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State-Dependant Pricing and Optimal Monetary Policy

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Abstract

This paper studies optimal monetary policy under precommitment in a state-dependent pricing (SDP) environment, in contrast to the standard assumption of time-dependent pricing (TDP). I show that the endogenous timing of price adjustment under SDP importantly alters the policy tradeoffs faced by the monetary authority, due to lower cost of inflation variation on the relative-price distortion. It is thus desirable under SDP for the monetary authority to put less weight on inflation stabilization, relative to other stabilization goals. The optimal Ramsey policy under SDP delivers a 24 percent higher standard deviation of inflation, but with 26 percent and 6 percent lower standard deviations of output gap and nominal interest rate, respectively. Within a simple, Taylor-like policy rule, the change in the policy tradeoffs is manifested in higher feedback response coefficients on the output gap and the lagged nominal interest rate deviation under SDP. Additionally, this paper studies the optimal policy start-up problem related to the cost of adopting the timeless perspective policy instead of the true Ramsey policy. The SDP assumption leads to different start-up dynamics compared to the dynamics under the TDP assumption in several important ways. In particular, the change in the policy tradeoffs gives rise to much higher start-up inflation under SDP.

JEL Classification: E31, E52, E61

Keywords: optimal monetary policy, state-dependent pricing, start-up problem, policy trade-off, Ramsey policy, simple policy rule.

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1 Introduction

This paper studies optimal monetary policy under precommitment in a state-dependent pricing environment, in contrast to the standard assumption of time-dependent pricing. A common denominator of virtually all studies on optimal monetary policy design is that the nature of the pricing friction responsible for the presence of predetermined prices is time dependent. That is, firms have no choice about the timing of their price adjustments, as in the familiar work of Calvo (1983) and Taylor (1980). One reason why time-dependent pricing (henceforth, TDP) models are popular is that there is no need to track the price distribution of firms and the frequency of price adjustment. For example, under Calvo, a single parameter, the probability of price adjustment, is a sufficient statistic to summarize the entire price distribution, given the reset price and the lagged aggregate price level. In a state-dependent pricing (henceforth, SDP) environment, one typically needs to track many state variables involving predetermined prices and the distribution of firms according to the time since the last price adjustment, further complicating the models and their analyses.

Yet, there are several reasons why it is desirable to move away from the realm of TDP in optimal monetary policy analysis. Chiefly among them is the observation that the endogenous timing of price adjustment under SDP may alter the inflation-output tradeoff faced by the monetary authority. Since monetary policy works primarily because of the existence of this tradeoff, such a modification in the tradeoff may in turn lead to a different prescription as to the optimal conduct of monetary policy. In such an environment, the monetary authority needs to take into account the effect of its policy on the frequency of price adjustment, since this also affects the course of real economic activity. The literature ignores the incorporation of SDP arguably because there is a belief that TDP is a good approximation to the more-realistic assumption of SDP (see e.g. Klenow and Kryvtsov, 2008). However, this is only true in an economy with small and stable inflation. When inflation is high and highly variable, SDP is a better and more accurate representation of the true firms' price adjustment decision. How should monetary policy be conducted in such an environment is therefore an important question for academics and practitioners alike. In terms of data evidence, there is growing evidence documenting state dependence in firms' price adjustment activities. For example, Nakamura and Steinsson (2008), among others, show that it is the frequency, rather than the size of price adjustment, that has a strong positive correlation with inflation — this evidence is more consistent with the SDP assumption.

The state-dependent pricing framework used in this paper is the model of Dotsey, King, and

Wolman (1999), in which firms are allowed to endogenously change their timing of price adjustments by paying a small fixed menu cost.[1] In terms of the optimal monetary policy approach, this paper follows the common practice in the public finance literature—for instance used in Erceg, Henderson, and Levin (2000), Khan, King, and Wolman (2003) and Schmitt-Grohe and Uribe (2005, 2007)—by assuming that the objective function of the monetary authority is the households' lifetime utility function. The conditional or the unconditional expectation of the lifetime utility function thus becomes a direct and natural criterion for welfare evaluation. In this approach, market distortions or inefficiencies are identified and monetary policy affects welfare through its influence on the variations in these distortions.

Specifically, there are four set of distortions present in the model: (i) the markup distortion that arises from firms' monopoly power, which causes market-generated output level to be inefficient; (ii) the relative-price distortion arising from firms' asynchronous price-adjustment process; (iii) the monetary distortions due to the use of money and credit to purchase final consumption goods; and (iv) the menu cost distortion due to the fixed cost of price adjustment. There are generally tradeoffs between these distortions requiring the monetary authority to balance the extent of these distortions in the process of achieving the socially optimum allocation. The monetary authority is assumed to solve a precommitment Ramsey problem in a decentralized economy setting where the private sector's efficiency conditions must be respected at all times.

The first finding of this paper involves the Ramsey timeless perspective responses to exogenous shocks. I show that the optimal response to either shock can be characterized as an approximate price stability rule, in a sense that the price level is still largely stabilized around its deterministic trend. Hence, the optimal policy under SDP is to closely replicate the dynamics under the TDP assumption previously found in past studies such as Khan, King, and Wolman (2003) and Schmitt-Grohe and Uribe (2007). Despite the close association to optimal response under TDP, the presence of endogenous timing of price adjustment under SDP importantly alters the policy tradeoffs faced by the monetary authority. In particular, I show that it is optimal for the monetary authority to let inflation vary *more* under SDP. The lower cost of inflation variation on the relative-price distortion under SDP allows for smaller focus on inflation stabilization, relative to other stabilization goals involving the output gap and the nominal interest rate.

[1] The choice of which SDP model to use matters little for the general results in this paper. One can use, for example, the model of Golosov and Lucas (2007) or Midrigan (2011) instead of Dotsey, King, and Wolman (1999). The key ingredient for the results is that firms are allowed to respond to shocks and changing states of the economy by adjusting their prices, subject to some regularity conditions, e.g. the optimal reset price is increasing with inflation.

The modification of the policy tradeoffs under SDP is further demonstrated through the calculated second moments of various variables and the optimal response coefficients within a simple policy rule supporting the optimal Ramsey allocation. I find that based on the benchmark calibration, compared to TDP, the optimal Ramsey policy under SDP delivers a 24 percent higher standard deviation of inflation, but with 26 percent and 6 percent lower standard deviations of output gap and nominal interest rate, respectively. And while the Ramsey allocations under both TDP and SDP can be supported and implemented by way of a simple, Taylor-like, policy rule, the feedback coefficients are shown to be different. In particular, the optimum feedback response coefficients on the output gap and the lagged nominal interest rate are both higher under SDP.

The result above stands in contrast to the finding in Nakov and Thomas (2014), which conclude that the incorporation of SDP instead of TDP does not alter much the optimal Ramsey policy responses. This different finding stems from their assumption of a cashless model, with only relative-prive and markup distortions, and their focus on a productivity shock. Under this environment, the monetary authority is not faced by a policy tradeoff in the first place, which makes it impossible to observe the change in the policy tradeoffs under SDP. In this paper, the policy tradeoffs are provided through the additional presence of monetary distortions, thus allowing such an observation to be made.

