation to a weighted average of the two recent lags of inflation. The weight parameter, given by $\tau \in [0,1]$, represents the importance given to $t-1$ aggregate inflation relative to $t-2$ aggregate inflation. In this framework, where trend inflation is zero, the NKPC takes the form

$$\pi_t = \left[\frac{\rho\tau - \beta\rho(1-\tau)}{1+\beta\rho\tau}\right]\pi_{t-1} + \left[\frac{\rho(1-\tau)}{1+\beta\rho\tau}\right]\pi_{t-2} + \left[\frac{\beta}{1+\beta\rho\tau}\right]E_t\pi_{t+1} + \left[\frac{\lambda}{1+\beta\rho\tau}\right]mc_t + u_t, \quad (1)$$

where π is inflation, mc is real marginal costs, and $0 < \beta < 1$ is the subjective discount factor. The coefficient λ is a function of the model's deep parameters, where $\lambda = (1-\alpha)(1-\alpha\beta)/(\alpha + \alpha\theta\omega)$. The parameter $\theta > 1$ is the constant elasticity of substitution across differentiated goods, and ω

is the elasticity of firms' marginal cost with respect to their own output (a measure of strategic complementarity in pricing).

The above expression also nests a more familiar NKPC specification. When $\tau = 1$ only $t-1$ aggregate inflation is relevant, and as such, the specification collapses into

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} + \tilde{\lambda} mc_t,$$

where $\gamma_b = \rho/(1+\beta\rho)$, $\gamma_f = \beta/(1+\beta\rho)$ and $\tilde{\lambda} = \lambda/(1+\beta\rho)$. This generalisation is commonly referred to as the hybrid NKPC and is of the same form as the reduced-form NKPC derived in Gali and Gertler (1999) who introduce lagged inflation through rule-of-thumb pricing, rather than an indexation mechanism. The coefficients γ_b and γ_f measure the backward-looking and forward-looking components of inflation, respectively. The inclusion of τ in the subsequent estimations is significant from an empirical perspective as it may act to reduce the effect of misspecification bias in the structural estimates.

From the *difference-equation* specification of the NKPC in (1) it is possible to solve the NKPC forward so as to obtain a closed-form expression for inflation. Since (1) holds in every period, future expectations of inflation are constrained so as to follow the structure implied by this equation. Iterating forwards yields the *closed-form* expression of the NKPC,

$$\pi_t = \rho\tau\pi_{t-1} + \rho(1-\tau)\pi_{t-2} + \lambda \sum_{k=0}^{\infty} \beta^k E_t mc_{t+k} + u_t. \tag{2}$$

2.1 Open Economy Dimensions

To model the small open-economy aspects we follow the framework developed in Monacelli (2005), which is also used in Nimark (2009) in the context of deriving the NKPC for Australia. The domestic economy is populated by two types of firms: domestic producers and importers. Both domestic producers and importers set prices according to the augmented Calvo mechanism detailed above, where a fraction α^d of firms producing domestically and a fraction α^m of importing firms cannot adjust their prices optimally in a given period. The two-sector generalisation yields two Phillips curves of the form

$$\pi_t^d = \left[\frac{\rho^d\tau^d - \beta\rho^d(1-\tau^d)}{1+\beta\rho^d\tau^d}\right]\pi_{t-1}^d + \left[\frac{\rho^d(1-\tau^d)}{1+\beta\rho^d\tau^d}\right]\pi_{t-2}^d + \left[\frac{\beta}{1+\beta\rho^d\tau^d}\right]E_t\pi_{t+1}^d + \left[\frac{\lambda^d}{1+\beta\rho^d\tau^d}\right]mc_t^d + \upsilon_{t,d}, \tag{3}$$

and

$$\pi_t^m = \left[\frac{\rho^m\tau^m - \beta\rho^m(1-\tau^m)}{1+\beta\rho^m\tau^m}\right]\pi_{t-1}^m + \left[\frac{\rho^m(1-\tau^m)}{1+\beta\rho^m\tau^m}\right]\pi_{t-2}^m + \left[\frac{\beta}{1+\beta\rho^m\tau^m}\right]E_t\pi_{t+1}^m + \left[\frac{\lambda^m}{1+\beta\rho^m\tau^m}\right]mc_t^m + \upsilon_{t,m}. \tag{4}$$

The CPI inflation is simply the weighted average of inflation in both sectors:

$$\pi_t = (1-\phi)\pi_t^d + \phi\pi_t^m, \tag{5}$$

where ϕ is the share of imports in consumption, and the superscripts d and m denote domestically produced goods and imports respectively.[6]

The Australian models employed by Justiniano and Preston (2010) and Jaaskela and Nimark (2011) measure the marginal cost of importers as the relative price of imported goods at the dock (where the law of one price holds) to the retail price of imported goods. In accordance with Monacelli (2005), these models assume that in setting the retail price of their goods, the importers solve a dynamic markup problem (à la Calvo), thus providing a short-run channel for deviations from the law of one price. However, in estimating such models difficulties arise due to the fact that there is no direct measure of the retail price of imported goods. Accordingly, Justiniano and Preston (2010) and Jaaskela and Nimark (2011) treat these prices as an unobserved variable and estimates are achieved using the Kalman filter. Here, we follow the derivation in Kuttner and Robinson (2010), who make the simplifying assumption that the Calvo parameter governing price-stickiness is constant across both the domestic and importing sector, that is $\alpha^d = \alpha^m = \alpha$. This is a strong assumption that is not supported in either Nimark (2009) or Justiniano and Preston (2010), however, given the mixed evidence on the relative duration of prices it is an appropriate compromise for this exercise.[7] The simplifying assumptions allow for the derivation of a single

[6] Note that in the above Phillips curves the slope coefficients, λ^d and λ^m, are functions of their respective sector-specific structural parameters.

[7] Nimark (2009) finds the duration of prices to be less for domestic goods relative to imported goods, whereas

equation aggregate Phillips curve.[8]

Based on the above assumptions and combining (3), (4) and (5), we obtain an open-economy NKPC equation in *difference-equation* form,

$$\pi_t = \left[\frac{\rho\tau - \beta\rho(1-\tau)}{1+\beta\rho\tau}\right]\pi_{t-1} + \left[\frac{\rho(1-\tau)}{1+\beta\rho\tau}\right]\pi_{t-2} + \left[\frac{\beta}{1+\beta\rho\tau}\right]E_t\pi_{t+1}$$
$$+ \left[\frac{\lambda}{1+\beta\rho\tau}\right]\left[(1-\phi)mc_t^d + \phi mc_t^m\right]. \tag{6}$$

In a similar vein to that described previously, (6) can be solved forward to obtain the *closed-form* representation of the Australian NKPC,

$$\pi_t = \rho\tau\pi_{t-1} + \rho(1-\tau)\pi_{t-2} + \lambda\sum_{k=0}^{\infty}\beta^k E_t\left[(1-\phi)mc_{t+k}^d + \phi mc_{t+k}^m\right] + u_t. \tag{7}$$

2.2 Econometric Methodology

We follow the approach in Cogley and Sbordone (2008) in estimating the deep structural parameters of the NKPC: α, θ, ρ and τ. Since the main interest is assessing the importance of the inertial component of inflation the parameters β and ω are pinned down for ease of estimation.[9] The strategic complementarity parameter ω is calibrated at a value of 0.429, while β is constrained to equal 0.99 in all estimations[10]. Thus, we focus on providing inferences about the elasticity of substitution across differentiated goods, θ, the frequency of optimal price readjustment, reflected in the estimates of α, and of most relevance, the extent of indexation to past inflation, ρ. The estimation procedure employs a two-stage method that exploits cross-equation restrictions between the structural parameters of the Calvo model and those of an unrestricted reduced-form VAR. The VAR estimated in the first stage of the procedure is used to proxy for agents' expectations about future inflation and real marginal costs.

Consider a time series vector $\mathbf{x}_t$ that includes n variables. For now, $\mathbf{x}_t$ is constrained to include period t inflation and real marginal costs of both domestic producers and importers[11], so that

Justiniano and Preston (2010) find the opposite.

[8] It is also assumed that the degree of indexation is common across both sectors, i.e. $\rho^d = \rho^m = \rho$. This assumption is similarly imposed in Kuttner and Robinson (2010) and Nimark (2009) who assume that the share of firms that use rule-of-thumb pricing is the same for domestic producers and importers. As a corollary it is also assumed that $\tau^d = \tau^m = \tau$.

[9] Based on the NKPC equation in (6), these two parameters are also not identified separately from the other parameters.

[10] The strategic complementarity parameter is defined as $\omega = 1/(1-\delta)$, where $1-\delta$ is the Cobb-Douglas labour elasticity. Thus, $\omega = 0.429$ is consistent with a value of $\delta = 0.3$.

[11] In subsequent sections with time-varying trend inflation $\mathbf{x}_t$ will be extended to include other variables, such as

$n = 3$. It is assumed that the law of motion of $\mathbf{x}_t$ can be expressed as a reduced-form VAR(p). Then, defining a vector $\mathbf{z}_t = (\mathbf{x}'_t, \mathbf{x}'_{t-1}, ..., \mathbf{x}'_{t-p+1})'$, the VAR(p) may be expressed as

$$\mathbf{z}_t = \mathbf{A}\mathbf{z}_{t-1} + \epsilon_{z,t}, \tag{8}$$

where $\mathbf{A}$ is a square matrix containing the VAR coefficients in the first n rows.[12,13] The conditional expectation of a variable $y_{t+j} \in \mathbf{x}_{t+j}$ at time t can be obtained from the first-stage VAR, so that

$$E_t y_{t+j} = \mathbf{e}'_y \mathbf{A}^j \mathbf{z}_t. \tag{9}$$

The vector $\mathbf{e}'_y$ is a selection vector that isolates y_t in $\mathbf{z}_t$.

The above forecasting rule is used in the second stage to proxy for expectations present in the NKPC. Below, we detail the minimum-distance estimator used in the second stage, which could differ in form, depending on whether the difference-equation (henceforth, DE) specification in (6) or the closed form (henceforth, CF) specification in (7) is used at this estimation stage.

2.2.1 The Difference Equation Specification

Taking expectations conditional on information at $t-2$ of the NKPC in (6) and using the forecasting rule in (9) yields

$$\begin{aligned} \mathbf{e}'_\pi \mathbf{A}^2 \mathbf{z}_{t-2} = &\left[\frac{\rho\tau - \beta\rho(1-\tau)}{1+\beta\rho\tau}\right] \mathbf{e}'_\pi \mathbf{A} \mathbf{z}_{t-2} + \left[\frac{\rho(1-\tau)}{1+\beta\rho\tau}\right] \mathbf{e}'_\pi \mathbf{I} \mathbf{z}_{t-2} + \left[\frac{\beta}{1+\beta\rho\tau}\right] \mathbf{e}'_\pi \mathbf{A}^3 \mathbf{z}_{t-2} \\ &+ (1-\phi)\left[\frac{\lambda}{1+\beta\rho\tau}\right] \mathbf{e}'_{mcdom} \mathbf{A}^2 \mathbf{z}_{t-2} + \phi\left[\frac{\lambda}{1+\beta\rho\tau}\right] \mathbf{e}'_{mcim} \mathbf{A}^2 \mathbf{z}_{t-2}, \end{aligned} \tag{10}$$

where $\mathbf{I}$ denotes an identity matrix of the same dimension as $\mathbf{A}$.[14] The above restriction captures the essence of the minimum distance problem, where the left-hand side of (10) represents the conditional expectation of inflation obtained from the reduced-form VAR, and the right-hand side represents inflation expectations as derived from the structural NKPC. If inflation is truly deter-

output growth and a nominal discount factor.

[12] The coefficient matrix $\mathbf{A}$ has all roots inside the unit circle; $\epsilon_{z,t}$ is a vector of i.i.d. residuals. For simplicity the regression intercepts are omitted here, however, they play an essential role in the NKPC with time-varying trend inflation and will be appropriately introduced in the pursuant sections.

[13] In order to exploit the cross-equation restrictions detailed below it is implicitly assumed that the solution to the structural NKPC model for the variables in $\mathbf{x}_t$ can be well represented, asymptotically, by the reduced-form representation captured by the VAR in (8).

[14] The subscripts *mcdom* and *mcim* denote real marginal costs corresponding to domestic producers and importers respectively.

mined according to the NKPC then the reduced-form VAR forecast of inflation and the forecast according to the NKPC must be equivalent. Imposing that (10) holds for all realisations of $\mathbf{z}_t$ yields a vector of cross-equation restrictions involving the VAR coefficient matrix $\mathbf{A}$ and the structural parameters of the NKPC, which are collected in the vector $\boldsymbol{\psi} = [\alpha, \theta, \rho, \tau]$:

$$\begin{aligned}\mathbf{e}'_\pi \mathbf{A}^2 = & \left[\frac{\rho\tau - \beta\rho(1-\tau)}{1+\beta\rho\tau}\right] \mathbf{e}'_\pi \mathbf{A} + \left[\frac{\rho(1-\tau)}{1+\beta\rho\tau}\right] \mathbf{e}'_\pi \mathbf{I} + \left[\frac{\beta}{1+\beta\rho\tau}\right] \mathbf{e}'_\pi \mathbf{A}^3 \\ & + (1-\phi)\left[\frac{\lambda}{1+\beta\rho\tau}\right] \mathbf{e}'_{mcdom}\mathbf{A}^2 + \phi\left[\frac{\lambda}{1+\beta\rho\tau}\right] \mathbf{e}'_{mcim}\mathbf{A}^2 \\ \equiv & \mathbf{g}^{DE}(\mathbf{A}, \boldsymbol{\psi}).\end{aligned}$$

Or, equivalently

$$\mathbf{F}^{DE}(\mathbf{A}, \boldsymbol{\psi}) \equiv \mathbf{e}'_\pi \mathbf{A}^2 - \mathbf{g}^{DE}(\mathbf{A}, \boldsymbol{\psi}) = \underline{0}',$$

where $\underline{0}$ is a vector of zeroes, the same size as $\mathbf{e}'_\pi$ and the superscript DE indicates that the expression applies to the NKPC in its difference equation form.

Thus, the two-stage minimum distance estimation procedure may be summarised as follows. In the first stage the data, contained in the vector $\mathbf{x}_t$, is fitted to an unrestricted reduced-form VAR as specified in (8). This step yields an estimated coefficient matrix $\hat{\mathbf{A}}$. The second stage of the estimation procedure exploits the cross-equation restrictions described above and involves searching for values of the parameters in $\boldsymbol{\psi}$ that constrain $\mathbf{F}^{DE}(\hat{\mathbf{A}}, \boldsymbol{\psi})$ as being close to zero. Specifically, the parameters $\boldsymbol{\psi}$ are estimated so as to minimise the squared deviation of $\mathbf{g}^{DE}(\hat{\mathbf{A}}, \boldsymbol{\psi})$ from $\mathbf{e}'_\pi \hat{\mathbf{A}}^2$:

$$\widehat{\boldsymbol{\psi}}^{DE} \equiv \arg_{\psi} \mathbf{F}^{DE}(\hat{\mathbf{A}}, \boldsymbol{\psi}) \cdot \mathbf{F}^{DE}(\hat{\mathbf{A}}, \boldsymbol{\psi})' \tag{11}$$

2.2.2 The Closed Form Specification

Estimation of the CF specification of the Australian NKPC, represented in equation (7), is achieved in the same manner as described above. Again, taking expectations at time $t-2$ conditional on the forecasting rule (9) yields

$$\begin{aligned}\mathbf{e}'_\pi \mathbf{A}^2 \mathbf{z}_{t-2} = & \rho\tau \mathbf{e}'_\pi \mathbf{A} \mathbf{z}_{t-2} + \rho(1-\tau)\mathbf{e}'_\pi \mathbf{I} \mathbf{z}_{t-2} + (1-\phi)\lambda \mathbf{e}'_{mcdom}(\mathbf{I} - \beta\mathbf{A})^{-1}\mathbf{A}^2\mathbf{z}_{t-2} \\ & + \phi\lambda \mathbf{e}'_{mcim}(\mathbf{I} - \beta\mathbf{A})^{-1}\mathbf{A}^2\mathbf{z}_{t-2}.\end{aligned}$$

The corresponding vector of cross-equation restrictions is given by

$$\begin{aligned}\mathbf{e}_\pi'\mathbf{A}^2 &= \rho\tau\mathbf{e}_\pi'\mathbf{A} + \rho(1-\tau)\mathbf{e}_\pi'\mathbf{I} + (1-\phi)\lambda\mathbf{e}_{mcdom}'(\mathbf{I}-\beta\mathbf{A})^{-1}\mathbf{A}^2 \\ &\quad +\phi\lambda\mathbf{e}_{mcim}'(\mathbf{I}-\beta\mathbf{A})^{-1}\mathbf{A}^2 \\ &\equiv \mathbf{g}^{CF}(\mathbf{A},\boldsymbol{\psi}),\end{aligned}$$

which may be equivalently expressed as

$$\mathbf{F}^{CF}(\mathbf{A},\boldsymbol{\psi}) \equiv \mathbf{e}_\pi'\mathbf{A}^2 - \mathbf{g}^{CF}(\mathbf{A},\boldsymbol{\psi}) = \underline{0}'.$$

The superscripts CF denote expressions which correspond to the closed form NKPC. The first stage VAR is identical to that described for the difference equation specification. The second stage of the estimation is also analogous and involves determining the values of the structural parameters in $\boldsymbol{\psi}$ that minimise the squared deviation of $\mathbf{g}^{CF}(\mathbf{A},\boldsymbol{\psi})$ from $\mathbf{e}_\pi'\mathbf{A}^2$. That is,

$$\widehat{\boldsymbol{\psi}}^{CF} \equiv \arg_{\psi} \mathbf{F}^{CF}(\hat{\mathbf{A}},\boldsymbol{\psi}) \cdot \mathbf{F}^{CF}(\hat{\mathbf{A}},\boldsymbol{\psi})'. \tag{12}$$

2.3 Difference Equation vs. Closed Form Estimation

Estimation of the NKPC is typically carried out in its difference equation (DE) form as specified in equation (6). Indeed Cogley and Sbordone's (2008) focus purely on the estimation of the NKPC with time-varying trend inflation expressed in its DE form. In a partial-information method as in the two-stage procedure describe above, however, the estimates may differ if the CF specification in (7) is used instead. The difference and the relationship between the estimates based on the DE and the CF specifications are explored deeply in BGLO (2011) and Gumbau-Brisa, Lie, and Olivei (2015). Rudd and Whelan (2005) also find that the estimates of the NKPC coefficients based on the closed-form specification may be significantly different than the estimates arising from the difference-equation specification, in the context of the GMM estimates in Gali and Gertler (1999).

To understand the differences in the structural estimates, recall that inflation expectations are generated by the forecasting rule (9), estimated from the first-stage VAR. Conditional on the estimated inflation forecasts, the second stage uses the minimum distance methods described above to estimate the parameters in $\boldsymbol{\psi}$. Since the first-stage VAR estimates are the same across both specifications (DE and CF), the disparity in the parameter estimates arise from the way inflation expectations enter the NKPC. In the DE specification, inflation expectations are left unconstrained. In particular, the VAR forecasts of inflation are taken as the only available information on inflation expectations and the structural relationship implied by the difference equation is estimated directly. On the other hand, in the CF representation of the NKPC the expectations of inflation themselves are constrained so as to follow the structure implied by the difference equation NKPC in every period. In this way the recursive closed-form solution adds model-consistent discipline to the evolution of inflation expectations.

Given that the structural relationship implied by the NKPC can only be considered as an approximation of the true data generating process for inflation, the DE and CF formulations will not be equivalent.[15] Thus, it will be of interest to examine how the parameter estimates, $\boldsymbol{\psi}$, given $\hat{\mathbf{A}}$, are effected when one compare the estimates based on the difference equation NKPC to the estimates implied by the closed form specification, with its additional model-consistent discipline on inflation expectations. The discussion in Gumbau-Brisa, Lie, and Olivei (2015) however makes clear that as long as the NKPC is close to the true data generating process for inflation, imposing additional model discipline on expectations will always yield parameter estimates that are closer to their true values.

2.4 Data

In line with many of the Australian studies on inflation dynamics in the literature, an underlying measure of inflation is used, as opposed to headline CPI.[16] Specifically, inflation is measured as the quarterly percentage change in the trimmed-mean CPI adjusted for the introduction of the GST.[17] Although empirical papers on inflation modelling in different countries generally use headline measures of inflation (typically based on the consumer price index), an underlying measure such as trimmed-mean CPI may be preferable as it precludes much of the noise and variation in the CPI. Headline CPI encompasses several components (such as food and energy prices) that are subject to large transitory fluctuations. Such transitory fluctuations may have significant ramifications for consumers, however, they are not easily captured in a structural model.

When modelling CPI inflation the labour share (which is often used as a proxy for real marginal

[15] If the true $\mathbf{A}$ is known, or $\hat{\mathbf{A}}$ is estimated without sampling errors, the estimates based on the DE and the CF specifications are identical.

[16] See, for example: de Brouwer and Ericsson (1998), Norman and Richards (2010) and Kuttner and Robinson (2010). This paper uses data from Kuttner and Robinson (2010).

[17] Note that prior to March 1982 the trimmed-mean CPI is not official data and have been constructed by the Reserve Bank of Australia for research purposes.

costs) will be an inappropriate measure of real marginal costs as it deflates nominal marginal costs by the GDP deflator (rather than the CPI). Accordingly, a measure of nominal unit labour costs deflated by the CPI is used as a proxy for real marginal costs of domestic producers.[18] As noted by Norman and Richards (2010), deflating nominal unit labour costs by trimmed-mean CPI has the added benefit of abstracting from the substantial influence that commodity prices have on the GDP deflator (and hence labour's share) in Australia. As discussed above, marginal costs for the import sector are measured as import prices relative to consumer prices:[19]

$$mc_t^m = p_t^m - p_t.$$

For the constant trend inflation case in this section, the reduced-form VAR is of order 2, with the ordering of the variables given by: mc_t^d, π_t and mc_t^m. The foreign real marginal costs, mc_t^m (as proxied by real import prices), are regarded as exogenous and it is therefore the last variable in the VAR(2) ordering. The sample covers the period 1960:Q1 to 2007:Q2. The end of the sample is chosen so that we exclude the recent global financial crisis period.

2.5 Estimation Results

Table 1 presents median estimates of the deep structural parameters $\boldsymbol{\psi} = [\alpha, \rho, \tau]$ for the full sample period: 1960:Q1-2007:Q2.[20] The results are presented for four specifications of the NKPC: the difference equation (DE) form and the closed form (CF) representation, with τ constrained to equal one, and unconstrained τ. Given identification issues, θ was calibrated to 5 during all estimations, implying a desired steady-state price markup of 25%.[21] In addition, given that the weight of imported goods in consumption, ϕ, has been a difficult parameter to estimate, the estimates presented in Table 1 correspond to the case where ϕ is calibrated to 0.2. This calibrated value is approximately equal to the average share of imports in Australian GDP since 1993 and is also used by Kuttner and Robinson (2010) in their analysis of the Australian NKPC.[22]

[18] Nominal unit labour costs are for the non-farm sector. The series is calculated as Compensation of Employees (ABS Table 41 National Accounts) + Payroll Tax (Table 39) less Subsidies (Table 39) divided by seasonally adjusted real non-farm GDP (Table 41).

[19] Import prices are measured as the implicit price deflator (ABS Table 5 National Accounts) adjusted for the declining rate of tariff protection on imports into Australia over much of the sample period. Following Beechy, Bharucha, Cagliarini, Gruen and Thompson (2000) log import prices are defined as: $p_t^{m,tar} = p_t^m + (1 + \textit{tariff}_t)$, where $\textit{tariff}_t$ is the average tariff rate on Australian imports (not its log).

[20] The 90% confidence intervals were derived using bootstrapping procedures.

[21] This calibrated value is consistent with the subsequent estimates of θ in the model with time-varying trend (see Table 6 in Section 5.4), which does not suffer from the same identification problem.

[22] This value is also consistent with Nimark (2009), who calibrates the import share as 0.18.

Table 1
Structural parameter estimates (baseline NKPC with zero trend inflation)
Sample period: 1960:Q1 - 2007:Q2
$\phi = 0.2$; $\theta = 5$

	ρ	α	τ
DE_{con}	0.005	0.999	1
	(0, 0.317)	(0.678, 1)	-
DE_{uncon}	0.874	0.771	0.706
	(0.401, 1)	(0.623, 1)	(0.385, 0.990)
CF_{con}	0.396	0.896	1
	(0.150, 0.562)	(0.824, 1)	-
CF_{uncon}	0.795	0.943	0.801
	(0.373, 0.861)	(0.854, 1)	(0.310, 0.992)

Notes: numbers in parentheses are 90% confidence intervals; DE_{con} and CF_{con} correspond to the difference equation and closed from specifications respectively, with $\tau = 1$; DE_{uncon} and CF_{uncon} correspond to the difference equation and closed form specifications with unconstrained τ, respectively.

The first row of Table 1 reports the DE parameter estimates with τ constrained to equal 1. For this specification (denoted DE_{con}) the indexation parameter, ρ, is estimated at 0.005, indicating that the role of lagged inflation in determining current inflation is negligible, which implies a purely forward-looking NKPC. However, when the DE specification is considered without constraining τ to unity (i.e. DE_{uncon}), the median estimate for ρ increases dramatically to 0.87, with its 90% confidence interval bounded well above zero. Thus, allowing for two lags of inflation in the indexation mechanism produces a much higher (and economically significant) estimate for ρ, suggesting the possibility of misspecification bias in the DE_{con} estimates. Furthermore, τ is estimated at 0.706, with the upper bound of its confidence interval lying below 1. In all, the DE_{uncon} estimates indicate that the indexation to the last two lags of inflation is more appropriate for characterising inflation persistence in Australia.

Focusing on the CF estimates it is clear that in the constrained case (denoted CF_{con}) the median estimate for ρ is noticeably higher than its DE counterpart. However, for the unconstrained specification (denoted CF_{uncon}) the estimated value for ρ is slightly lower than in the corresponding DE case. Nevertheless, the estimates for ρ are clearly indicative of a backward-looking component in Australia's NKPC given that the (likely misspecified) DE_{con} specification is the only case where

ρ is statistically and economically insignificant.

2.5.1 Implied NKPC coefficients

From (6), the NKPC may be expressed in its reduced-form,

$$\pi_t = \gamma_{b,1}\pi_{t-1} + \gamma_{b,2}\pi_{t-2} + \gamma_f E_t \pi_{t+1} + \tilde{\lambda} mc_t,$$

where

$$\gamma_{b,1} = \frac{\rho\tau - \beta\rho(1-\tau)}{1+\beta\rho\tau}$$
$$\gamma_{b,2} = \frac{\rho(1-\tau)}{1+\beta\rho\tau}$$
$$\gamma_f = \frac{\beta}{1+\beta\rho\tau} \ .$$

In the constrained cases, where $\tau = 1$, the coefficients collapse such that $\gamma_{b,1} = \rho/(1+\beta\rho)$, $\gamma_f = \beta/(1+\beta\rho)$ and $t-2$ inflation does not enter the specification.

Table 2
Implied NKPC coefficients
Sample: 1960:Q1 - 2007:Q2

	γ_b	$\gamma_{b,2}$	γ_f
DE_{con}	0.005	-	0.985
DE_{uncon}	0.225	0.155	0.615
CF_{con}	0.284	-	0.711
CF_{uncon}	0.294	0.097	0.607

Notes: The implied NKPC coefficients are conditional on the median parameter estimates from Table 1.

Conditioning on the point estimates of the structural parameters, Table 2 displays the implied NKPC coefficients for the full sample period. Unsurprisingly the DE_{con} specification implies essentially a purely-forward looking NKPC, with the coefficient on expected future inflation near unity. However, all three remaining specifications suggest that there is an important backward-looking component in Australia's NKPC. The results seem to indicate an enhanced role for backward-looking behaviour in comparison to the conventional GMM estimates of the Australian NKPC, .e.g. as presented in Kuttner and Robinson (2010). Kuttner and Robinson estimate $\gamma_f = 0.806$ and $\gamma_b = 0.166$ for the same sample period. Comparing the CF estimates to the DE estimates seems to suggest that when all model-consistent restrictions are placed on the evolution of inflation expectations the weight given to lagged inflation tends to increase. Despite the apparent increased role for lagged inflation, the estimates in Table 2 continue to suggest that the Australian NKPC is predominantly forward-looking.

The aforementioned results are largely illustrative but nonetheless display the clear disparity between the DE and CF estimates. The estimates reported for the DE_{uncon} specification and the closed form suggest the presence of an important backward-looking component in the Australian NKPC. In the next section these issues are explored in greater detail in the context of an extended Calvo model.[23] Trend inflation is incorporated into the Calvo model yielding an extended NKPC with time-varying coefficients.

3 The Australian NKPC with Time-Varying Trend Inflation

The conventional NKPC framework, as analysed in the previous section, relies on the assumption that inflation in the steady-state is zero. This section introduces an adapted version of the NKPC for Australia, taking into account the time variation in trend inflation, in the vein of Cogley and Sbordone (2008). In contrast to the baseline NKPC framework in Section 2, the model takes the log-linearisation of the equilibrium conditions of the Calvo model around a steady-state associated with *drifting* trend inflation. In the baseline NKPC, the log-linearisation is taken around a constant trend ($\overline{\pi} = 0$), which is the same in every period. In the context of this framework trend inflation is assumed to be an exogenous process that is modelled as a random walk. Unlike the conventional framework the NKPC coefficients in this setup will evolve over time according to the evolution of trend inflation. We augment their model to capture the small open-economy dimensions and, in the same vein as described in Section 2, also allow for two lags of inflation in the indexation mechanism. We leave the full detail of the derivation of the NKPC in Appendix A. Here, we just mention the two resulting primary equations, required for estimation.

The first primary equilibrium relationship is the restriction between trend inflation and steady-

[23] Discussion of the estimates for α and θ are reserved until Section 3.4.

state real marginal costs, which is of the form

$$\left(1-\alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}\right)^{\frac{1+\theta\omega}{1-\theta}}\left[\frac{1-\alpha\bar{q}\bar{g}^y\overline{\Pi}_t^{:\theta(1+\omega)(1-\rho)}}{1-\alpha\bar{q}\bar{g}^y\overline{\Pi}_t^{:(1-\rho)(\theta-1)}}\right]=(1-\alpha)^{\frac{1+\theta\omega}{1-\theta}}\frac{\theta}{\theta-1}(\overline{mc}_t^d)^{1-\phi}(\overline{mc}_t^m)^{\phi}, \tag{13}$$

where $\bar{q}$ denotes the steady-state real discount factor, $\bar{g}^y$ is steady-state output growth, $\overline{\Pi}_t$ is gross trend inflation at time t. The structural parameters, $\alpha, \rho, \theta, \omega$ and ϕ, retain their definitions from the previous section. The above restriction differs slightly, but in an important way, from the equivalent expression in Cogley and Sbordone (2008). In our case, the right hand side includes both $\overline{mc}_t^d$ and $\overline{mc}_t^m$, which represent trend real marginal costs of domestic producers and importers, respectively. Both terms enter the restriction according to the respective weights of domestically produced goods and imports in domestic consumption. In this way (13) provides a simple mechanism for capturing the open economy aspects of the inflationary process in Australia.[24] As in the constant trend case, it is assumed that the deep structural parameters are equivalent across both the domestic and foreign sectors.