In addition to the timeless perspective Ramsey policy, I also analyze the optimal monetary policy start-up problem. As described in Woodford (2003), a timeless perspective policy is a policy that is assumed to have been long implemented. Most studies on optimal precommitment policy have focused exclusively on timeless-perspective policies. However, even though a precommitment policy is optimal from the timeless perspective, it is not the true Ramsey solution that maximizes the representative agent's welfare. The true Ramsey solution specifies that the monetary authority should treat the starting period of precommitment policy implementation differently than subsequent periods. This is because in the starting period, there is no past commitment that the monetary authority must follow through. With monopolistically-competitive firms and nominal price rigidity as in the model in this paper, this so-called *start-up problem* is manifested in the optimal decision of the monetary authority to temporarily stimulate the economy by generating surprise inflation in the starting period. It is optimal to do so since the economy operates inefficiently due to the presence of firms' monopoly power. It is of interest here to analyze the start-up problem since it allows us to more clearly highlight the modification in the policy tradeoffs under SDP, since in such a case the aggregate price level deviates further away from its deterministic path.

I find that incorporating SDP in the model economy leads to different start-up dynamics compared to the dynamics under the standard TDP assumption along several interesting and important dimensions. In particular, it is optimal to generate much *higher* start-up inflation under SDP in spite of the monetary authority having *less* leverage over real activities in the presence of SDP. This result is once again due to the subtle modification to the policy tradeoffs involving the lower cost of inflation variation on the relative-price distortion.

The rest of this paper is organized as follows. Section 2 presents the model and describes the equilibrium allocation under the Ramsey optimal policy. Section 3 describes the calibration of the model parameters, the solution methodology, and the welfare cost measure that can used to evaluate the cost of adopting a certain policy instead of the Ramsey policy. Section 4 presents and analyzes the steady state and the near-steady-state dynamics under the timeless-perspective Ramsey policy. I also present several robustness analyses and calculate the second moments of various variables. Section 5 investigates the start-up problem and the implementability of the Ramsey allocations, under both TDP and SDP, by way of a simple policy rule. Section 6 concludes.

2 The model

I first present the private sector's efficiency conditions of the model economy. The description of the optimal precommitment policy problem, in which all the private sector's efficiency conditions must be respected at all times, follows next.

2.1 The private sector

The private sector consists of two sets of optimizing agents, a representative household and a continuum of intermediate-good firms on a unit measure. These agents solve dynamic optimization problems given the state of the economy and the knowledge of the policy rule employed by a fully committed monetary policy authority. The economy has two sets of state variables. The first set, $s_t^{(1)}$, contains the "non-policy" state variables, further decomposed into the endogenous state vector, k_t, and the exogenous state vector, ς_t. ς_t evolves according to the process $\varsigma_{t+1} = \nu(\varsigma_t, \eta_{t+1})$, where η_t is a vector of serially independent shocks. Lagged optimal policy multipliers, collected in vector $s_t^{(2)}$, are the additional set of state variables. These multipliers summarize the past policy plans of the monetary authority that must be respected under the precommitment policy. The entire set of state variables is $s_t = (s_t^{(1)}, s_t^{(2)})$. Later in the section the individual elements of these state vectors will be clarified. Aggregate fluctuations are driven by three exogenous shocks: productivity, government purchase, and markup shocks.

2.1.1 Intermediate-good firms

There is a continuum of monopolistically competitive firms in the economy, indexed by $i \in [0, 1]$. Each firm i produces a differentiated variety. These varieties of intermediate goods are bundled together into a final consumption good according to

$$c_t = \left[\int_0^1 c_t(i)^{\frac{\varepsilon_t - 1}{\varepsilon}} \right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}} ,$$

The elasticity of substitution across goods varieties (or equivalently, the price elasticity of demand for any variety i), ε_t, is potentially time varying and subject to a shock. As ε_t governs the markup that firm i charges over its production cost, a shock to ε_t can be interpreted as a markup shock. There is also a government that purchases final consumption goods, g_t, aggregated according to the same process.[2] Let $p_t(i)$ be the relative price of variety i. The demand for variety i is then given by $p_t(i)^{-\varepsilon} y_t = p_t(i)^{-\varepsilon_t}(c_t + g_t)$, where $y_t = c_t + g_t$ is the aggregate economy-wide output.

Each firm i is subject to a fixed price-adjustment cost that needs to be paid every time it adjusts the price of its product. Following Dotsey, King, and Wolman (1999), the fixed adjustment costs (in labor units) are heterogenous across firms and are drawn each period independently from a time-invariant distribution with cumulative distribution function (CDF) $G(\cdot)$. Firms that choose not to adjust prices, after observing the fixed cost draw and the state of the economy, do not need to pay these fixed costs but must keep the nominal price from the previous period. This specification means that at any given period firms will be distributed according to the *time since the last price adjustment* j, with $j = 0$ indicating those firms that adjust in the current period. Let $\omega_{j,t}$ be the end-of-period fraction of firms that last adjusted their prices j periods ago.[3] These variables summarize the distribution of firms according to the time since the last price adjustment and evolve according to

$$\omega_{j,t} = (1 - \alpha_{jt})\omega_{j-1,t-1} \ , \tag{1}$$

for $j = 1, ..., J - 1$, where J is the maximum possible period of price fixity. $\alpha_{j,t}$ is the fraction of firms j—firms that last adjusted prices j periods ago—that decide to adjust at the beginning of

[2] Government purchases are assumed to be financed by lump-sum taxes.

[3] By end-of-period, I mean that these fractions are observed in the current period t after firms' production and pricing decisions are made.

period t right after the fixed cost draw. The fraction of adjusting firms in the current period t is then given by

$$\omega_{0,t} = 1 - \sum_{j=1}^{J-1} \omega_{j,t} \ . \tag{2}$$

Note that in the above SDP specification, both $\{\alpha_{jt}\}_{j=1}^{J-1}$ and $\{\omega_{j,t}\}_{j=0}^{J-1}$ are determined endogenously and depend on aggregate variables and firms' prices. Under the TDP assumption as in Levin (1991) and Khan, King, and Wolman (2003), these variables are exogenous and become parameters of the model.[4]

Next, I describe the maximization problem of adjusting firms. The production function for any firm i is given by $y_t(i) = a_t n_t(i)$, where a_t is the exogenous aggregate productivity level and $n_t(i)$ is the production labor used by firm i. The labor market is global so that all firms faced the same real marginal cost w_t/a_t, where w_t is the aggregate real wage. Let $p_{j,t}$ be the relative price of firms that have not adjusted for j period(s) at time t. Given the form of the consumption aggregator above, the real profit of a firm with price $p_{j,t}$ is given by

$$z(p_{j,t}, s_t) = [p_{j,t} - \frac{w_t}{a_t}] \cdot p_{j,t}^{-\varepsilon_t}(c_t + g_t) \ .$$