The second primary equilibrium relationship is the NKPC equation, given by

$$\begin{aligned}\widehat{\pi}_t &= \rho\tau(\widehat{\pi}_{t-1}-\widehat{g}_t^{\pi})+\rho(1-\tau)(\widehat{\pi}_{t-2}-\widehat{g}_{t-1}^{\pi}-\widehat{g}_t^{\pi})+\Omega_t E_t[\widehat{\pi}_{t+1}-\rho\tau\widehat{\pi}_t-\rho(1-\tau)(\widehat{\pi}_{t-1}-\widehat{g}_t^{\pi})]\\ &\quad+\lambda_t\left[(1-\phi)\widehat{mc}_t^d+\phi\widehat{mc}_t^m\right]+\gamma_t\widehat{D}_t+u_{\pi,t},\end{aligned} \tag{14}$$

where hatted variables represent log-deviations of stationary variables from their (shifting) steady-state values.[25] $\widehat{D_t}$ is defined recursively as[26]

$$\begin{aligned}\widehat{D}_t &= \varphi_{1,t}E_t(\widehat{q}_{t,t+1}+\widehat{g}_{t+1}^y)+\varphi_{1,t}(\theta-1)E_t\{\widehat{\pi}_{t+1}-\rho\tau\widehat{\pi}_t-\rho(1-\tau)(\widehat{\pi}_{t-1}-\widehat{g}_t^{\pi})\}\\ &\quad+\varphi_{1,t}E_t\widehat{D}_{t+1}.\end{aligned} \tag{15}$$

---

[24] For the relationship (13) to be well defined, we assume the following two inequalities hold: $\varphi_{1,t}=\alpha\bar{q}\bar{g}^y\overline{\Pi}_t^{(1-\rho)(\theta-1)}<1$ and $\varphi_{2,t}=\alpha\bar{q}\bar{g}^y\overline{\Pi}_t^{\theta(1-\omega)(1-\rho)}<1$. These inequalities hold in 99% of the sample estimates.

[25] In particular, $\widehat{\pi}_t=\ln(\Pi_t/\overline{\Pi}_t)$, $\widehat{mc}_t^d=\ln(mc_t^d/\overline{mc}_t^d)$, $\widehat{mc}_t^m=\ln(mc_t^m/\overline{mc}_t^m)$ and $\widehat{g}_t^{\pi}=\ln(\overline{\Pi}_t/\overline{\Pi}_{t-1})$ is the growth rate of trend inflation; $u_{\pi,t}$ is a structural shock.

[26] Here $\widehat{g}_t^y=\ln(g_t^y/\bar{g}^y)$ and $\widehat{q}_{t,t+1}=\ln(q_{t,t+1}/\bar{q}_{t,t+1})$. Also note that $\varphi_{0,t}=\left[\frac{1-\alpha\overline{\Pi}_t^{:(1-\rho)(\theta-1)}}{\alpha\overline{\Pi}_t^{:(1-\rho)(\theta-1)}}\right]$, and $\Omega_t=\varphi_{2,t}(1+\varphi_{0,t})$.

Combining (14) and (15) yields the NKPC in its difference-equation (DE) form,

$$
\begin{aligned}
\widehat{\pi}_t \;=\;& \widetilde{\rho}_{1,t}^{DE}(\widehat{\pi}_{t-1} - \widehat{g}_t^{\pi}) + (1-\tau)\widetilde{\rho}_{2,t}^{DE}(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\pi} - \widehat{g}_t^{\pi}) \\
& + \widetilde{\lambda}_t^{DE}\left[(1-\phi)\widehat{mc}_t^d + \phi\widehat{mc}_t^m\right] \\
& + b_{1,t}^{DE} E_t \widehat{\pi}_{t+1} \\
& + b_{2,t}^{DE} E_t \sum_{j=2}^{\infty} \varphi_{1,t}^{j-1} \widehat{\pi}_{t+j} \\
& + b_{3,t}^{DE} E_t \sum_{j=0}^{\infty} \varphi_{1,t}^{j} \left[\widehat{Q}_{t+j,t+j+1} + \widehat{g}_{t+j+1}^{y}\right] + \tilde{u}_{\pi,t}.
\end{aligned} \tag{16}
$$

While the NKPC in (16) retains a similar form to the conventional NKPC in Section 2, the two variations differ in some important respects. Firstly, the extended NKPC contains additional variables, such as the growth rate in trend inflation ($\widehat{g}_t^{\pi}$), as well as terms involving the nominal discount factor ($\widehat{Q}_t$), real output growth ($\widehat{g}_t^{y}$) and higher-order leads of inflation. While these additional terms may suggest the presence of omitted variable bias in the traditional NKPC, perhaps a more important distinguishing feature of the extended NKPC is the fact that the NKPC coefficients in (16) are non-linear functions of trend inflation and the structural parameters of the model. Consequently these coefficients, which are of interest to policy makers, are subject to variation over time and evolve according to the drift in trend inflation.

It is possible to also obtain a closed-form (CF) representation of the NKPC by iterating equations (16) and (15) forward:[27]

$$
\begin{aligned}
\widehat{\pi}_t \;=\;& \rho\tau(\widehat{\pi}_{t-1} - \widehat{g}_t^{\pi}) + \rho(1-\tau)(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\pi} - \widehat{g}_t^{\pi}) + \lambda_t \sum_{k=0}^{\infty} \Omega_t^k E_t \left[(1-\phi)\widehat{mc}_{t+k}^d + \phi\widehat{mc}_{t+k}^m\right] \\
& + \gamma_t \sum_{k=0}^{\infty} \Omega_t^k E_t \widehat{D}_{t+k} + u_{\pi,t}.
\end{aligned} \tag{17}
$$

To maintain consistency with the constant trend case in Section 2, in the pursuant analysis the same four specifications of the NKPC are estimated. In particular, the DE specification (16) is estimated for the case where τ is constrained to equal one and for the case where τ is left unconstrained. Similarly, the NKPC in its CF representation (17) is estimated for both the constrained

[27] See Appendix B for derivation. As stipulated in BGLO (2011) equation (17) is more appropriately referred to as the "quasi-closed form" NKPC as $\widehat{\pi}_t$ remains a function of higher-order leads of inflation. It is nevertheles possible to obtain an exact closed form solution, however, BGLO (2011) show that the US estimates of the exact closed form are very similar to the quasi-closed form. Their results indicate that the additional restrictions that the exact closed form solution imposes over (17) are not critical for estimating the structural parameters of the model.

and unconstrained cases.

3.1 Estimating the NKPC Structural Parameters

As in Section 2 the objective is estimate the structural parameters of the NKPC model, $\boldsymbol{\psi} = [\alpha, \rho, \theta, \tau]$. The same two-step estimation approach, as described in Section 2.2, is adopted here. In the present context the time series vector $\mathbf{x}_t$ is extended so as to include not only inflation and real marginal costs, but also a nominal discount factor and output growth, so that $n = 5$. The reduced-form VAR is now written as

$$\mathbf{z}_t = \boldsymbol{\mu}_t + \mathbf{A}_t \mathbf{z}_{t-1} + \epsilon_{z,t}, \tag{18}$$

where $\epsilon_{z,t}$ is a vector of serially uncorrelated shocks. In contrast to the estimation of the baseline NKPC, the reduced-form VAR now has drifting coefficients, captured in $\boldsymbol{\mu}_t$ and $\mathbf{A}_t$.

As detailed previously, if the extended NKPC is the true data generating process for inflation, then the forecasts from the reduced-form VAR in (18) and the structural forecasts from (16) should be equivalent. The forecasting rule in equation (9) is augmented such that the conditional expectation of a variable $\widehat{y}_{t+j} \in \mathbf{x}_{t+j}$ at time t is now written as

$$E_t \widehat{y}_{t+j} = \mathbf{e}_y' \mathbf{A}_t^j \widehat{\mathbf{z}}_t, \tag{19}$$

where $\mathbf{e}_y'$ is the selection vector as defined previously. The above condition differs from the original forecasting rule (9) as it defines expectations of $\widehat{y}_{t+j}$, rather than y_{t+j}. As such $\widehat{\mathbf{z}}_t$ represents the vector of variables expressed in deviations from their time-varying trend levels at time t

$$\widehat{\mathbf{z}}_t \equiv \mathbf{z}_t - (\mathbf{I} - \mathbf{A}_t)^{-1} \boldsymbol{\mu}_t.$$

3.1.1 The Difference-Equation Specification (*DE*)

Given the structural representations of the NKPC in its difference-equation form (16), and the forecasting rule (19), the $t-2$ conditional expectation of inflation is written as

$$
\begin{aligned}
\mathbf{e}'_\pi \mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} &= \widetilde{\rho}^{DE}_{1,t-2}\mathbf{e}'_\pi \mathbf{A}_{t-2}\widehat{\mathbf{z}}_{t-2} + (1-\tau)\widetilde{\rho}^{DE}_{2,t-2}\mathbf{e}'_\pi \widehat{\mathbf{z}}_{t-2} \\
&\quad +(1-\phi)\widetilde{\lambda}^{DE}_{t-2}\mathbf{e}'_{mcdom}\mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} + \phi\widetilde{\lambda}^{DE}_{t-2}\mathbf{e}'_{mcim}\mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} \\
&\quad +b^{DE}_{1,t-2}\mathbf{e}'_\pi \mathbf{A}^3_{t-2}\widehat{\mathbf{z}}_{t-2} + b^{DE}_{2,t-2}\varphi_{1,t}\mathbf{e}'_\pi \mathbf{M}_{t-2}\mathbf{A}^4_{t-2}\widehat{\mathbf{z}}_{t-2} \\
&\quad +b^{DE}_{3,t-2}(\mathbf{e}'_q \mathbf{M}_{t-2}\mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} + \mathbf{e}'_{g^y}\mathbf{M}_{t-2}\mathbf{A}^3_{t-2}\widehat{\mathbf{z}}_{t-2}),
\end{aligned}
$$

where

$$
\mathbf{M}_t \equiv (\mathbf{I} - \varphi_{1,t}\mathbf{A}_t)^{-1}.
$$

The assumption that trend inflation follows a driftless random walk implies that the expected future growth rate of trend inflation is zero, thus all terms involving $\widehat{g}^\pi_t$ are ignored.[28] The vector of cross-equation restrictions is thus given by

$$
\begin{aligned}
\mathbf{e}'_\pi \mathbf{A}^2_{t-2} &= \widetilde{\rho}^{DE}_{1,t-2}\mathbf{e}'_\pi \mathbf{A}_{t-2} + (1-\tau)\widetilde{\rho}^{DE}_{2,t-2}\mathbf{e}'_\pi \mathbf{I} \\
&\quad +(1-\phi)\widetilde{\lambda}^{DE}_{t-2}\mathbf{e}'_{mcdom}\mathbf{A}^2_{t-2} + \phi\widetilde{\lambda}^{DE}_{t-2}\mathbf{e}'_{mcim}\mathbf{A}^2_{t-2} \\
&\quad +b^{DE}_{1,t-2}\mathbf{e}'_\pi \mathbf{A}^3_{t-2} + b^{DE}_{2,t-2}\varphi_{1,t}\mathbf{e}'_\pi \mathbf{M}_{t-2}\mathbf{A}^4_{t-2} \\
&\quad +b^{DE}_{3,t-2}(\mathbf{e}'_q \mathbf{M}_{t-2}\mathbf{A}^2_{t-2} + \mathbf{e}'_{g^y}\mathbf{M}_{t-2}\mathbf{A}^3_{t-2}) \\
&\equiv \mathbf{g}^{DE}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}).
\end{aligned}
$$

Equivalently,

$$
\mathbf{F}^{DE}_1(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) \equiv \mathbf{e}'_\pi \mathbf{A}^2_{t-2} - \mathbf{g}^{DE}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) = \underline{0}',\ \forall t. \tag{20}
$$

When inflation has a time-varying trend the parameters must also satisfy the steady-state restriction between trend inflation and marginal costs given by (13), which can be rewritten as

$$
\begin{aligned}
\mathbf{F}_2(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) &\equiv \left(1 - \alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}\right)^{\frac{1+\theta\omega}{1-\theta}} \left[\frac{1 - \alpha\bar{q}\bar{g}^y\overline{\Pi}_t^{:\theta(1+\omega)(1-\rho)}}{1 - \alpha\bar{q}\bar{g}^y\overline{\Pi}_t^{:(1-\rho)(\theta-1)}}\right] \\
&\quad -(1-\alpha)^{\frac{1+\theta\omega}{1-\theta}}\frac{\theta}{\theta-1}(\overline{mc}^d_t)^{1-\phi}(\overline{mc}^m_t)^{\phi} \\
&= \underline{0}'
\end{aligned} \tag{21}
$$

[28] The assumption that $|\varphi_{1,t}| < 1$, combined with the fact that the coefficient matrix $\mathbf{A}_t$ is constrained such that the roots of $\mathbf{A}_t$ lie inside the unit circle at each period in time, implies that the series $\mathbf{I} + \varphi_{1,t}\mathbf{A}_t + \varphi^2_{1,t}\mathbf{A}^2_t + \ldots$, converges and can be expressed as $\mathbf{M}_t \equiv (\mathbf{I} - \varphi_{1,t}\mathbf{A}_t)^{-1}$.

Thus, $\mathbf{F}_2(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi})$ must also be minimised at each period of the estimation sample. The two moment conditions (20) and (21) are combined by defining the vector

$$\mathcal{F}_t^{DE} = [\mathbf{F}_1^{DE}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) \;\; \mathbf{F}_2(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi})].$$

The complete set of cross-equation restrictions that must be satisfied is represented as the long vector,

$$\mathcal{F}^{DE}(\boldsymbol{\Theta}) = \left[\mathcal{F}_1^{DE}, \mathcal{F}_2^{DE}, ..., \mathcal{F}_T^{DE}\right],$$

where

$$\boldsymbol{\Theta} \equiv \{\boldsymbol{\mu}_t, \mathbf{A}_t\}_{t=1}^{T}.$$

As in the conventional NKPC case the first stage of the estimation procedure involves fitting the data to an unrestricted reduced-form VAR. However, in the presence of time-varying trend inflation, estimation of the first-stage VAR is achieved by using the Bayesian methods, as detailed in Cogley and Sargent (2005), to deliver the posterior distribution of $\boldsymbol{\Theta}$ from a set of N estimates or ensembles, $\{\widehat{\boldsymbol{\Theta}}_i\}_{i=1}^{N}$. Conditional on the estimates $\{\widehat{\boldsymbol{\Theta}}_i\}$, the structural parameters $\widehat{\boldsymbol{\psi}}_i^D$ are estimated in similar fashion to the conventional NKPC case:

$$\widehat{\boldsymbol{\psi}}_i^D = \arg_{\psi} \mathcal{F}^D(\widehat{\boldsymbol{\Theta}}_i) \cdot \mathcal{F}^D(\widehat{\boldsymbol{\Theta}}_i)' \quad \text{for } i = 1, ..., N. \tag{22}$$

3.1.2 The Closed Form Specification (*CF*)

Given the closed form specification of the NKPC in (17) and the forecasting rule (19), the $t-2$ conditional expectation of inflation is expressed as[29]

$$\begin{aligned}
\mathbf{e}'_\pi \mathbf{A}_{t-2}^2 \widehat{\mathbf{z}}_{t-2} \;\; = \;\; & \widetilde{\rho}_{2,t-2}^{CF} \mathbf{e}'_\pi \mathbf{A}_{t-2} \widehat{\mathbf{z}}_{t-2} + (1-\tau)\widetilde{\rho}_{2,t-2}^{CF} \mathbf{e}'_\pi \widehat{\mathbf{z}}_{t-2} + (1-\phi)\widetilde{\lambda}_{t-2}^{CF} \mathbf{e}'_{mcdom} \mathbf{K}_{t-2} \mathbf{A}_{t-2}^2 \widehat{\mathbf{z}}_{t-2} \\
& + \phi \widetilde{\lambda}_{t-2}^{CF} \mathbf{e}'_{mcim} \mathbf{K}_{t-2} \mathbf{A}_{t-2}^2 \widehat{\mathbf{z}}_{t-2} + b_{0,t-2}^{CF} \mathbf{e}'_\pi \mathbf{K}_{t-2} \mathbf{A}_{t-2} \widehat{\mathbf{z}}_{t-2} \\
& + b_{1,t-2}^{CF} \mathbf{e}'_\pi \mathbf{K}_{t-2} \mathbf{A}_{t-2}^2 \widehat{\mathbf{z}}_{t-2} + b_{2,t-2}^{CF} \mathbf{e}'_\pi \mathbf{K}_{t-2} \mathbf{A}_{t-2}^3 \widehat{\mathbf{z}}_{t-2} \\
& + b_{2,t-2}^{CF} \varphi_{1,t-2} \mathbf{e}'_\pi \mathbf{K}_{t-2} \mathbf{M}_{t-2} \mathbf{A}_{t-2}^4 \widehat{\mathbf{z}}_{t-2} \\
& + b_{3,t-2}^{CF} (\mathbf{e}'_q \mathbf{K}_{t-2} \mathbf{M}_{t-2} \mathbf{A}_{t-2}^2 \widehat{\mathbf{z}}_{t-2} + \mathbf{e}'_{g^y} \mathbf{K}_{t-2} \mathbf{M}_{t-2} \mathbf{A}_{t-2}^3 \widehat{\mathbf{z}}_{t-2}),
\end{aligned}$$

[29] For a complete derivation see Appendix B.

where

$$\mathbf{K}_t \equiv (\mathbf{I} - \Omega_t \mathbf{A}_t)^{-1}. \tag{23}$$

In order for the series $\mathbf{I} + \Omega_t \mathbf{A}_t + \Omega_t^2 \mathbf{A}_t^2 + ...$ to converge and be represented as in (23) the roots of $\Omega_t \mathbf{A}_t$ need to lie inside the unit circle, i.e. $|\Omega_t \mathbf{A}_t| < 1$. This condition is not guaranteed by the conditions of the model, and thus is an important empirical issue. Essentially, the estimation procedure implicitly assumes that the NKPC has a reduced-form VAR representation as in (18), which implies $|\Omega_t \mathbf{A}_t| < 1$. This issue is explored further in the robustness analysis conducted in Section 4.

The vector of non-linear cross-equation restrictions derived from the above conditional expectation is now given by

$$\begin{aligned}
\mathbf{e}'_\pi \mathbf{A}_{t-2}^2 &= \widetilde{\rho}_{2,t-2}^{CF} \mathbf{e}'_\pi \mathbf{A}_{t-2} + (1-\tau)\widetilde{\rho}_{2,t-2}^{CF} \mathbf{e}'_\pi \mathbf{I} + (1-\phi)\widetilde{\lambda}_{t-2}^{CF} \mathbf{e}'_{mcdom} \mathbf{K}_{t-2} \mathbf{A}_{t-2}^2 \\
&\quad + \phi \widetilde{\lambda}_{t-2}^{CF} \mathbf{e}'_{mcim} \mathbf{K}_{t-2} \mathbf{A}_{t-2}^2 + b_{0,t-2}^{CF} \mathbf{e}'_\pi \mathbf{K}_{t-2} \mathbf{A}_{t-2} \\
&\quad + b_{1,t-2}^{CF} \mathbf{e}'_\pi \mathbf{K}_{t-2} \mathbf{A}_{t-2}^2 + b_{2,t-2}^{CF} \mathbf{e}'_\pi \mathbf{K}_{t-2} \mathbf{A}_{t-2}^3 \\
&\quad + b_{2,t-2}^{CF} \varphi_{1,t-2} \mathbf{e}'_\pi \mathbf{K}_{t-2} \mathbf{M}_{t-2} \mathbf{A}_{t-2}^4 \\
&\quad + b_{3,t-2}^{CF} (\mathbf{e}'_q \mathbf{K}_{t-2} \mathbf{M}_{t-2} \mathbf{A}_{t-2}^2 + \mathbf{e}'_{g^y} \mathbf{K}_{t-2} \mathbf{M}_{t-2} \mathbf{A}_{t-2}^3) \\
&\equiv \mathbf{g}^{CF}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}).
\end{aligned}$$

The corresponding minimum distance problem is given by

$$\mathbf{F}_1^{CF}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) \equiv \mathbf{e}'_\pi \mathbf{A}_{t-2}^2 - \mathbf{g}^{CF}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) = \underline{0}', \quad \forall t.$$

As in the DE specification the trend restriction (13) must be satisfied at all stages of the estimation. Thus, the condition (21) also applies for the closed-form specification. Combining, we have the long vector,

$$\mathcal{F}_t^{CF} = [\mathbf{F}_1^{CF}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}) \;\; \mathbf{F}_2(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi})] \;.$$

The complete set of cross-equation restrictions is thus given by

$$\mathcal{F}^{CF}(\boldsymbol{\Theta}) = \left[\mathcal{F}_1^{CF}, \mathcal{F}_2^{CF}, ..., \mathcal{F}_T^{CF}\right].$$

The first-stage VAR that provides the Bayesian posterior for $\boldsymbol{\Theta}$ is identical to the VAR estimated for the DE specification. The estimates of the model's deep parameters are obtained in the second stage according to

$$\widehat{\boldsymbol{\psi}}_i^{CF} = \arg_{\psi} \mathcal{F}^{CF}(\widehat{\boldsymbol{\Theta}}_i) \cdot \mathcal{F}^{CF}(\widehat{\boldsymbol{\Theta}}_i)' \quad \text{for } i = 1, ..., N.$$

3.2 Data

The data for inflation and real marginal costs used in the estimation of the first-stage VAR is the same as in the conventional case, detailed in Section 2.4. However, in the time-varying trend case the time series vector $\mathbf{x}_t$ is extended to include measures of output growth and a nominal discount factor. Output growth is calculated using quarterly non-farm real GDP, seasonally adjusted at an annual rate.[30] The nominal discount factor, Q_t, is constructed by expressing the Reserve Bank's cash rate on a quarterly discount basis :[31]

$$Q_t = (1 + r_t)^{-\frac{1}{4}},$$

where r_t is the RBA's cash rate expressed as a decimal. The monthly cash rate data, as published by the RBA, was converted to quarterly values by point-sampling the first month of each quarter.

The reduced-form time-varying VAR is of order 2. However, in order to capture open economy effects, this analysis requires the inclusion of real marginal costs of importers, resulting in a five-variable VAR as opposed to Cogley and Sbordone's (2008) four. The ordering of the variables given by: g_t^y, mc_t^d, π_t, Q_t and mc_t^m. Again, given the exogeneity of foreign real marginal costs, mc_t^m is the last variable in the VAR(2) ordering. The sample encompasses the period 1960:Q1 - 2007:Q2. However, the time-varying trend estimation requires that data from 1960:Q1 - 1965:Q4 be used to initialise the prior, thus the model is estimated for the sample period 1966:Q1 - 2007:Q2. The number of ensembles is $N = 3000$. In Section 4, we increase the number of ensembles to $N = 5000$ and show that the estimates are largely unaltered.[32]

[30] Real non-farm GDP is the same series as used in Kuttner and Robinson (2010) (ABS National Accounts, Table 41).

[31] Prior to 1990 the Reserve Bank did not publish a cash rate target, thus the cash rate measures prior to 1990 are proxies of the current measure, and were obtained upon request from the Reserve Bank. The Reserve Bank previously used other measures of the short-term interest rates, such as the unofficial 11am call rate and the official (authorised dealers') rate, as its tool of conducting monetary policy.

[32] Cogley and Sbordone (2008) also use $N = 5000$. Here, we use $N = 3000$ for most specifications initially due to computer memory constraint (due to drifting VAR coefficients).

3.3 Trend Inflation and Persistence

Prior to the exposition of the estimation results in the next section, it is worthwhile to present some preliminary evidence of the importance of trend inflation and in particular its relevance as a possible explanation of persistence. The measure for trend inflation is obtained from the first-stage VAR estimates, given by

$$\overline{\pi}_t = \mathbf{e}'_\pi(\mathbf{I} - \widehat{\mathbf{A}}_t)^{-1}\widehat{\boldsymbol{\mu}}_t.$$

Figure 1 displays the median estimate of trend inflation at each period for the sample 1966:Q1 - 2007:Q2. For comparison, actual inflation, as measured by the trimmed-mean CPI, and mean inflation for the full sample are also presented.

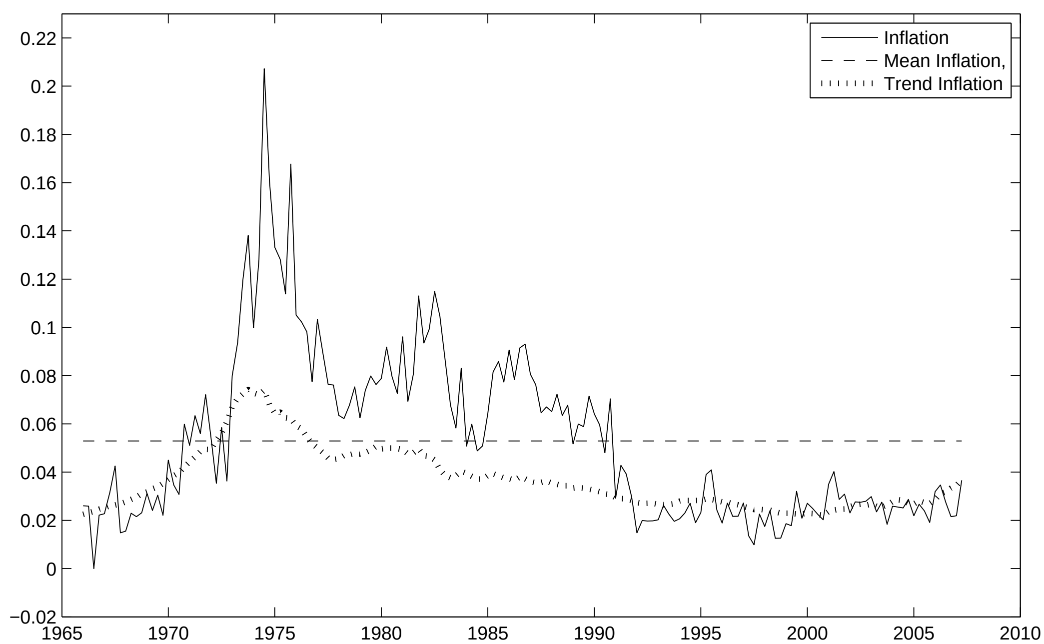


Figure 1
Trend Inflation in Australia, 1966:Q1 - 2007:Q2

The figure provides a depiction of some of the more important features of trend inflation, and its significant role in explaining the persistence of inflation. The first notable feature of the plot is, of course, the fact that the estimated trend in inflation exhibits variation over time. The estimates indicate that trend inflation rose from 2.3% per annum in the late 1960s to approximately 7.8% in the 1970s, and then fell back to levels below 3% during the 1990s. The latter part of the sample is perhaps of most interest in terms of its relevance to policy measures enacted by the Reserve Bank of Australia. Trend inflation can be thought as the central bank's long-run inflation target.[33] Accordingly, movements in $\overline{\pi}_t$ can be interpreted as reflecting shifts in that target. Figure 1 clearly depicts a reduction in the level of trend inflation during the 1990s. Furthermore, trend inflation from 1993 - 2007 hovers between 2-3%, which coincides with the Reserve Bank's targeted inflation level.

The time-varying nature of trend inflation also has significant implications for the evolution of the inflation gap. As stressed in Cogley, Primiceri and Sargent (2010) the mean-based measures of the inflation gap will display greater persistence than their trend-based counterparts. Examination of Figure 1 seems to support such a conclusion as there are long periods – in particular, during the 1960s, then from the 1970s through to the late 1980s and during the 1990s – where inflation does not cross its sample mean. In contrast, inflation intersects its time-varying trend more frequently, especially during the inflation targeting period. In the context of inflation persistence as predicted by the NKPC, this observation begs the question of whether mean-based measures reflect an exaggeration of persistence, rather than there being a lack of persistence implied by the purely forward-looking NKPC.

Table 3 presents the first-order autocorrelations of the two measures of the inflation gap. The

Table 3
First-order Autocorrelations of the Inflation Gap

	1966:Q1 - 2007:Q2	1993:Q1 - 2007:Q2
Inflation	0.8935	0.3386
Trend-Based Gap	0.8088	0.3627

first row refers to the deviation of inflation from its constant mean. The second row displays the autocorrelations of the trend-based gap, $\widehat{\pi}_t = \pi_t - \overline{\pi}_t$. For the long sample both measures display high degrees of persistence. Importantly, the trend-based measure is indeed less persistent compared to the conventional mean-based measure. However, the trend-based inflation gap displays slightly higher persistence than the mean-based gap after the onset of inflation-targeting policy in Australia (1993:Q1-2007:Q2). But this is perhaps not surprising since during the inflation-targeting period,

[33] Or, the public expectations of that target.

the (constant) mean inflation is largely identical to the very-stable trend inflation. According to both measures the Australian NKPC only needs to account for a relatively modest degree of persistence post-inflation targeting, indicating that the backward-looking inflation term(s) should play a smaller role.

3.4 Estimation Results

Table 4 displays the benchmark estimates of the structural parameters $\boldsymbol{\psi} = [\alpha, \rho, \theta, \tau]$ for the full-sample (1966:Q1 - 2007:Q2). As in the constant trend case, the discount factor, β, and the strategic complementary parameter, ω, are pinned down to values of 0.99 and 0.429 respectively. Again, given the difficulty in estimating the weight of imported goods in consumption, ϕ, the estimates reported in Table 4 correspond to the case where ϕ is calibrated to 0.2. For robustness, the model is also estimated for the case where ϕ is left unconstrained, presented in Section 4.

Table 4
Structural parameter estimates (median and 90% trust region)
Sample period: 1966:Q1 - 2007:Q2
$\phi = 0.2$

	ρ	α	θ	τ
DE_{con}	0.158	0.654	5.150	1
	(0, 0.784)	(0.509, 0.765)	(4.614, 6.502)	-
DE_{uncon}	0.828	0.635	5.002	0.597
	(0.390, 1)	(0.421, 0.734)	(4.550, 7.899)	(0.256, 0.873)
CF_{con}	0.794	0.892	6.015	1
	(0.480, 1)	(0.255, 0.927)	(5.011, 9.782)	-
CF_{uncon}	0.890	0.889	4.644	0.709
	(0.646, 1)	(0.157, 0.925)	(4.238, 5.923)	(0.378, 0.955)

Notes: numbers in parentheses are 90% trust regions; DE_{con} and CF_{con} correspond to the difference equation and closed from specifications respectively, with $\tau = 1$; DE_{uncon} and CF_{uncon} correspond to the difference equation and closed form specifications with unconstrained τ, respectively.

The first row of the table corresponds to the estimates based on the difference-equation specification (16) when the indexation is based on one lag of inflation (τ is constrained to one), denoted as DE_{con} — this specification is the one presented in Cogley and Sbordone (2008). Interestingly, the Australian data implies a non-zero median estimate for the indexation parameter ρ. This result is in contrast to Cogley and Sbordone, who estimated ρ at zero, implying a purely forward-looking NKPC for the US. However, as is evident from the confidence interval (which includes zero), ρ is not significant at the 10% level, suggesting that a purely forward-looking NKPC may also be applicable for Australia. The Calvo parameter, α, governing the degree of price-stickiness implies a median duration of prices of 1.63 quarters – a value very similar to that implied by Cogley and Sbordone's estimates and the US micro evidence presented in Nakamura and Steinsson (2007).[34].