Define $v_0(s_t)$ as the value function of a typical adjusting firm, net of adjustment cost. Given the state vector s_t, the maximization problem is

$$\begin{aligned} v_0(s_t) \quad = \quad & \max_{p_{0,t}} \ \{z(p_{0,t}, s_t) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(1-\alpha_{1,t+1}) v_1(p_{1,t+1}, s_{t+1}) \\ & + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \alpha_{1,t+1} v_0(s_{t+1}) - \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \alpha_{1,t+1} w_{t+1} \Xi_{1,t+1}\} \ , \end{aligned} \tag{3}$$

where the choice variable $p_{0,t}$ is the optimal relative price. The above expression says that an adjusting firm chooses the optimal price to maximize its expected present discounted values of current and future profits. In doing so, it has to take into account the possibility of price adjustment in future periods. There is an expected probability of $(1 - \alpha_{1,t+1})$ that it will choose not to adjust so that its expected value in the next period is $v_1(p_{1,t+1}, s_{t+1})$. With expected probability $\alpha_{1,t+1}$, it will optimally choose to adjust so that its value becomes $v_0(s_{t+1})$. The last term in the second line of (3) reflects the fact that if the firm decides to adjust in the next period, it must also pay the fixed adjustment cost. $\Xi_{1,t+1}$ is the expected fixed adjustment cost in the next period, conditional on adjustment. Since the fixed costs are in terms of labor unit, the expected cost in consumption

[4] This TDP specification nests two popular price/wage rigidity specifications. When $J \to \infty$ and α_j are identical across j, the specification collapses to Calvo (1983). For any finite J and $\{\alpha_j\}_{j=1}^{J-1} = 0$ with $\alpha_J = 1$, we have the contracting model of Taylor (1980).

units is then given by $w_{t+1}\Xi_{1,t+1}$. Note that since households own the firms, future periods are discounted by the effective discount factor $\beta E_t \frac{\lambda_{t+1}}{\lambda_t}$, where λ_t is the shadow value of households' income.

For firms that do not adjust ($j = 1, ..., J-1$) and hence simply apply prices from the previous period, the value functions can be expressed as[5]

$$\begin{aligned} v_j(p_{j,t}, s_t) \quad = \quad & z(p_{j,t}, s_t) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}(1-\alpha_{j+1,t+1}) v_{j+1}(p_{j+1,t+1}, s_{t+1}) \\ & + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \alpha_{j+1,t+1} v_0(s_{t+1}) - \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \alpha_{j+1,t+1} w_{t+1} \Xi_{j+1,t+1} \ . \end{aligned} \tag{4}$$

The first order necessary condition of the the dynamic problem (3) and recursive differentiation of (4) lead to a formula for the optimal nominal price,

$$P_{0,t} = p_{0,t} \cdot P_t = \frac{\varepsilon_t}{\varepsilon_t - 1} \frac{\sum_{j=0}^{J-1} \beta^j E_t \frac{\omega_{j,t+j}}{\omega_{0,t}} \frac{\lambda_{t+j}}{\lambda_t} \frac{W_{t+j}}{a_{t+1}} (P_{t+j})^{\varepsilon} y_{t+j}}{\sum_{j=0}^{J-1} \beta^j E_t \frac{\omega_{j,t+j}}{\omega_{0,t}} \frac{\lambda_{t+j}}{\lambda_t} (P_{t+j})^{\varepsilon - 1} y_{t+j}} \ , \tag{5}$$

where P_t and W_t are the aggregate price level and nominal wage at time t, respectively. The consumption aggregator implies the aggregate price level is given by

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \ .$$

On the equilibrium determination of the probability of price adjustment, firms only adjust if there is a positive benefit of doing so — that is, if the value of adjustment outweighs the fixed cost associated with adjustment. Given the continuous distribution of the fixed adjustment cost, there will be a mass of firms at the margin for each j that are indifferent between adjusting their prices or keeping the prices from the previous period. For these firms, there is a zero benefit to adjust, so that $(v_{0,t} - v_{j,t}) = w_t \bar{\epsilon}_{j,t}$, where $\bar{\epsilon}_{j,t}$ is the fixed cost at the margin for bin j.[6] Hence, the proportion or probability of firms adjusting for each j is given by

$$\alpha_{j,t} = G\left(\frac{v_{0,t} - v_{j,t}}{w_t} \right) \tag{6}$$

for $j = 1, ..., J-1$. For $j = J$, we have $\alpha_{J,t} = 1$ since all firms will find it optimal to adjust after J periods. Finally, the average or expected adjustment cost conditional on adjustment is given by

$$\Xi_{j,t} = \frac{1}{\alpha_{j,t}} \int_0^{G^{-1}(\alpha_{j,t})} x dG(x) \tag{7}$$

[5]There is no max operator in this expression since the only decision made by non-adjusting firms is the input decision for production to meet demand.

[6]This fixed cost at the margin is the largest cost that is actually paid by adjusting firms.

for $j = 1, ..., J$.

As a preview of the optimal monetary approach used in this paper, all the efficiency conditions of the firms' problem above become relevant constraints that must be respected by the committed monetary authority. Specifically, the monetary authority must respect the optimal price decisions by adjusting firms given by equation (5).[7] It must also respect the firms' adjustment decision, which are summarized by (3) and (6). The evolution of the distribution of firms in (1) and (2) must also be respected. In particular, note that some of these constraints are forward-looking, which is why there needs to be a commitment mechanism in the optimal policy problem. This commitment mechanism is summarized by the lagged policy multipliers in a recursive Lagrangian problem.

2.1.2 Households

Most elements of the households' optimization problem are identical to those in Khan, King, and Wolman (2003). Hence, I simply mention the core assumptions and proceed directly to the resulting efficiency conditions. Appendix A provides additional details on the households' optimization problem.

Households choose the amount of final goods consumption (c_t) and leisure (l_t) to maximize a lifetime utility function subject to a budget constraint. The instantaneous utility function is assumed to be given by $u(c_t, l_t) = \frac{1}{1-\sigma} c_t^{1-\sigma} + \chi \frac{1}{1-\phi} l_t^{1-\phi}$. Final consumption goods can be purchased using credit or money (cash). Specifically, households choose to purchase a fraction of goods ξ_t with credit and the balance $1 - \xi_t$ with money. There is a cost associated with using credit, as in Dotsey and Ireland (1996). These transaction time costs of using credit (in terms of labor time units) are heterogenous across goods and are randomly drawn from a continuous distribution with CDF $F(.)$. Since credit use is costly, money can be used to facilitate the purchase of final consumption goods. However, there is an opportunity cost to using money in the form of foregone interest (with rate R_t) from purchasing one-period nominal discount bond. Households thus must balance the costs associated with holding cash and using credit in purchasing final consumption goods. In the absence of costly transaction time, households would purchase all goods using credit, as in the cashless model of Woodford (2003). At the other extreme where no credit is allowed, we have a familiar cash-in-advance model as in Lucas (1980). Finally, households also derive income from their labor effort in the amount of $w_t n_t$.

[7] In terms of actual computation of the optimal policy, equation (5) must be written in a recursive form. This can be done by reformulating this constraint using the marginal value recursions, i.e. the derivatives of the value functions (3) and (4) with respect to the optimal relative price $p_{0,t}$.