The second row of Table 4 considers the DE specification adjusted so that τ is no longer constrained to equal 1, i.e. the indexation is based on two recent lags of inflation. This is a minor but important refinement, as the misspecification bias that arises from constraining τ to equal unity can be large. The estimation results (denoted as DE_{uncon}) indicate that this modification produces a large disparity in the estimation of ρ, indeed suggesting the presence of misspecification bias in the DE_{con} case. The median estimate of ρ increases dramatically from a value of 0.158 in the DE_{con} case to 0.828, implying that the Australian NKPC includes a backward-looking component, even after accounting for time-varying inflation trend. Although the estimate of ρ is relatively imprecise, the 90% confidence interval does not include zero. Furthermore, τ is estimated as 0.597, with the upper bound of its confidence interval below unity. This result is in line with the estimates presented in BGLO (2011) for the US data and implies that average inflation over the previous six months is more relevant in determining current inflation than merely the inflation rate in the most recent quarter.

The final two rows in Table 4 present the estimates implied by the closed-form (CF) specification of the NKPC. In the constrained case (denoted CF_{con}) the median estimate of ρ is 0.794, and retains a 90% confidence interval bounded well away from zero. The unconstrained case (denoted CF_{uncon}) implies a median estimate of ρ equal to 0.890. Thus, the CF estimates of ρ are larger than their DE counterparts, implying a greater degree of backward-looking behaviour (and are also estimated somewhat more precisely). The estimated value of τ in the CF_{uncon} case, 0.709, implies that last quarter's inflation rate is given a greater weight in the indexation mechanism, compared to its DE counterpart. However, the confidence interval is still bounded away from 1, suggesting that $t-2$ inflation is still relevant in explaining the current inflation rate and consequently should be included in the model.

In both CF specifications α is estimated at approximately 0.89, suggesting that prices are re-

[34] For a purely forward-looking Calvo model, the time between re-optimisation can be approximated as α^t. Thus, the median duration of prices is given by $-\ln(2)/\ln(\alpha)$.

optimised every 5 quarters, on average. However, in the presence of backward-indexation (implied by a non-zero estimate for ρ) α^t now refers to the approximate time elapsed between price *re-optimisation*, rather than a price change. Backward-indexation implies that prices are changed much more frequently (in fact, prices are changed every period) than they are actually being re-optimised. That the frequency of price adjustment is relatively greater than the frequency of optimal price resets may be reflective of the micro evidence which suggests that the information required to set the optimal markup is costly to obtain (see Zbaracki, et al. 2004).

The median estimates of θ are fairly consistent across all four specifications and imply a steady-state markup price of 20% - 27% when prices are flexible.[35] It is worth noting that the above estimates for θ are markedly lower than those found by both Cogley and Sbordone (2008) and BGLO (2011) for the US economy, implying that the desired steady-state markup in Australia is approximately double that suggested by the US data. In the context of general equilibrium models, US estimates of the steady-state markup range from approximately 6 to 23 percent.[36] Therefore, one can conclude that the estimates for θ presented in Table 4 imply that the steady-state markup in Australia lies at least towards the upper limit of the US estimates. Estimation of θ in the Australian empirical literature is relatively sparse. In their medium-scale DSGE model for Australia, Jaaksela and Nimark (2011) estimate $\theta \approx 1.10$, implying an implausibly high domestic markup. However, the authors also consider a model specification in which the domestic markup is calibrated at 20 percent – a value which is consistent with the estimates presented in Table 4. Adolfson, *et al.* (2007) estimate a steady-state markup of 17.4 percent in their open economy DSGE model for Europe, suggesting that domestic price markups in Australia are more comparable to those estimated for the Euro area, rather than in the US.

What can one conclude from the estimates reported in Table 4? When one compares the results presented in the table to those for the baseline NKPC, with zero trend inflation, reported in Table 1, it is evident that the estimates of ρ have not been greatly affected. Thus, it would seem that even once time-varying trend inflation is incorporated into the Calvo model, there is no meaningful change in the degree of autonomous inflation required to match the Australian data. This conclusion suggests that time-varying trend inflation is, at best, only contributes marginally for explaining inflation persistence in Australia.

[35] In a Calvo setting the desired markup is given by $\frac{\theta}{\theta-1}$ in a flexible price environment.

[36] See, for example: Christiano, Eichenbaum and Evans (2005) whose estimates range from 6.35 to 20 percent depending on the specification of their model; Rotemberg and Woodford (1997) estimate a steady-state markup of 15 percent ($\theta \approx 7.8$).

3.4.1 Parameter estimates during the inflation targeting period

As evidenced by Figure 1, the inflation targeting period has been characterised by low levels of inflation and reduced volatility. It is therefore relevant to ask whether such an environment of low and stable inflation has lead to any changes in the structural parameters of the NKPC. This subsample analysis considers the same specifications of the NKPC examined in Table 4, with the only change being the chosen sample period. Table 5 presents the structural parameter estimates for the inflation targeting period (1993:Q1 - 2007:Q2).

Table 5
Structural parameter estimates (median and 90% trust region)
Sample period: 1993:Q1 - 2007:Q2
$\phi = 0.2$

	ρ	α	θ	τ
DE_{con}	0.00	0.702	4.268	1
	(0, 0.427)	(0.457, 0.820)	(3.796, 29.290)	-
DE_{uncon}	0.740	0.698	4.171	0.598
	(0.213, 1)	(0.132, 0.829)	(3.775, 4.705)	(0.232, 1)
CF_{con}	0.424	0.882	5.291	1
	(0.180, 0.863)	(0.789, 0.921)	(4.140, 20.052)	-
CF_{uncon}	0.789	0.850	4.899	0.706
	(0.350, 1)	(0.162, 0.924)	(4.137, 25.702)	(0.174, 1)

Notes: numbers in parentheses are 90% trust regions; DE_{con} and CF_{con} correspond to the difference equation and closed from specifications respectively, with $\tau = 1$; DE_{uncon} and CF_{uncon} correspond to the difference equation and closed form specifications with unconstrained τ, respectively.

The estimates of α and τ are left largely unchanged when compared to the longer sample – suggesting that the time between optimal price readjustment and the weight given to the first lag of inflation in the indexation mechanism, have not changed in any meaningful sense since the introduction of an inflation targeting regime. The median estimates of θ are marginally smaller than those presented in Table 6 indicating that steady-state price markups may have increased slightly during the inflation targeting period. Perhaps of the greatest relevance in the present context are the changes exhibited in the estimates of ρ. Although the pattern of the findings is largely the same as in Table 4, for all four specifications the median estimates of ρ are lower than their corresponding values in the full sample. Indeed, the DE_{con} specification now yields an estimated ρ centred at zero. That the median estimated values for ρ are lower during the inflation targeting period reflects the decrease in the reduced-form persistence presented in Table 3. Nonetheless, given that the DE_{con} case is the only specification that implies a zero value for ρ, despite the decline in backward-looking behaviour, a purely forward-looking NKPC is still insufficient to explain inflation dynamics in Australia.

3.5 NKPC Coefficients

Recall the expression of the extended NKPC in its DE form (16):[37]

$$\begin{aligned}
\widehat{\pi}_t =& \widetilde{\rho}_{1,t}^{DE}(\widehat{\pi}_{t-1} - \widehat{g}_t^{\pi}) + (1-\tau)\widetilde{\rho}_{2,t}^{DE}(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\pi} - \widehat{g}_t^{\pi}) \\
&+ \widetilde{\lambda}_t^{DE}\left[(1-\phi)\widehat{mc}_t^d + \phi\widehat{mc}_t^m\right] \\
&+ b_{1,t}^{DE} E_t \widehat{\pi}_{t+1} \\
&+ b_{2,t}^{DE} E_t \sum_{j=2}^{\infty} \varphi_{1,t}^{j-1} \widehat{\pi}_{t+j} \\
&+ b_{3,t}^{DE} E_t \sum_{j=0}^{\infty} \varphi_{1,t}^{j} \left[\widehat{Q}_{t+j,t+j+1} + \widehat{g}_{t+j+1}^{y}\right] + \tilde{u}_{\pi,t}.
\end{aligned}$$

Of interest to policy makers is how trend inflation, through its interaction with the structural parameters of the Calvo model, affects the NKPC coefficients $\widetilde{\rho}_{1,t}$, $\widetilde{\lambda}_t$, $b_{1,t}$, $b_{2,t}$ and $b_{3,t}$. Conditioning on median estimates of the VAR and the DE_{con} parameter estimates, Figure 2 displays the NKPC coefficients (see Appendix A for the detail on the coefficients). Dashed lines represent conventional estimates, which assume zero trend inflation, while the solid lines represent approximations based on the extended model with time-varying trend inflation.

The evolution of the NKPC coefficients are clearly contingent on the level of trend inflation, $\overline{\pi}_t$, and are very similar to the corresponding time paths presented in Cogley and Sbordone (2008) for the US economy. The coefficient $\widetilde{\lambda}_t$, which represents the weight given to current marginal costs of domestic producers and importers, varies inversely with the level of trend inflation. Similarly, the backward-looking coefficient, $\widetilde{\rho}_{1,t}$, moves in the opposite direction to trend inflation. In contrast, the three forward-looking coefficients – $b_{1,t}$, $b_{2,t}$ and $b_{3,t}$ – evolve directly with the level of trend inflation. As described by Cogley and Sbordone, this variation in price-setting dynamics follows

[37] See Appendix A for definitions of the NKPC coefficients.

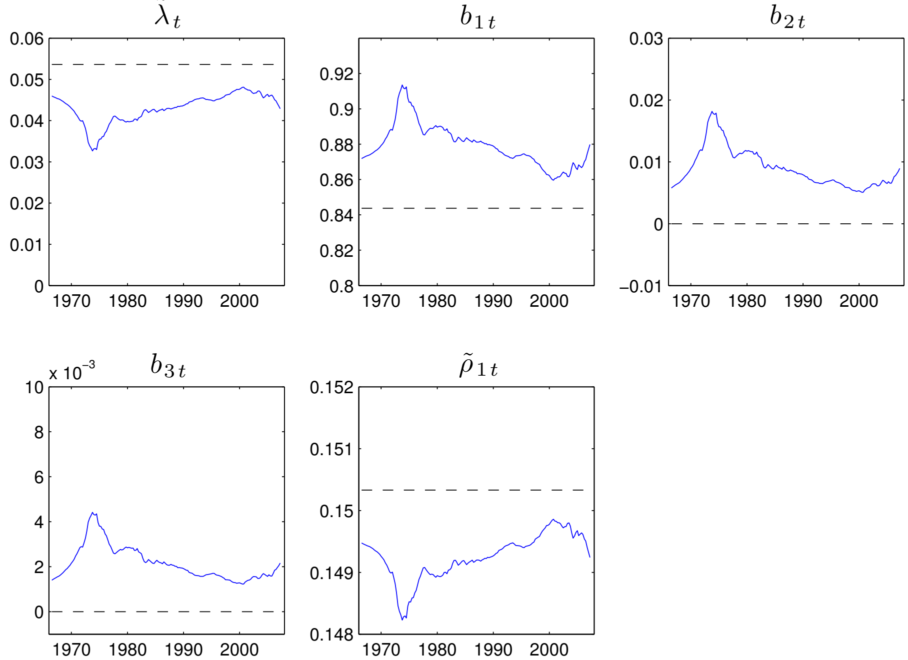


Figure 2
NKPC Coefficients – DE_{con}

from the fact that a high level of trend inflation accelerates the rate at which a firm's relative price is eroded when it lacks the opportunity to re-optimise its price. Accordingly, when trend inflation is high firms become more vulnerable to contingencies that may prevail in the future if their price remains fixed for some period of time. As such, when trend inflation rises, the backward-looking component and the weight given to current marginal costs both decrease, and the relative influence of the forward-looking terms are enhanced.

The coefficients implied by the conventional approximations (dashed lines) accord well with the reduced-form GMM estimates of Kuttner and Robinson (2010). Interestingly, when one compares the coefficients implied by the extended model (solid lines) to those implied by the conventional approximation, current costs and lagged inflation matter less and future expectations matter more. Focusing on the forward-looking coefficients, it is evident that the coefficient $b_{3,t}$ is always close to zero. Thus, terms involving forecasts of output growth and the discount factor do not contribute to inflation in any meaningful sense.

The above analysis draws on the evolution of the NKPC coefficients as derived from the DE_{con} specification. As a point of comparison, Figure 3 portrays the NKPC coefficients based on the CF

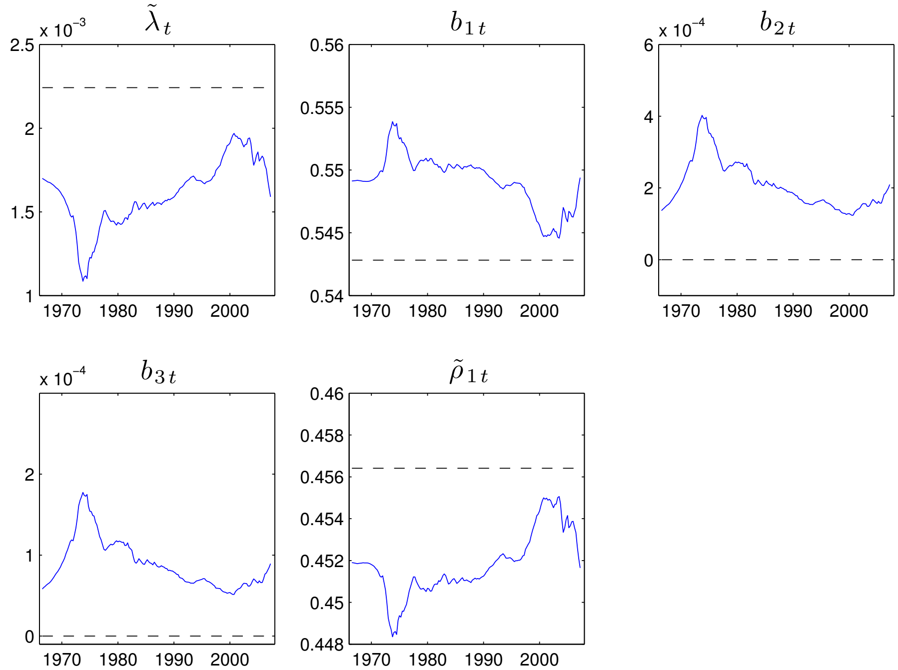


Figure 3
NKPC Coefficients – CF_{con}

specification in (17), contingent on the median estimates of the CF_{con} in Table 4. Although the shape of the coefficients' time paths are nearly identical to those derived from the DE_{con} case, it is clear that their relative magnitudes are significantly different.

Figure 3 shows that when model-consistent restrictions are placed on the evolution of inflation expectations, the backward-looking component, $\widetilde{\rho}_{1,t}$, increases dramatically (reflecting the increased value of the indexation parameter ρ). Indeed, according to the CF_{con} specification lagged inflation and future expectations of inflation enter the NKPC with almost equal weights, despite the time-variation in trend inflation. Furthermore, the CF_{con} estimates suggest that the link between marginal costs and current inflation, captured by $\widetilde{\lambda}_t$, is significantly weaker than that based on the DE_{con} estimates. These results could be of particular relevance to policy makers and imply that when all model restrictions on expectations are taken into account, the Australian NKPC is *not* predominantly forward-looking in the sense implied by the theory and the previous empirical literature. The relative magnitude of the backward-looking component has important implications not only for the dynamics of inflation but also in determining the appropriate inflation management policies to be employed by the central bank.

4 Robustness Analysis

This section reports the results from four robustness exercises. The first involves estimation of the second stage with an unconstrained import share parameter, ϕ. The second considers an alternative specification of the first-stage VAR, with four variables. Thirdly, the parameter estimates are presented for the case where an extended number of ensembles are used in the Bayesian estimation of the first stage VAR. Finally, the validity of the parameter estimates are considered, with reference to the necessary condition $|\Omega_t \mathbf{A}_t| < 1$.