Proceeding to households' efficiency conditions, the labor-leisure optimal choice implies the equality between the utility cost of foregone leisure and the value of income gained by working; thus,

$$u_l(c_t, l_t) = w_t \lambda_t \ . \tag{8}$$

The holdings of one-period nominal discount bonds require that

$$\frac{\lambda_t}{1+R_t} = \beta E_t \frac{\lambda_{t+1}}{1+\pi_{t+1}} \ , \tag{9}$$

where π_t is the net inflation rate. Next, the marginal utility of consumption must equal to the full price of a unit of final consumption good,

$$u_c(c_t, l_t) = \lambda_t \left(1 + R_t \left(1 - \xi_t\right)\right) \ . \tag{10}$$

Note that since there is an opportunity cost of using money, the full consumption price above involves the nominal interest rate R_t, multiplied by the fraction of goods bought using money, $(1 - \xi_t)$. The largest credit time cost that the representative household will choose to pay is $R_t c_t / w_t$, so that the fraction of goods purchased using credit is given by

$$\xi_t = F\left(\frac{R_t c_t}{w_t}\right) \ , \tag{11}$$

Finally, the time use constraint for the economy is given by

$$l_t + n_t + \int_0^{(R_t c_t / w_t)} x dF(x) = 1 \ , \tag{12}$$

where $\int_0^{(R_t c_t / w_t)} x dF(x)$ is the aggregate time costs of credit. Given the above specification, the implied real money demand depends positively on c_t and negatively on R_t and ξ_t.[8]

2.1.3 Other equilibrium conditions and the state vectors

Several aggregate equilibrium condition need to hold in the economy. First, the aggregate production of goods must be equal to the aggregate demand:

$$a_t n_t^y = \left[\sum_{j=0}^{J-1} \omega_{j,t} p_{j,t}^{-\varepsilon_t}\right] (c_t + g_t) \ , \tag{13}$$

[8] The demand for nominal money is $M = (1-\xi)\bar{P}c$, where $\bar{P} = (1+R)P$ is the unit price of the final consumption good. Combining this with $\xi = F(Rc/w)$, we have $\frac{M}{Pc} = 1 - F(\frac{Rc}{w})$.

where n_t^y represents the total labor used in production. Since the fixed adjustment costs are in term of labor costs, there is also the aggregate labor used for this price adjustment process given by

$$n_t^p = \sum_{j=0}^{J-1} \omega_{j,t-1} \left(\alpha_{j+1,t} \Xi_{j+1,t}\right) \ . \tag{14}$$

The sum of the aggregate production labor and pricing labor must be equal to the total labor supply so that we have

$$n_t^y + n_t^p = n_t \ . \tag{15}$$

The relative price aggregation implied by the consumption aggregator is

$$\sum_{j=0}^{J-1} \omega_{j,t} p_{j,t}^{1-\varepsilon_t} = 1 \ . \tag{16}$$

Given the gross inflation $(1+\pi_t)$, the evolution of predetermined relative prices (for $j = 1, ..., J-1$) can be expressed by[9]

$$p_{jt} = p_{j-1,t-1} \frac{1}{1+\pi_t} \ . \tag{17}$$

Together with the lagged fractions of firms, $\{\omega_{j,t-1}\}_{j=0}^{J-1}$, the lagged relative prices, $\{p_{j,t-1}\}_{j=0}^{J-2}$, are elements of the endogenous state vector k_t in $s_t^{(1)}$.

Finally, I assume simple driving processes for the three exogenous variables:

$$\begin{aligned} \ln(a_t/\bar{a}) &= \rho_a \ln(a_{t-1}/\bar{a}) + \sigma_a e_{z,t} \ , & (18) \\ \ln(g_t/\bar{g}) &= \rho_g \ln(g_{t-1}/\bar{g}) + \sigma_g e_{g,t} \ , & (19) \\ \ln(\varepsilon_t/\bar{\varepsilon}) &= \rho_\varepsilon \ln(\varepsilon_{t-1}/\bar{\varepsilon}) + \sigma_\varepsilon e_{\varepsilon,t} \ , & (20) \end{aligned}$$

where $[e_{z,t}, e_{g,t}, e_{\varepsilon,t}] \in \eta_t$ are i.i.d. shocks, normalized to have unit variance. σ_a, σ_g, and σ_ε and are the standard deviations of shocks to productivity, government purchase, and firms' markup, respectively. The exogenous state vector ς_t thus consists of a_t, g_t, and ε_t. This completes the definition for the state vector $s_t^{(1)}$.

2.2 Identifying the market distortions

Since monetary policy operates through its influences on various distortions in the economy it is useful to explicitly identify these distortions. There are four sets of distortions present in the model

[9] Specifically, suppose that $P_{j-1,t-1}$ is the nominal price of the intermediate-goods for firms that have not adjusted their prices for $j-1$ periods at time $t-1$, with $j \leq J-1$. If a firm in this bin chooses not to adjust at time t, then $P_{j,t} = P_{j-1,t-1}$, so that $P_{j,t}/P_t = P_{j-1,t-1}/P_t \Rightarrow p_{j,t} = (P_{j-1,t-1}/P_{t-1})(P_{t-1}/P_t) \ \Rightarrow p_{j,t} = p_{j-1,t-1}\frac{1}{1+\pi_t}$.

economy. As in Schmitt-Grohe and Uribe (2005, 2007), I assume fiscal subsidies are unavailable to be used to eliminate these distortions.

First, there is a distortion due to firms' monopoly power that is frequently termed the *markup* distortion and is captured by the reciprocal of the real marginal cost,

$$\psi_t = \frac{a_t}{w_t} \ . \tag{21}$$

If firms are perfectly competitive $\psi_t = 1$ since the real wage is always equal to productivity. This distortion influences, for example, the firms' value functions (3), (4) and the optimal price chosen by adjusting firms (5).

Second, there is a distortion arising from infrequent price adjustments,

$$\Delta_t = \frac{a_t n_t^y}{(c_t + g_t)} = \sum\nolimits_{j=0}^{J-1} \omega_{j,t} p_{j,t}^{-\varepsilon_t} \ . \tag{22}$$

This *relative-price* distortion measures the extent of lost aggregate output due to sticky prices—if Δ_t is greater than unity, then it takes more labor to produce a given volume of output as suggested by (13). Both the markup and relative-price distortions are eliminated at zero inflation where both ψ_t and Δ_t are equal to unity. The model then collapses to a standard RBC framework if monetary frictions are absent.

The third set of distortions involve the use of money and credit in purchasing consumption goods. Frequently termed the monetary distortions, these distortions can be decomposed further into the *monetary wedge* and *shopping-time* distortions. The monetary wedge distortion is the inefficiency that arises because the full cost of consumption is augmented by the requirement that money must be held to finance a portion of consumption purchase. It can be expressed as

$$\Phi_t = R_t(1 - \xi_t) \ . \tag{23}$$

As suggested by (10), this distortion drives a wedge between the marginal utility of consumption and the shadow value of wealth. The shopping-time distortion arises because the use of credit in purchasing consumption goods requires some use of labor; this distortion affects the total available resources in (12). It is governed by the expression

$$h_t = \int_0^{(R_t c_t / w_t)} x dF(x) \ . \tag{24}$$

Both the monetary wedge and the shopping-time distortions are minimized at zero nominal interest rate as prescribed by the Friedman rule, so that the inflation rate must be negative (deflation).

In the time-dependent-pricing version of the model, all three distortions mentioned above are present. Under SDP, there exists another distortion involving the costs associated with price adjustments, represented by (14):

$$n_t^p = \sum\nolimits_{j=0}^{J-1} \omega_{j,t-1} \left(\alpha_{j+1,t} \Xi_{j+1,t}\right) \ .$$

Like the shopping-time distortion, this *menu cost* distortion is best viewed as a drain on the economy's resources.

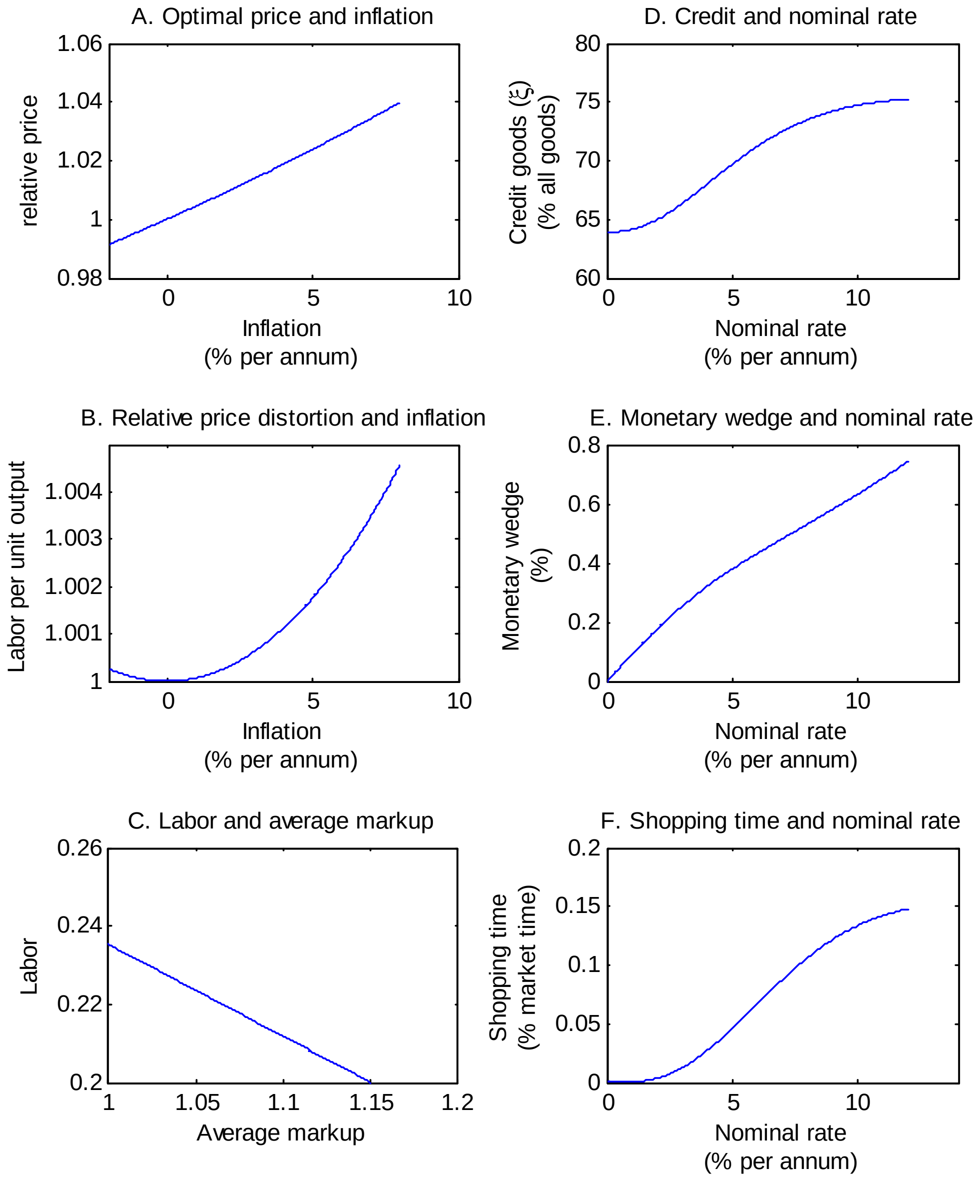
To gain more insight into the distortions, Figure 1 looks at the steady-state implications of these various inefficiencies in the model economy. First, we look at the relationship between inflation and the relative price distortion in panels A and B. For simplicity and comparability to the literature, these two panels are produced under the assumptions that there are no monetary distortions and firms' pricing decisions are time-dependent.[10] Looking first at panel A, the optimal relative price (p_0) is equal to one under zero inflation and is increasing with inflation. When inflation is positive, price-adjusting firms will charge a higher price since they anticipate that the relative price will be eroded by future inflation. On the contrary, under negative inflation the optimal relative price is lower since the relative price will increase if firms do not adjust in future periods. That is, those firms which do not adjust will be faced with a decreasing relative price under positive inflation and an increasing relative price under deflation—as a consequence, price dispersion is increasing with both non zero inflation and deflation (panel B). Only under zero inflation is the relative price distortion is eliminated and a unit of labor will exactly produce a unit of output. Panel C looks at the implication of varying degrees of average markup on market activity (production labor). As the average markup increases, there is a corresponding decrease in the real wage paid by firms so that households substitute away from market activity. The monetary authority can temporarily erode this average markup by generating surprise inflation, a topic that will be discussed in Section 5.

Moving on to the monetary distortions in the right-side panels of Figure 1, we see that households substitute away from cash transactions into costly credit since the opportunity cost of holding money becomes higher as the nominal interest rate increases.[11] As a consequence, the fraction of goods bought using credit increases with the nominal rate (panel D). Higher nominal rates are also

[10] Specifically, I assume that $J = 6$, with $\{\omega_j\}_{j=0}^{J-1}$ and $\{\alpha_j\}_{j=1}^{J-1}$ equal to their steady-state values under the optimal policy presented in figure 2. For any given inflation rate, we can compute the optimal reset price and the corresponding relative price distortion at the steady state. Assuming state dependence does not change the figures in panels A and B.

[11] To generate panels D, E, and F, I assume that the credit cost distribution and its parameters are as in Table 1. I also assume that the production labor is $n^y = 0.2$, and the real wage $w = a = 1$, with consumption $c = a \cdot n^y = 0.2$. For any given nominal interest R, we can then compute the credit good fraction ξ and the two monetary distortions.

Figure 1: Steady state implications of various distortions



Notes: (1) panels A, B, and C are generated using $J = 6$ with $\{\omega_j\}_{j=0}^{J-1}$ and $\{\alpha_j\}_{j=1}^{J-1}$ equal to their steady-state values under the optimal policy based on the calibration in Table 1, with monetary and menu cost distortions are assumed to be absent; (2) panels D, E, and F are constructed using Uniform distribution for the credit cost distribution, with parameter values as in Table 1—further, I assume constant $n^y = 0.2$ and $w = a = 1$ so that $c = a \cdot ny = 0.2$, as the nominal interest rate (R) varies.

associated with higher monetary wedge distortions (panel E), which in turn create a substitution away from consumption since this wedge affects the full cost of consumption as shown in equation (10). Finally, as displayed in panel E, the higher fraction of credit goods associated with higher nominal rates will in turn increase the transaction (shopping) time associated with costly credit.

Figure 1 also shows that there are tradeoffs among the distortions, even in this simple static illustration. As an illustration, we know from panel B that the relative price distortion is minimized at zero steady-state inflation. However, at zero inflation, the nominal interest rate must be positive for the Fisher equation (9) to be satisfied—hence, the monetary wedge distortion (panel E) is not minimized. On the contrary, if the nominal interest rate is zero so that there is a small deflation at the steady state, the relative price distortion will not be minimized. The task of the monetary authority then is to balance the costs of these various distortions, both in the steady state and in the near-steady-state dynamics in response aggregate shocks.