As alluded to in Section 3.4, given the difficulty in estimating the import share parameter, ϕ, the benchmark estimates in Table 4 correspond to the case where ϕ is calibrated as 0.2. Table 6 presents the median parameter estimates when ϕ is left unconstrained.

Table 6
Structural parameter estimates (median and 90% trust region)
Sample period: 1966:Q1 - 2007:Q2
Unconstrained ϕ (import share).

	ρ	α	θ	τ	ϕ
DE_{con}	0.108	0.635	4.756	1	0.00
	(0, 0.992)	(0.018, 0.759)	(4.177, 12.277)	-	(0, 1)
DE_{uncon}	0.858	0.654	4.768	0.585	0.100
	(0.405, 1)	(0.418, 0.762)	(4.1317, 13.330)	(0.266, 0.865)	(0, 1)
CF_{con}	0.80	0.888	6.289	1	0.259
	(0.441, 0.983)	(0.836, 0.923)	(4.555, 18.750)	-	(0, 1)
CF_{uncon}	0.934	0.886	4.990	0.701	0.478
	(0.625, 0.996)	(0.800, 0.925)	(4.291, 20.609)	(0.423, 0.964)	(0, 1)

Notes: This table presents the structural parameter estimates of the extended Calvo model for the case where the import share ϕ is also estimated.

The overall effect of leaving ϕ unconstrained has a seemingly negligible effect on the estimates of the other parameters. In fact, the inclusion of ϕ in the estimation seems to improve the precision of the CF estimates (with the exception of θ). However, that the estimates of ϕ itself are extremely imprecise in all specifications of the NKPC is an indication that the parameter is weakly identified, and is perhaps better suited for calibration.

The next robustness exercise also follows from the difficulty in capturing the effects of marginal

Table 7
Structural parameter estimates (median and 90% trust region)
Sample period: 1966:Q1 - 2007:Q2
4-variable VAR

	ρ	α	θ	τ
DE_{con}	0.00	0.649	7.041	1
	(0, 0.265)	(0.510, 0.755)	(6.021, 8.642)	-
DE_{uncon}	0.834	0.634	6.982	0.584
	(0.471, 1)	(0.485, 0.728)	(5.840, 11.904)	(0.383, 0.797)
CF_{con}	0.869	0.919	9.539	1
	(0.645, 1)	(0.470, 0.944)	(6.850, 19.623)	-
CF_{uncon}	0.892	0.917	8.005	0.646
	(0.763, 1)	(0.269, 0.940)	(6.156, 15.265)	(0.474, 0.860)

Notes: This table presents estimates based on a different VAR specification used in the first stage. Here the sector-specific marginal cost data was combined to yield a single variable, such that the first stage VAR comprised of four variables as opposed to five. Since there is only a single marginal cost variable, ϕ is set as 0.

costs in the importing sector. An alternative four-variable VAR is considered so as to mechanically capture the inflationary effect derived from the importing sector. Specifically, data on the marginal costs of domestic producers, mc_t^d, and importers, mc_t^m, are aggregated to create a single marginal cost variable:

$$mc_{t,total} = (1 - 0.2) \times mc_t^d + 0.2 \times mc_t^m.$$

Thus, the first stage VAR collapses to the 4-variable case (with their ordering given by: g_t^y, $mc_{t,total}$, π_t, Q_t), as opposed the 5-variable VAR detailed in Section 3. Table 7 reports the resulting parameter estimates.

The parameter estimates are marginally affected by the alternative VAR specification. Most notably, the median estimate of ρ for the DE_{con} case is now zero, in line with Cogley and Sbordone's results using US data. The median estimates of θ are also somewhat higher than their corresponding values in Table 4. Nonetheless, the conclusions drawn from Table 4 remain robust to this alternative VAR specification. Namely, the estimates of ρ in Table 7, with the exception of the DE_{con} specification, continue to suggest an important role for backward-looking indexation.

The third robustness exercise extends the number of ensembles used to deliver the Bayesian

Table 8
Structural parameter estimates (median and 90% trust region)
Sample period: 1966:Q1 - 2007:Q2
$\phi = 0.2$, $N = 5000$ ensembles

	ρ	α	θ	τ
DE_{con}	0.165	0.663	5.156	1
	(0, 0.785)	(0.492, 0.764)	(4.604, 6.581)	-
DE_{uncon}	0.843	0.636	4.997	0.613
	(0.396, 1)	(0.137, 0.736)	(4.545, 6.233)	(0.286, 0.873)
CF_{con}	0.799	0.892	5.978	1
	(0.486, 1)	(0.270, 0.927)	(5.020, 9.350)	-
CF_{uncon}	0.890	0.894	5.653	0.707
	(0.623, 1)	(0.177, 0.927)	(4.877, 7.655)	(0.419, 0.968)

Notes: This table presents the parameter estimates when $N = 5000$ ensembles are used in the Bayesian estimation of the first stage VAR, as opposed to $N = 3000$ in Table 4.

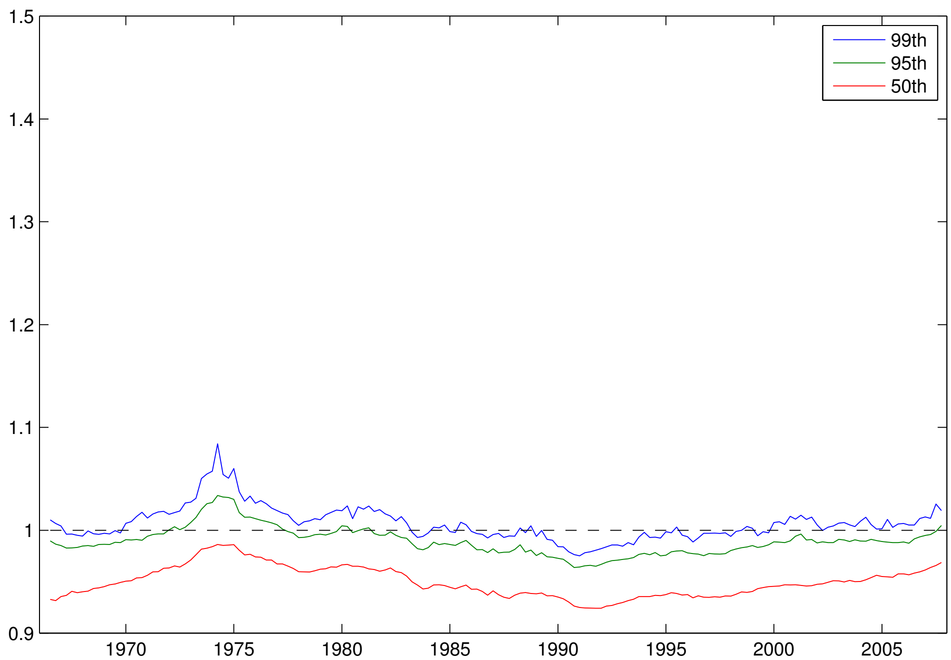
posterior for the VAR parameters $\{\boldsymbol{\mu}_t, \mathbf{A}_t\}$. Cogley and Sbordone (2008) and BGLO (2011) characterise the distribution of the posterior for $\{\boldsymbol{\mu}_t, \mathbf{A}_t\}$ from a set of $N = 5000$ estimates. However, due to computational limitations, N is initially set to 3000 in our open-economy case in order to deliver the estimates presented in Table 4. Table 8 reports the results with N extended to 5000, thereby providing a larger distribution of parameter estimates in the second stage. The inclusion of greater number of ensembles does not affect the parameter estimates in any meaningful sense. Thus, the benchmark estimates presented in Table 4 are robust despite the use of 3000 ensembles instead of 5000.

Finally, compliance with the necessary condition $|\Omega_t \mathbf{A}_t| < 1$ is analysed. As emphasised in BGLO (2011) and Gumbau-Brisa, Lie, and Olivei (2015), violation of this condition would render the estimates presented in Table 4 invalid as this condition is necessary for the first stage and second stage estimates to be compatible with each other.[38] Panel A in Figure 4 depicts the distribution of the largest estimated root of $\widehat{\Omega}_t \cdot \hat{\mathbf{A}}_t$ for the CF_{uncon} specification, while Panel B displays the corresponding distribution for the DE_{con} case (note that $\hat{\mathbf{A}}_t$ is the same in both specifications). The figure captures the sharp contrast between the compatibility of two specifications with the first

[38]When the condition $|\Omega_t \mathbf{A}_t| < 1$ is violated, the NKPC solution is indeterminate. This means that the NKPC solution cannot be represented by a reduced-form VAR with finite lags, as implicitly assumed in the two-stage econometric procedure. See BGLO (2011) for an in depth discussion.

stage VAR. Although the 99th and 95th percentiles for the CF_{uncon} specification do occasionally rise above unity, it is strikingly clear that $|\widehat{\Omega}_t^{CF}\hat{\mathbf{A}}_t| < 1$ is satisfied much more readily than in the DE_{con} case. Indeed, in the DE_{con} case, the 99th and 95th percentiles lie almost completely above unity. These findings strongly suggest an inconsistency between the assumption of a VAR representation in the first stage, and the DE estimates obtained in the second stage. Figure 4 indicates that the minimum-distance estimation procedure is better suited for the CF specification.

Panel A: Closed Form (CF), distribution of $|\widehat{\Omega}_t^{CF}\hat{A}_t|$



Panel B: Difference Equation (DE) Form, distribution of $|\widehat{\Omega}_t^{DE}\hat{A}_t|$

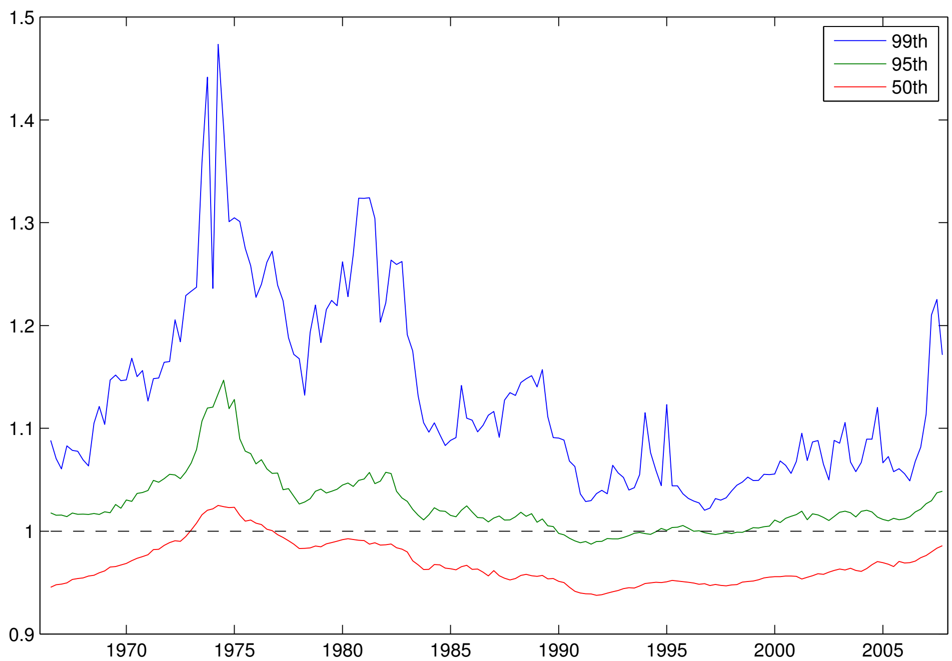


Figure 4

Distribution of the largest root of $\widehat{\Omega}_t \cdot \hat{\mathbf{A}}_t$ in absolute value — Median, 95th and 99th percentiles

5 Conclusion

This paper addresses the question of whether the time-varying and the persistence nature of trend inflation can explain the dynamics and the persistence of inflation in Australia, within the context of the New Keynesian Phillips curve (NKPC) equation. We derive and estimate an extended open-economy NKPC equation with past indexation, accounting explicitly for time-varying trend inflation. Based on this extended NKPC, where the coefficients are functions of both the model's structural parameters and the trend inflation, we investigate whether accounting for the time variation in trend inflation can explain away the need for the backward-looking inflation terms in the NKPC equation. The results illustrate the substantial difference in estimates when the NKPC is expressed in its closed form (CF) rather than its typical difference-equation (DE) form. While estimations of all NKPC specifications seem to suggest at least some role for backward-looking indexation, its estimated importance is contingent on the particular specification. In line with Cogley and Sbordone's (2008) US results, the backward-looking indexation parameter is negligible in some DE specifications. In contrast, when a more plausible indexation mechanism is imposed on the DE, and when using the CF specification, the backward-looking indexation parameter is estimated at values closer to one. Thus, in light of the evidence presented, and especially when one considers estimation of the closed form, the results suggest that accounting for time-varying trend inflation in the NKPC cannot explain the apparent inertia present in the Australian inflation data.

Analysis of the NKPC coefficients implied by the closed-form parameter estimates has shown that lagged inflation and future expectations of inflation enter the NKPC with almost equal weights. Such a finding is in stark contrast to the results of Cogley and Sbordone (2008) who find in favour of a purely forward-looking NKPC. In the context of Australia, the traditionally accepted GMM estimates of the NKPC suggest that inflation is predominantly forward-looking (see e.g. Kuttner and Robinson, 2010). However, an almost equal split between past and future inflation when analysing inflation dynamics according to the NKPC is common when estimation procedures take into account the model-consistent constraints placed by the NKPC on all future expectations of inflation (e.g. Fuhrer and Moore, 1995, Linde, 2005, and Barnes, et.al, 2011). Notwithstanding the historical performance of the NKPC in Australia, this paper also shows a marked decline in the role of the backward-looking inflation terms since the adoption of an inflation targeting regime by the Reserve Bank in 1993. Accordingly, it is possible that the characterisation of the Australian NKPC as being predominantly forward-looking is more applicable in recent years.

Our analysis has shown that in order to explain inflation dynamics in Australia using the NKPC, ad-hoc backward-looking terms are required even when shifts in trend inflation are accounted for. Thus, when using the NKPC, a specification that has become commonplace in modern macroeconomic analysis (including for the Australian economy), there is a need to assume some degree of autonomous inertia in inflation as there are no existing sound microfounded mechanisms which can capture such persistence. While backward-indexation or rule-of-thumb behaviour may help, to capture the persistence in inflation, there is a need to develop more comprehensive mechanisms which can explain the complex structural behaviours underlying inflation dynamics. Whether such mechanisms as time-variance in the Calvo pricing parameters, the impact of learning on pricing, sticky-information models or state-dependent menu-cost models provide avenues to stronger microfoundations remains to be seen. Nonetheless, the quest to uncover the true nature of inflation dynamics is an important exercise that requires ongoing research and investigation.

References

[1] Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2007). "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through," *Journal of International Economics*, 72(2), 481–511.

[2] Barnes, M. L., F. Gumbau-Brisa, D. Lie, and G. P. Olivei (2011). "Estimation of Forward-Looking Relationships in Closed Form: An Application to the New Keynesian Phillips Curve," Federal Reserve Bank of Boston Working Paper No. 11-3.

[3] Beechey, M., N. Bharucha, A. Cagliarini, D. Gruen, and C. Thompson (2000). "A Small Model of the Australian Macroeconomy," Research Discussion Paper 2000-05, Reserve Bank of Australia.

[4] Benati, L. (2008). "Investigating Inflation Persistence Across Monetary Regimes," *The Quarterly Journal of Economics*, MIT Press, vol. 123(3), 1005-1060, August.

[5] Bernanke, B., T. Laubach, F.S. Mishkin, and A.S. Posen (2001). *Inflation Targeting: Lessons from the International Experience*, Princeton University Press.

[6] Calvo, G. A. (1983). "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12(3), 383–398.