2.3 The equilibrium under Ramsey optimal policy

I define the equilibrium under Ramsey optimal policy as a collection of stationary processes for all the endogenous variables $(c_t,\ n_t,\ \{\alpha_{j,t}\}_{j=1}^{J-1},\ ...)$ in which the benevolent, fully-committed policy authority maximizes the expected lifetime utility (welfare) of the representative agent,

$$V_0 = E_0 \sum\nolimits_{t=0}^{\infty} \beta^t u(c_t, l_t) \ ,$$

under the decentralized economy setting where all the private sector efficiency conditions (e.g. equations (6), (8), and (9)) and the aggregate market clearing conditions are respected. Without any loss of generality, here I assume that the starting period of the Ramsey policy is at $t = 0$.

Following Woodford (2003) the policy is optimal from the timeless perspective, i.e. the policy authority is assumed to have committed to the state-contingent policy rule since time immemorial. Although the true Ramsey solution involves a policy authority that treat the initial period $(t = 0)$ differently due to the absence of past commitments, I assume that the monetary authority ignores this "start-up problem." The cost of ignoring these initial conditions is analyzed in Section 5. In finding the Ramsey policy, I also assume that it is not bound by any specific policy rule. That is, rather than searching for the optimal policy rule, I search for the optimal allocation consistent with the Ramsey equilibrium. I will however address the optimal policy rule, within a family of simple rules, later in Section 5. For the full recursive Lagrangian of the Ramsey problem, see Appendix B.

3 Calibration, solution methodology, and welfare measure

3.1 Calibration

Table 1 presents the calibration of the structural parameters in the model. The time unit is meant to be a quarter. Households' preference follows

$$u(c_t, l_t) = \frac{1}{1-\sigma} c_t^{1-\sigma} + \chi \frac{1}{1-\phi} l_t^{1-\phi}$$

The choice of σ and ϕ equal unity means that the utility function involves log function in consumption and leisure. The parameter χ is chosen so that the steady-state labor in production is equal to 0.2. The choice of the steady-state demand elasticity implies that the firms' markup is about 11 percent in the flexible-price equilibrium. Following various estimates in the real business cycle literature, the aggregate productivity is very persistent ($\rho_a = 0.95$), with the standard deviation (σ_a) equal to 0.0072.[12] The choices of $\{\rho_g, \sigma_g\}$ and $\{\rho_\varepsilon, \sigma_\varepsilon\}$ are in line with the estimates in Ravn (2007) and Bhattarai, et al. (2012), respectively.

In terms of the fixed adjustment cost distribution, I use a generalized Tangent distribution, used in Dotsey and King (2005). The results in this paper are not sensitive to the choice of this fixed cost distribution, including its parameter values.[13] I calibrate the largest possible fixed cost paid by firms, B, so that the maximum number of quarters of price fixity (J) is 6 quarters. Figure 2 displays the steady-state probability (α_j) and firms' distribution (ω_j) of price adjustment based on these calibrated parameters under the Ramsey optimal policy. The price adjustment hazard is increasing with the time since last price adjustment, unlike the constant hazard implied by the standard Calvo model. The implied mean and median durations of price fixity are 4.02 quarters and 3 quarters, respectively, and the average frequency of price adjustment is 24 percent per quarter. This degree of price rigidity is in the ballpark of available empirical estimates using macroeconomic data such as Christiano, Eichenbaum, and Evans (2005). It is higher than the evidence based on microeconomic data reported in Bils and Klenow (2004), but still close to the finding in Nakamura and Steinsson (2008).

For the credit cost distribution, I assume a generalized Uniform distribution with CDF

$$F(x) = \bar{\kappa} + (1-\bar{\kappa}) \left[\frac{(x/K) - \kappa_1}{\kappa_2 - \kappa_1} \right] \ ,$$

[12] The values of ρ_a and σ_a are consistent with King and Rebelo (1999).

[13] Of course, certain quantitative implications may be sensitive to the choice of the fixed cost distribution, e.g. the calculated second moments in Table 3. However, the general message about the difference between the optimal policies under SDP vs. TDP remains unaltered.

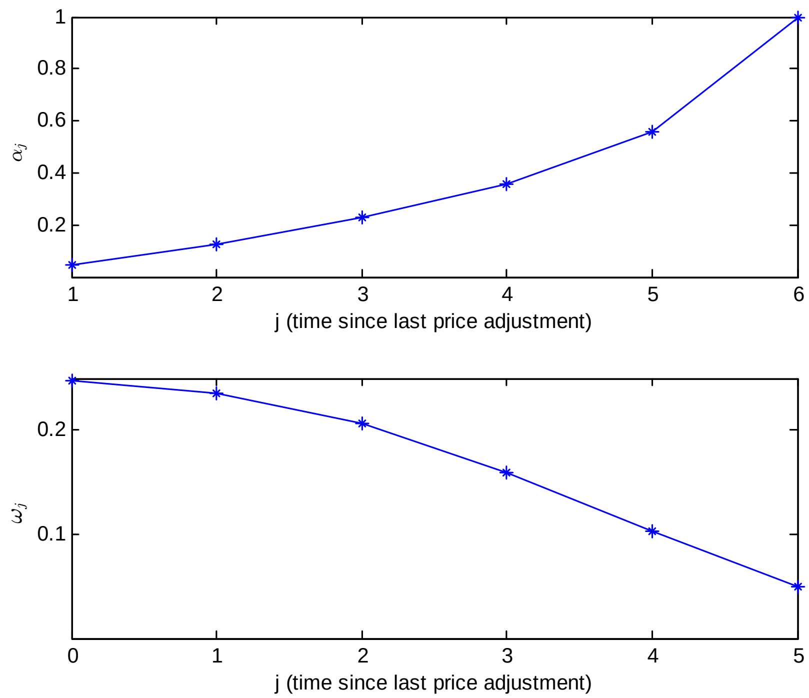
TABLE 1: STRUCTURAL PARAMETERS CALIBRATION

Parameter	Description	Value
β	Discount factor	0.99
σ	Preference parameter	1
ϕ	Preference parameter	1
χ	Preference parameter	3.59
$\bar{a}$	Steady state productivity level	1
$\bar{g}$	Steady state government purchase	0
$\bar{\varepsilon}$	Steady state price elasticity of demand of intermediate goods	10
ρ_a	Productivity AR(1) persistence parameter	0.95
ρ_g	Government purchase AR(1) persistence parameter	0.90
ρ_ε	Demand elasticity AR(1) persistence parameter	0.25
σ_a	Standard deviation of productivity shock	0.0072
σ_g	Standard deviation of government purchase shock	0.0080
σ_ε	Standard deviation of demand elasticity (markup) shock	0.0016
	Fixed adjustment cost distribution	
	CDF: $G(x) = H_1 \tan\left(b_1 + (b_2 - b_1)\frac{x}{B}\right) + H_2$, $H_1 = \frac{1}{\tan(b_2)-\tan(b_1)}$, $H_2 = \frac{-\tan(b_1)}{\tan(b_2)-\tan(b_1)}$.	
b_1	Parameter 1	-0.0157
b_2	Parameter 2	0.0157
B	Maximum cost	0.00075
	Credit cost distribution (Uniform)	
	CDF: $F(x) = \bar{\kappa} + (1-\bar{\kappa})\left[\frac{(x/K)-\kappa_1}{\kappa_2-\kappa_1}\right]$	
$\bar{\kappa}$	Fraction with zero cost	0.62
κ_1	Parameter 1	0
κ_2	Parameter 2	1
K	Maximum cost	0.0127

where κ_1 and κ_2 are the left and the right parameters of the Uniform distribution. It is generalized in a sense that I allow a fraction $\bar{\kappa}$ of goods to have zero transaction cost and allow it to have another parameter involving the maximum cost (K). The choice of $\kappa_1 = 0$ and $\kappa_2 = 1$ implies a Standard Uniform distribution (when $\bar{\kappa} = 0$). The choice of K is taken from the estimate in Khan, King, and Wolman (2003).[14] Finally, the value of $\bar{\kappa}$ is chosen such that in the steady state households buy 65% of consumption goods using credit. Similarly to the fixed cost distribution, the choice of the credit cost distribution and its parameter values are not crucial to the results in this paper.

[14] Khan, King, and Wolman use a generalized Beta instead of a uniform distribution.

Figure 2: Steady-state probability (α_j) and distribution (ω_j) of price adjustment
Optimal policy under SDP



Notes: (1) the longest period of price fixity is $J = 6$; (2) all market distortions are present (full distortions); (3) the horizontal (j) axis is in quarters.

3.2 The solution methodology and the welfare cost measure

The Ramsey policy equilibrium is solved using the perturbation approximation solution method described in Johnston, King, and Lie (2014). Using the method, I solve for a second-order accurate solution to the equilibrium policy functions, which is needed for welfare to be accurately computed up to a second-order approximation.[15] The resulting welfare computation can be used to search for the coefficients of the optimal simple policy rule and the cost of implementing alternative policy rules other than the Ramsey policy, presented in Section 5.

To quantify the cost of implementing an alternative policy rule instead of the Ramsey policy there needs to be a welfare cost measure. I follow the approach of Schmitt-Grohe and Uribe (2006) by defining this welfare cost as the fraction of consumption that the representative household would be willing to give up under the benchmark Ramsey policy environment to be equally well

[15] See Schmitt-Grohe and Uribe (2004) for a discussion on why a second-order approximation to the equilibrium solution is needed for an accurate welfare computation.

off as under the alternative policy environment. Specifically, let $\{c_t^r,\, l_t^r\}$ and $\{c_t^a,\, l_t^a\}$ be the state-contingent plans for consumption and leisure under the Ramsey policy and under the alternative policy, respectively. The *conditional* welfare cost, λ^c, is implicit in the expression

$$E_0 \sum_{t=0}^{\infty} \beta^t u((1-\lambda^c)c_t^r, l_t^r) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^a, l_t^a) \ .$$

The expectation operator above makes clear that the welfare cost is conditional on the initial state at $t = 0$, which I assume to be the deterministic steady state under the Ramsey policy. The deterministic steady state under the alternative policy is assumed to be identical to that under the Ramsey policy. Similarly, the *unconditional* welfare cost, λ^u, can be obtained from

$$E\left\{\sum_{t=0}^{\infty} \beta^t u((1-\lambda^u)c_t^r, l_t^r)\right\} = E\left\{\sum_{t=0}^{\infty} \beta^t u(c_t^a, l_t^a)\ \right\}.$$

Here, E is the unconditional expectation operator.

We can then obtain second-order accurate approximations to λ^c and λ^u by following the procedure described in Schmitt-Grohe and Uribe (2006). For the specific form of the utility function and the calibrated parameters in Table 1, I obtain

$$\lambda^c \approx (1-\beta)\left(V^{rc}_{\sigma_\eta\sigma_\eta} - V^{ac}_{\sigma_\eta\sigma_\eta}\right)\frac{\sigma_\eta^2}{2},$$

where σ_η is the perturbation parameter and $V^{rc}_{\sigma_\eta\sigma_\eta}$ and $V^{ac}_{\sigma_\eta\sigma_\eta}$ denote the second-order expansions with respect to σ_η of the conditional welfare functions $V_0^r = E_0 \sum\limits_{t=0}^{\infty} \beta^t u(c_t^r, l_t^r)$ and $V_0^a = E_0 \sum\limits_{t=0}^{\infty} \beta^t u(c_t^a, l_t^a)$, respectively. Similarly,

$$\lambda^u \approx (1-\beta)\left(V^{ru}_{\sigma_\eta\sigma_\eta} - V^{au}_{\sigma_\eta\sigma_\eta}\right)\frac{\sigma_\eta^2}{2} \ .$$

Here, $V^{ru}_{\sigma_\eta\sigma_\eta}$ and $V^{au}_{\sigma_\eta\sigma_\eta}$ are the unconditional expectations of the welfare function under the Ramsey policy, $V_t^r = E_t \sum\limits_{t=0}^{\infty} \beta^t u(c_t^r, l_t^r)$, and the welfare function under the alternative policy, $V_t^a = E_t \sum\limits_{t=0}^{\infty} \beta^t u(c_t^a, l_t^a)$, respectively.

4 The optimal Ramsey policy: The steady state and the dynamics

In this section I characterize the optimal Ramsey policy both in the steady state and the approximate near steady-state dynamics. Throughout the analysis the implications based on the SDP model are compared to those resulting from its TDP model counterpart. The TDP model is constructed under the assumption that the $\{\alpha_j\}_{j=1}^{J}$ and $\{\omega_j\}_{j=0}^{J-1}$ parameters are equal to the steady-state values under the SDP model, with all other structural parameters are set as in Table 1.[16]

4.1 The optimal steady-state inflation

Consider first the steady-state inflation under the optimal policy. The task of the benevolent policy authority is to balance the costs of various distortions in the economy. As shown by various studies, e.g. King and Wolman (1999) and Woodford (2002), the optimal steady-state inflation is zero when only relative-price and markup distortions are present. Both distortions are eliminated under zero steady-state inflation and there is no long-run tradeoff between them. The presence of the menu cost distortion in the current SDP model does not alter this conclusion since this distortion is also eliminated under zero inflation, i.e. firms would find no need to change prices under zero inflation and hence would never have to pay the adjustment costs. However, as shown by Khan, King, and Wolman (2003) and Schmitt-Grohe and Uribe (2007) under TDP, the presence of monetary distortions alters the policy tradeoffs faced by the monetary authority. As prescribed by the Friedman rule, these monetary distortions are eliminated when the opportunity cost of holding money—the nominal interest rate—is zero. This requirement implies that inflation must be negative (deflation). It follows that the optimal steady-state inflation when all distortions are present should also be negative, but not as negative as implied by the Friedman rule. Put another way, the monetary authority compromises between maintaining price stability and following the Friedman rule.

Table 2 shows that the optimal steady-state inflation rate in the SDP model reflects the above discussion. The steady-state (net) inflation is negative (-1.07 percent per annum) but not as negative as the Friedman rule (-2.88 percent with the calibrated discount rate β). Under a lower elasticity of demand ($\varepsilon = 6$), the steady state deflation increases to -1.94 percent per annum. This higher deflation is associated with the lower cost of relative price distortion implied by the lower elasticity of demand.