[7] Christiano, L., M. Eichenbaum, and C. Evans (2005). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

[8] Cogley, T., G. Primiceri, and T. Sargent (2010). "Infation-Gap Persistence in the US," *American Economic Journal: Macroeconomics*, 2(1), 43–69.

[9] Cogley, T., and T. Sargent (2005). "Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII U.S," *Review of Economic Dynamics*, 8(2), 262–302.

[10] Cogley, T., and A. Sbordone (2008). "Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve," *The American Economic Review*, 98(5), 2101–2126.

[11] De Brouwer, G., and N. Ericsson (1998). "Modeling inflation in Australia," *Journal of Business and Economic Statistics*, 16(4), 433–449.

[12] Fuhrer, J. (2010). "Inflation Persistence," in B.M. Friedman & M. Woodford (ed.), *Handbook of Monetary Economics*, Elsevier, 1(3), 3.

[13] Fuhrer, J., and G. Moore (1995). "Inflation Persistence," *The Quarterly Journal of Economics*, 110(1), 127–159.

[14] Galí, J., and M. Gertler (1999). "Infation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics*, 44(2), 195–222.

[15] Galí, J., and T. Monacelli (2005). "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 72(3), 707–734.

[16] Gumbau-Brisa, F., D. Lie, and G. P. Olivei (2015). "Comments on 'Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve'," manuscript.

[17] Gruen, D., A. Pagan, and C. Thompson (1999). "The Phillips Curve in Australia," *Journal of Monetary Economics*, 44(2), 223–258.

[18] Jääskelä, J., and K. Nimark (2011). “A Medium-Scale New Keynesian Open Economy Model of Australia,” *Economic Record*, 87(276), 11–36.

[19] Justiniano, A., and B. Preston (2010). “Monetary Policy and Uncertainty in an Empirical Small Open Economy Model,” *Journal of Applied Econometrics*, 25(1), 93–128.

[20] Kreps, D. (1998). “Anticipated Utility and Dynamic Choice,” in *Frontiers of Research in Economic Theory: The Nancy L. Schwartz Memorial Lectures*, 1983–1997, D.P Jacobs, E. Kalai and M.Kamien (eds.), 242-274. Cambridge, MA: Cambridge University Press.

[21] Kuttner, K., and T. Robinson (2010). “Understanding the Flattening Phillips Curve,” *North American Journal of Economics and Finance*, 21(2), 110–125.

[22] Lindé, J. (2005). “Estimating New-Keynesian Phillips Curves: A Full Information Maximum Likelihood Approach,” *Journal of Monetary Economics*, 52(6), 1135–1149.

[23] Monacelli, T. (2005). “Monetary Policy in a Low Pass-Through Environment,” *Journal of Money, Credit, and Banking*, 37(6), 1047–1066.

[24] Nakamura, E., and J. Steinsson (2008). “Five Facts About Prices: A Reevaluation of Menu Cost Models,” *The Quarterly Journal of Economics*, 123(4), 1415–1464.

[25] Nimark, K. P. (2009). “A Structural Model of Australia as a Small Open Economy,” *Australian Economic Review*, 42(1), 24–41.

[26] Norman, D., and A. Richards (2010). “Modelling Inflation in Australia,” Research Discussion Paper 2010-03, Reserve Bank of Australia.

[27] Rotemberg, J., and M. Woodford (1997). “An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy,” *NBER Macroeconomics Annual 1997*, B.S. Bernanke and J.J Rotemberg (eds.), 297-346. Cambridge, MA: MIT Press.

[28] Rudd, J., and K. Whelan (2005). “New Tests of the New-Keynesian Phillips Curve,” *Journal of Monetary Economics*, 52(6), 1167–1181.

[29] Taylor, J. (1980). “Aggregate Dynamics and Staggered Contracts,” *The Journal of Political Economy*, 88(1), 1–23.

[30] Zbaracki, M., M. Ritson, D. Levy, S. Dutta, and M. Bergen (2004). “Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets,” *Review of Economics and Statistics*, 86(2), 514–533.

Appendix A: Derivation of the NKPC in difference equation (DE) form

This appendix details the derivation of the extended NKPC with time-varying trend inflation in its difference-equation (DE) form as described in (16).

First, the log-linear approximation of the evolution of aggregate prices is derived. Let X_t be the optimal nominal price at time t chosen by firms that are allowed to adjust their prices, which occurs with probability probability $(1-\alpha)$. Based on the indexation mechanism, the price of an individual firm i that is unable to adjust its price (with probability α) is determined according to

$$P_t(i) = (\Pi_{t-1}^{\tau}\Pi_{t-2}^{1-\tau})^{\rho} P_{t-1}(i) \ .$$

Hence, the aggregate price based on the CES aggregator is given by

$$P_t = \left[(1-\alpha)X_t^{1-\theta} + \alpha\left\{(\Pi_{t-1}^{\tau}\Pi_{t-2}^{1-\tau})^{\rho} P_{t-1}\right\}^{1-\theta}\right]^{\frac{1}{1-\theta}} \ .$$

Dividing by the price level P_t, yields

$$1 = (1-\alpha)x_t^{1-\theta} + \alpha\left\{(\Pi_{t-1}^{\tau}\Pi_{t-2}^{1-\tau})^{\rho}\Pi_t^{-1}\right\}^{1-\theta} \ , \tag{24}$$

where x_t is the optimal relative price at time t. Next define stationary variables $\widetilde{\Pi}_t = \Pi_t/\overline{\Pi}_t$, $g_t^{\bar{\pi}} = \overline{\Pi}_t/\overline{\Pi}_{t-1}$, $g_t^{y} = Y_t/Y_{t-1}$, and $\widetilde{x}_t = x_t/\overline{x}_t$. Here, for any variable k_t, $\overline{k}_t$ is its time-varying trend. Equation (24) can then be transformed in terms of these stationary variables to yield:

$$\begin{aligned} 1 \ &= \ (1-\alpha)\widetilde{x}_t^{1-\theta}\overline{x}_t^{1-\theta} \\ &+\alpha\left[\begin{array}{c} \widetilde{\Pi}_{t-2}^{\rho(1-\tau)(1-\theta)}\widetilde{\Pi}_{t-1}^{\rho\tau(1-\theta)}\widetilde{\Pi}_t^{-(1-\theta)}\overline{\Pi}_t^{(1-\rho)(\theta-1)}\cdot \\ (g_{t-1}^{\bar{\pi}})^{-\rho(1-\tau)(1-\theta)}(g_t^{\bar{\pi}})^{-\rho(1-\tau)(1-\theta)}(g_t^{\bar{\pi}})^{-\rho\tau(1-\theta)} \end{array}\right] \ . \end{aligned} \tag{25}$$

In the steady state where $\widetilde{x}_t = \widetilde{\Pi}_t = g_t^{\bar{\pi}} = 1$, (25) can be solved for $\overline{x}_t$ as a function of $\overline{\Pi}_t$:

$$\overline{x}_t = \left[\frac{1-\alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}}{1-\alpha}\right]^{\frac{1}{1-\theta}} \ . \tag{26}$$

Defining $\widehat{\pi}_t \equiv \ln\widetilde{\Pi}_t \equiv \ln(\Pi_t/\overline{\Pi}_t)$ and $\widehat{x}_t \equiv \ln\widetilde{x}_t$, imposing (26), and rearranging gives the log-linear approximation of (25) around the steady state, which can be expressed as

$$\begin{aligned} \widehat{x}_t \ &= \ -\frac{1}{\varphi_{0,t}}\rho(1-\tau)\left(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\bar{\pi}} - \widehat{g}_t^{\bar{\pi}}\right) \\ &-\frac{1}{\varphi_{0,t}}\rho\tau\left(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}\right) \\ &+\frac{1}{\varphi_{0,t}}\widehat{\pi}_t \ , \end{aligned} \tag{27}$$

where $\varphi_{0,t} = \frac{1-\alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}}{\alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}}$.

Next, take the log-linear approximation to the first-order condition (FOC) of firms' pricing problem. The firms' FOC can be expressed as

$$E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} Y_{t+j} P_{t+j} \Psi_{tj}^{1-\theta} \left(X_t^{(1+\theta\omega)} - \frac{\theta}{\theta-1} MC_{t+j} \Psi_{tj}^{-(1+\theta\omega)} P_{t+j}^{\theta\omega} \right) = 0 \ , \tag{28}$$

where $Q_{t,t+j}$ and MC_{t+j} are the nominal discount factor and average marginal cost at $t+j$, respectively. The variable Ψ_{tj} enters in the CES demand function for any good i, $Y_{t+j}(i) = Y_{t+j}\left(\frac{P_{t+j}(i)\Psi_{tj}}{P_{t+j}}\right)$, with

$$\Psi_{tj} = \begin{cases} 1 & j = 0 \\ \prod_{k=0}^{j-1} \left(\Pi_{t+k}^{\tau} \Pi_{t+k-1}^{1-\tau} \right)^{\rho} & j \geq 1 \end{cases} \tag{29}$$

Combining (28) and (29) and rearranging leads to

$$X_t^{1+\theta\omega} = \frac{C_t}{D_t} \ ,$$

where C_t and D_t are recursively defined by

$$\begin{aligned} C_t &= \frac{\theta}{\theta-1} Y_t P_t^{\theta(1+\omega)-1} MC_t \\ &+ E_t \left[\alpha q_{t,t+1} \Pi_t^{-\rho\tau\theta(1+\omega)} \Pi_{t-1}^{-\rho(1-\tau)\theta(1+\omega)} C_{t+1} \right] \end{aligned} \tag{30}$$

$$\begin{aligned} D_t &= Y_t P_t^{\theta-1} \\ &+ E_t \left[\alpha q_{t,t+1} \Pi_t^{\rho\tau(1-\theta)} \Pi_{t-1}^{\rho(1-\tau)(1-\theta)} D_{t+1} \right] \ , \end{aligned} \tag{31}$$

where $q_{t,t+1}$ now is the real discount factor. Defining the stationary variables $\widetilde{C}_t = \frac{C_t}{Y_t P_t^{\theta(1+\omega)}}$ and $\widetilde{D}_t = \frac{D_t}{Y_t P_t^{\theta-1}}$, yields the following two expressions, based on (30) and (31):

$$\begin{aligned} \widetilde{C}_t &= \frac{\theta}{\theta-1} mc_t \\ &+ E_t \left[\alpha q_{t,t+1} g_{t+1}^{y} \Pi_{t+1}^{\theta(1+\omega)} \Pi_t^{-\rho\tau\theta(1+\omega)} \Pi_{t-1}^{-\rho(1-\tau)\theta(1+\omega)} \widetilde{C}_{t+1} \right] \end{aligned} \tag{32}$$

$$\widetilde{D}_t = 1 + E_t \left[\alpha q_{t,t+1} g_{t+1}^{y} \Pi_{t+1}^{(\theta-1)} \Pi_t^{\rho\tau(1-\theta)} \Pi_{t-1}^{\rho(1-\tau)(1-\theta)} \widetilde{D}_{t+1} \right] \ . \tag{33}$$

Also note that

$$\frac{\widetilde{C}_t}{\widetilde{D}_t} = \frac{C_t}{D_t} \frac{1}{P_t^{(1+\theta\omega)}} = x_t^{1+\theta\omega} \ , \tag{34}$$

where $x_t \equiv X_t / P_t$. Evaluating (32) and (33) at the steady state leads to

$$\overline{C}_t = \frac{\frac{\theta}{\theta-1} \overline{mc}_t}{1 - \alpha \overline{q} \overline{g}^{y} \overline{\Pi}_t^{\theta(1+\omega)(1-\rho)}}$$

$$\overline{D}_t = \frac{1}{1 - \alpha \overline{q} \overline{g}^y \overline{\Pi}_t^{(\theta-1)(1-\rho)}}$$

Imposing the assumption that $MC_{t+j} = (MC^d_{t+j})^{1-\phi}(MC^m_{t+j})^{\phi}$ and combining the two expressions above with (26) while using (34) leads to the steady-state restriction (13). This restriction does not depend on τ and hence is identical to the case in Cogley and Sbordone with $\tau = 1$. Next, define $\widehat{C}_t = \ln \frac{\widetilde{C}_t}{\overline{C}_t}$, $\widehat{D}_t = \ln \frac{\widetilde{D}_t}{\overline{D}_t}$, and $\widehat{mc}_t = \ln \frac{mc_t}{\overline{mc}_t}$. Log-linearizing (34) yields

$$(1 + \theta\omega)\widehat{x}_t = (\widehat{C}_t - \widehat{D}_t) \ . \tag{35}$$

Combining (35) with (27) and rearranging leads to an intermediate expression for $\widehat{\pi}_t$:

$$\begin{aligned} \widehat{\pi}_t &= \rho\tau \left[\widehat{\pi}_{t-1} - \widehat{g}^{\overline{\pi}}_t\right] \\ &\quad +\rho(1-\tau)\left[\widehat{\pi}_{t-2} - \widehat{g}^{\overline{\pi}}_{t-1} - \widehat{g}^{\overline{\pi}}_t\right] \\ &\quad +\frac{\varphi_{0,t}}{(1+\theta\omega)}(\widehat{C}_t - \widehat{D}_t) \ . \end{aligned} \tag{36}$$

The expressions for $\widehat{C}_t$ and $\widehat{D}_t$ are obtained by log-linearizing (32) and (33). Combining the resulting expressions with (35) leads to an expression for $\widehat{\pi}_t$ similar to that in the main text

$$\begin{aligned} \widehat{\pi}_t &= \rho\tau(\widehat{\pi}_{t-1} - \widehat{g}^{\overline{\pi}}_t) + \rho(1-\tau)(\widehat{\pi}_{t-2} - \widehat{g}^{\overline{\pi}}_{t-1} - \widehat{g}^{\overline{\pi}}_t) \\ &\quad +\Omega_t E_t(\widehat{\pi}_{t+1} - \rho\tau\widehat{\pi}_t - \rho(1-\tau)(\widehat{\pi}_{t-1} - \widehat{g}^{\overline{\pi}}_t)) + \lambda_t\widehat{mc}_t + \gamma_t\widehat{D}_t + u_{\pi,t} \end{aligned} \tag{37}$$

$$\begin{aligned} \widehat{D}_t &= \varphi_{1,t}E_t(\widehat{q}_{t,t+1} + \widehat{g}^y_{t+1}) \\ &\quad +\varphi_{1,t}(\theta-1)E_t\left\{\widehat{\pi}_{t+1} - \rho\tau\widehat{\pi}_t - \rho(1-\tau)(\widehat{\pi}_{t-1} - \widehat{g}^{\overline{\pi}}_t)\right\} + \varphi_{1,t}E_t\widehat{D}_{t+1} \ . \end{aligned} \tag{38}$$

To derive the NKPC equations as expressed in (14) and (15) recall that the in order to capture open economy effects of the inflationary process it is assumed

$$MC_{t+j} = (MC^d_{t+j})^{1-\phi}(MC^m_{t+j})^{\phi},$$

where average aggregate marginal costs, MC_{t+j}, is a combination of average marginal costs of domestic producers, MC^d_{t+j} and importers, MC^m_{t+j}. As mentioned in the main text, it is assumed that the structural parameters of the Calvo model (α, ρ, θ, ω and τ) are identical across both sectors. Thus, given (37) and (38), imposing such assumptions leads to the formulation of an open-economy NKPC as expressed in (14) and (15)

$$\begin{aligned} \widehat{\pi}_t &= \rho\tau(\widehat{\pi}_{t-1} - \widehat{g}^{\overline{\pi}}_t) + \rho(1-\tau)(\widehat{\pi}_{t-2} - \widehat{g}^{\overline{\pi}}_{t-1} - \widehat{g}^{\overline{\pi}}_t) + \Omega_t E_t(\widehat{\pi}_{t+1} - \rho\tau\widehat{\pi}_t - \rho(1-\tau)(\widehat{\pi}_{t-1} - \widehat{g}^{\overline{\pi}}_t)) \\ &\quad +\lambda_t\left[(1-\phi)\widehat{mc}^d_t + \phi : \widehat{mc}^m_t\right] + \gamma_t\widehat{D}_t + u_{\pi,t} \end{aligned} \tag{39}$$

$$\begin{aligned} \widehat{D}_t &= \varphi_{1,t}E_t(\widehat{q}_{t,t+1} + \widehat{g}^y_{t+1}) \\ &\quad +\varphi_{1,t}(\theta-1)E_t\left\{\widehat{\pi}_{t+1} - \rho\tau\widehat{\pi}_t - \rho(1-\tau)(\widehat{\pi}_{t-1} - \widehat{g}^{\overline{\pi}}_t)\right\} + \varphi_{1,t}E_t\widehat{D}_{t+1} \ . \end{aligned} \tag{40}$$

with the time-varying coefficients given by

$$
\begin{aligned}
\lambda_t &= \chi_t \varphi_{3,t} \\
\Omega_t &= \varphi_{2,t}(1+\varphi_{0,t}) \\
\gamma_t &= \frac{\chi_t(\varphi_{2,t}-\varphi_{1,t})}{\varphi_{1,t}} \\
\chi_t &= \frac{\varphi_{0,t}}{1+\theta\omega} \\
\varphi_{1,t} &= \alpha \overline{qg}^y \overline{\Pi}_t^{(\theta-1)(1-\rho)} \\
\varphi_{2,t} &= \alpha \overline{qg}^y \overline{\Pi}_t^{\theta(1+\omega)(1-\rho)} \\
\varphi_{3,t} &= 1-\varphi_{2,t}.
\end{aligned}
$$

Finally, iterating $\widehat{D}_t$ in (40) forward, substituting the resulting expression for $\widehat{D}_t$ into (39), converting real discount factors $\widehat{q}_{t+j,t+j+1}$ into nominal discount factors $\widehat{Q}_{t+j,t+j+1}$ and rearranging terms yields the NKPC in DE form (as expressed in equation (16) in the main text):

$$
\begin{aligned}
\widehat{\pi}_t = {} & \widetilde{\rho}_{1,t}^{DE}\left(\widehat{\pi}_{t-1}-\widehat{g}_t^{\overline{\pi}}\right)+(1-\tau)\widetilde{\rho}_{2,t}^{DE}\left(\widehat{\pi}_{t-2}-\widehat{g}_{t-1}^{\overline{\pi}}-\widehat{g}_t^{\overline{\pi}}\right) \\
& +\widetilde{\lambda}_t^{DE}\left[(1-\phi)\widehat{mc}_t^d+\phi:\widehat{mc}_t^m\right] \\
& +b_{1,t}^{DE}E_t\widehat{\pi}_{t+1} \\
& +b_{2,t}^{DE}E_t\sum_{j=2}^{\infty}\varphi_{1,t}^{j-1}\widehat{\pi}_{t+j} \\
& +b_{3,t}^{DE}E_t\sum_{j=0}^{\infty}\varphi_{1,t}^{j}\left[\widehat{Q}_{t+j,t+j+1}+\widehat{g}_{t+j+1}^{y}\right] \; +\widetilde{u}_{\pi,t},
\end{aligned}
\tag{41}
$$

where the coefficients are defined by

$$
\begin{aligned}
\widetilde{\rho}_{1,t}^{DE} &= \left[\rho\tau-\lambda_t\rho(1-\tau)-\gamma_t(\theta-1)\rho(1-\tau)\varphi_{1,t}\right]/\Delta_t \\
\widetilde{\rho}_{2,t}^{DE} &= \rho/\Delta_t \\
b_{1,t}^{DE} &= \widetilde{b}_{1,t}^{D}+b_{3,t}^{D} \\
b_{2,t}^{DE} &= \widetilde{b}_{2,t}^{D}+b_{3,t}^{D} \\
b_{3,t}^{DE} &= [\gamma_t\varphi_{1,t}]/\Delta_t \\
\widetilde{\lambda}_t^{DE} &= \lambda_t/\Delta_t \\
\Delta_t &= 1+\rho\tau\Omega_t+\gamma_t(\theta-1)\rho\varphi_{1,t}\left\{\tau+(1-\tau)\varphi_{1,t}\right\} \\
\widetilde{b}_{1,t}^{DE} &= \left[\Omega_t+\gamma_t(\theta-1)\varphi_{1,t}\left\{1-\rho\tau\varphi_{1,t}-\rho(1-\tau)\varphi_{1,t}^2\right\}\right]/\Delta_t \\
\widetilde{b}_{2,t}^{DE} &= \left[\gamma_t(\theta-1)\varphi_{1,t}\left\{1-\rho\tau\varphi_{1,t}-\rho(1-\tau)\varphi_{1,t}^2\right\}\right]/\Delta_t
\end{aligned}
\tag{42}
$$

Note that as in Cogley and Sbordone (2008) and BGLO (2011), the "anticipated utility" assumption (Kreps, 1998) is used in deriving the NKPC in (41) such that $E_t\prod_{k=0}^{i}\varphi_{1,t+k}\widehat{h}_{t+i}=\varphi_{1,t}^{i+1}E_t\widehat{h}_{t+i}$ for any variable $\widehat{h}_{t+i}$.

Appendix B: Derivation of the CF specification

This appendix describes in detail the derivation of the closed-form (CF) representation of the NKPC. First, define the variable

$$\widehat{B}_t = \widehat{\pi}_t - \rho\tau(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}) - \rho(1-\tau)(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\bar{\pi}} - \widehat{g}_t^{\bar{\pi}}) \ ,$$

so that

$$E_t\widehat{B}_{t+1} = E_t\widehat{\pi}_{t+1} - \rho\tau\widehat{\pi}_t - \rho(1-\tau)(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}) \ .$$

Note that the expectation above reflects the fact that $\widehat{g}_t^{\bar{\pi}}$ is an innovation process so that $E_t\widehat{g}_{t+j}^{\bar{\pi}} = 0$ for $j \geq 1$. Using this definition, we can rewrite (39) as

$$\widehat{B}_t = \Omega_t E_t \widehat{B}_{t+1} + \lambda_t \left[(1-\phi)\widehat{mc}_t^d + \phi : \widehat{mc}_t^m\right] + \gamma_t \widehat{D}_t + u_{\pi,t}. \tag{43}$$

Solving (43) forwards

$$\widehat{B}_t = \lambda_t E_t \sum_{j=0}^{\infty} \Omega_t^j \left[(1-\phi)\widehat{mc}_{t+j}^d + \phi : \widehat{mc}_{t+j}^m\right] + \gamma_t E_t \sum_{j=0}^{\infty} \Omega_t^j \widehat{D}_{t+j} + u_{\pi,t}. \tag{44}$$

In deriving (44) (and (45) below), the “anticipated utility" assumption is used so that

$$\begin{aligned} E_t\lambda_{t+j}\prod_{k=0}^{j}\Omega_{t+k}\left[(1-\phi)\widehat{mc}_{t+j}^d + \phi : \widehat{mc}_{t+j}^m\right] &= \lambda_t\Omega_t^{j+1}E_t\left[(1-\phi)\widehat{mc}_{t+j}^d + \phi : \widehat{mc}_{t+j}^m\right] \\ E_t\gamma_{t+j}\prod_{k=0}^{j}\Omega_{t+k}\widehat{D}_{t+j} &= \gamma_t\Omega_t^{j+1}E_t\widehat{D}_{t+j} \end{aligned}$$

for any $j > 0$. Next, solving forward (40), converting real discount factors into nominal ones, and rearranging leads to

$$\begin{aligned} \widehat{D}_t \;=\; & \varphi_{1,t}E_t\sum_{j=0}^{\infty}\varphi_{1,t}^j\left[\widehat{Q}_{t+j,t+j+1} + \widehat{g}_{t+j+1}^y\right] \\ & -\kappa_{1,t}\left[\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}\right] + \kappa_{2,t}\widehat{\pi}_t + \kappa_{3,t}\widehat{\pi}_{t+1} \\ & +\kappa_{3,t}E_t\sum_{j=2}^{\infty}\varphi_{1,t}^{j-1}\widehat{\pi}_{t+j} \ , \end{aligned} \tag{45}$$

with the new coefficients defined by

$$\begin{aligned} \kappa_{1,t} &= (\theta-1)\rho(1-\tau)\varphi_{1,t} \\ \kappa_{2,t} &= (\theta-1)\rho\tau\varphi_{1,t} + (\theta-1)\rho(1-\tau)\varphi_{1,t}^2 \\ \kappa_{3,t} &= \theta\varphi_{1,t} - (\theta-1)\rho\tau\varphi_{1,t}^2 - (\theta-1)\rho(1-\tau)\varphi_{1,t}^3 \end{aligned}$$

Next, the auxiliary variables $\widehat{B}_t$ and $\widehat{D}_t$ are removed and the NKPC is derived. Using the definition of $\widehat{B}_t$, and substituting into (44),

$$\begin{aligned}\widehat{\pi}_t &= \rho\tau(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}) + \rho(1-\tau)(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\bar{\pi}} - \widehat{g}_t^{\bar{\pi}}) \\ &\quad + \lambda_t E_t \sum_{j=0}^{\infty} \Omega_t^j \left[(1-\phi)\widehat{mc}_{t+j}^d + \phi : \widehat{mc}_{t+j}^m\right] + \gamma_t E_t \sum_{k=0}^{\infty} \Omega_t^j \widehat{D}_{t+j} \ . \end{aligned} \tag{46}$$

Finally, substitute for $\widehat{D}_{t+j}$ terms in (46) using (45) and rearrange the resulting expression to obtain the CF representation of NKPC:

$$\begin{aligned}\widehat{\pi}_t &= \widetilde{\rho}_{1,t}^{CF}(\widehat{\pi}_{t-1} - \widehat{g}_t^{\bar{\pi}}) + (1-\tau)\widetilde{\rho}_{2,t}^{CF}(\widehat{\pi}_{t-2} - \widehat{g}_{t-1}^{\bar{\pi}} - \widehat{g}_t^{\bar{\pi}}) \\ &\quad + \widetilde{\lambda}_t^{CF} E_t \sum_{j=0}^{\infty} \Omega_t^j \left[(1-\phi)\widehat{mc}_{t+j}^d + \phi\widehat{mc}_{t+j}^m\right] \\ &\quad + b_{0,t}^{CF} E_t \sum_{k=0}^{\infty} \Omega_t^k \left[\widehat{\pi}_{t+k-1} - \widehat{g}_{t+k}^{\bar{\pi}}\right] \\ &\quad + b_{1,t}^{CF} E_t \sum_{k=0}^{\infty} \Omega_t^k \widehat{\pi}_{t+k} \\ &\quad + b_{2,t}^{CF} E_t \sum_{k=0}^{\infty} \Omega_t^k \widehat{\pi}_{t+k+1} \\ &\quad + b_{2,t}^{CF} E_t \sum_{k=0}^{\infty} \Omega_t^k \sum_{j=2}^{\infty} \varphi_{1,t}^{j-1} \widehat{\pi}_{t+j+k} \\ &\quad + b_{3,t}^{CF} E_t \sum_{k=0}^{\infty} \Omega_t^k \sum_{j=0}^{\infty} \varphi_{1,t}^{j} \left[\widehat{Q}_{t+j+k,t+j+k+1} + \widehat{g}_{t+j+k+1}^{y}\right] + \ u_{\pi,t}, \end{aligned} \tag{47}$$

with the new coefficients defined as

$$\begin{aligned}\widetilde{\rho}_{1,t}^{CF} &= \rho\tau \\ \widetilde{\rho}_{2,t}^{CF} &= \rho \\ \widetilde{\lambda}_t^{CF} &= \lambda_t \\ b_{0,t}^{CF} &= -\gamma_t\kappa_{1,t} \\ b_{1,t}^{CF} &= -\gamma_t\kappa_{2,t} \\ b_{2,t}^{CF} &= \gamma_t\kappa_{3,t} \\ b_{3,t}^{CF} &= \gamma_t\varphi_{1,t}\end{aligned}$$

Cross-equation restrictions

Using the forecasting rule (19), the $t-2$ conditional expectation of the CF specification (47) takes the form

$$\begin{aligned}
\mathbf{e}'_{\pi}\mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} \;=\;& \widetilde{\rho}^{CF}_{2,t-2}\mathbf{e}'_{\pi}\mathbf{A}_{t-2}\widehat{\mathbf{z}}_{t-2} + (1-\tau)\widetilde{\rho}^{CF}_{2,t-2}\mathbf{e}'_{\pi}\widehat{\mathbf{z}}_{t-2} + (1-\phi)\widetilde{\lambda}^{CF}_{t-2}\mathbf{e}'_{mcdom}\mathbf{K}_{t-2}\mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} \\
&+\phi\widetilde{\lambda}^{CF}_{t-2}\mathbf{e}'_{mcim}\mathbf{K}_{t-2}\mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} + b^{CF}_{0,t-2}\mathbf{e}'_{\pi}\mathbf{K}_{t-2}\mathbf{A}_{t-2}\widehat{\mathbf{z}}_{t-2} \\
&+b^{CF}_{1,t-2}\mathbf{e}'_{\pi}\mathbf{K}_{t-2}\mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} + b^{CF}_{2,t-2}\mathbf{e}'_{\pi}\mathbf{K}_{t-2}\mathbf{A}^3_{t-2}\widehat{\mathbf{z}}_{t-2} \\
&+b^{CF}_{2,t-2}\varphi_{1,t-2}\mathbf{e}'_{\pi}\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}^4_{t-2}\widehat{\mathbf{z}}_{t-2} \\
&+b^{CF}_{3,t-2}(\mathbf{e}'_{q}\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}^2_{t-2}\widehat{\mathbf{z}}_{t-2} + \mathbf{e}'_{g^y}\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}^3_{t-2}\widehat{\mathbf{z}}_{t-2}),
\end{aligned}$$

where

$$\mathbf{K}_t \equiv (\mathbf{I} - \Omega_t\mathbf{A}_t)^{-1}.$$

Hence, the vector of cross-equation restrictions is given by

$$\begin{aligned}
\mathbf{e}'_{\pi}\mathbf{A}^2_{t-2} \;=\;& \widetilde{\rho}^{CF}_{2,t-2}\mathbf{e}'_{\pi}\mathbf{A}_{t-2} + (1-\tau)\widetilde{\rho}^{CF}_{2,t-2}\mathbf{e}'_{\pi}\mathbf{I} + (1-\phi)\widetilde{\lambda}^{CF}_{t-2}\mathbf{e}'_{mcdom}\mathbf{K}_{t-2}\mathbf{A}^2_{t-2} \\
&+\phi\widetilde{\lambda}^{CF}_{t-2}\mathbf{e}'_{mcim}\mathbf{K}_{t-2}\mathbf{A}^2_{t-2} + b^{CF}_{0,t-2}\mathbf{e}'_{\pi}\mathbf{K}_{t-2}\mathbf{A}_{t-2} \\
&+b^{CF}_{1,t-2}\mathbf{e}'_{\pi}\mathbf{K}_{t-2}\mathbf{A}^2_{t-2} + b^{CF}_{2,t-2}\mathbf{e}'_{\pi}\mathbf{K}_{t-2}\mathbf{A}^3_{t-2} \\
&+b^{CF}_{2,t-2}\varphi_{1,t-2}\mathbf{e}'_{\pi}\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}^4_{t-2} \\
&+b^{CF}_{3,t-2}(\mathbf{e}'_{q}\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}^2_{t-2} + \mathbf{e}'_{g^y}\mathbf{K}_{t-2}\mathbf{M}_{t-2}\mathbf{A}^3_{t-2}) \\
\equiv\;& \mathbf{g}^{CF}(\boldsymbol{\mu}_{t-2}, \mathbf{A}_{t-2}, \boldsymbol{\psi}).
\end{aligned}$$