I next compare the optimal steady-state inflation under SDP and TDP. Under the benchmark calibration there is a slightly higher deflation rate under SDP than under TDP — this pattern is also apparent under lower ε. This can be understood as follows. Recall that the key difference

[16] Note that firms do not have to pay adjustment costs under the TDP version of the model — hence, I adjust down the available resources under the TDP model by the steady state value of the aggregate pricing labor (np) under SDP. The results are insensitive to whether this adjustment is made.

between SDP and TDP is that under SDP, firms can endogenously choose to adjust their prices if the economic condition warrants it. Under TDP, firms do not have this opportunity to freely adjust prices and will only adjust if they are given the exogenous signal to do so. We should then expect that for a given non-zero inflation rate the price dispersion, i.e. the relative-price distortion, is lower under SDP compared to that under TDP. Recognizing this fact, the policy authority will thus shift more attention towards minimizing the monetary distortions and choose a slightly higher steady-state deflation rate under SDP.

Table 2: Steady-state annualized inflation under Ramsey policy

	SDP	TDP
Benchmark calibration (Table 1)	-1.074%	-1.068%
Lower demand elasticity ($\varepsilon = 6$)	-1.944%	-1.929%

Note: The time-dependent pricing (TDP) model is constructed under the assumptions that the probabilities of price adjustments ($\{\alpha_j\}_{j=1}^{J-1}$) and firms' distribution ($\{\omega_j\}_{j=0}^{J-1}$) are equal to their steady state values under the state-dependent pricing (SDP) model, with the same $J = 6$; all other parameters are as in Table 1.

This optimal steady-state inflation result tells us that the incorporation of SDP instead of the standard TDP assumption changes the nature of policy tradeoffs faced by the monetary policy authority. Below I show that this modification in the policy tradeoffs also occurs dynamically outside the steady state.

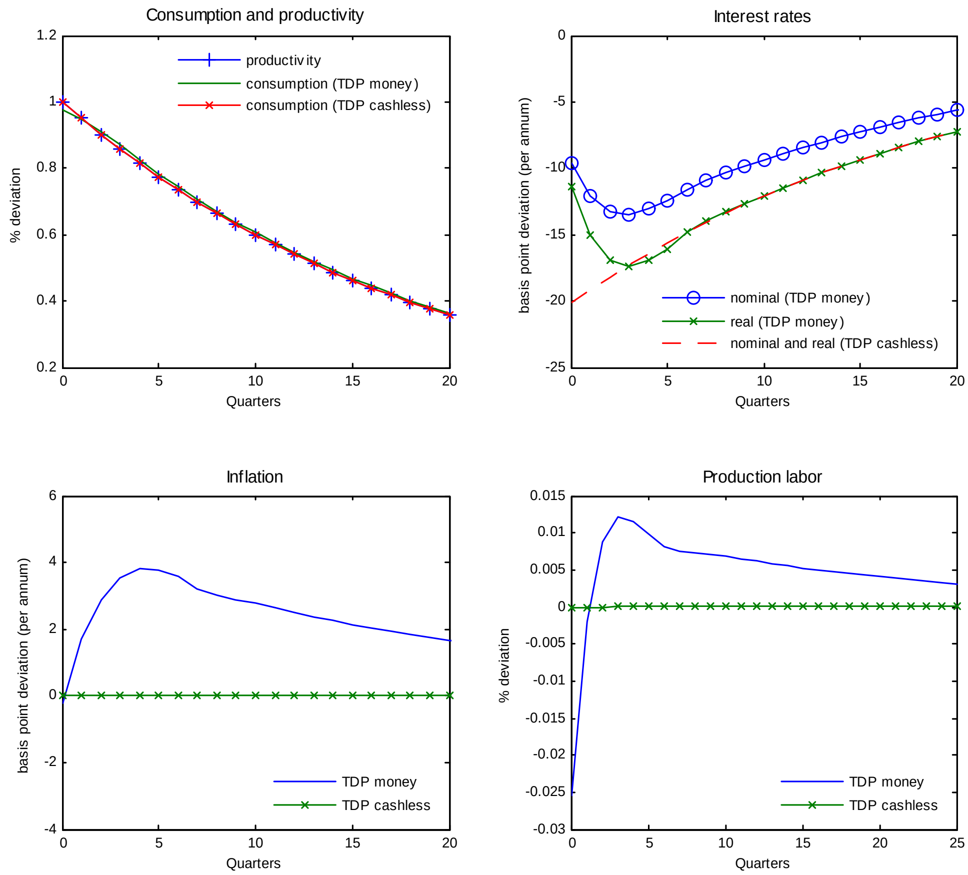
4.2 The near-steady-state dynamics

This section considers the Ramsey optimal policy responses to the exogenous shocks in the economy under the assumption of timeless perspective. Throughout the analysis below the economy is assumed to be always at the steady state prior to the shock. All the responses are based on the first-order approximate equilibrium solution. The responses based on second-order approximations are virtually identical to those under first order. Yet, the second-order solutions are needed in order to accurately characterize welfare costs of alternative policies, reported in Section 5.

4.2.1 Productivity shock

TDP with and without monetary distortions Consider first the optimal responses to a 1 percent productivity shock under the standard TDP assumption, depicted in Figure 3. For comparison, I also include and discuss first the dynamics when the monetary distortions are absent (the "cashless" model).

Figure 3: Optimal Ramsey policy responses to a productivity shock TDP cashless vs. TDP money



Notes: (1) *TDP cashless* refers to dynamics under TDP model when monetary distortions are absent; (2) *TDP money* refers to the dynamics under TDP model with monetary distortions; (3) the TDP model is constructed under the assumptions that the probabilities of price adjustments ($\{\alpha_j\}_{j=1}^{J-1}$) and firms' distribution ($\{\omega_j\}_{j=0}^{J-1}$) are equal to the steady-state values under TDP, with the same $J = 6$.

As is familiar, in the cashless model complete price stability is optimal in response to a productivity shock (see e.g. Goodfriend and King, 1997 and Benigno and Woodford, 2005).[17] It is optimal to fully accommodate the productivity shock and keep inflation constant at its steady

[17] In the cashless model, the optimal steady state inflation is zero. Hence, complete price stability is equivalent to constant inflation.

state level. Consumption increases one-to-one (1 percent on impact) with the shock throughout the whole period of higher productivity. Labor is essentially constant: there is an exact offset of wealth and substitution effects reflecting the households' preference specification.[18] The average markup and the relative-price distortions continue to be minimized and hence there are zero variations in each of these distortions (not shown). Regarding the interest rate's movements, initially there is a decrease in the real rate when consumption is high relative to the steady state, but it grows over time to its steady-state level as consumption is expected to decrease to its steady-state level. Since inflation is zero, the nominal rate and the real rate responses exactly coincide. Borrowing terms from the Linear-Quadratic (LQ) optimal policy framework (e.g. Clarida, Gali, and Gertler, 1999 and Benigno and Woodford, 2005), we say that in response to a productivity shock there is no tradeoff between inflation and output gap stabilizations.[19] Minimizing inflation variation also minimizes the variation in the output gap.

Moving on to the full distortion case ("TDP money"), the presence of monetary distortions alters the tradeoffs faced by the monetary authority. In addition to the costs of variations in the relative-price and markup distortions, the monetary authority also needs to manage the cost of variations in the nominal interest rate affecting the opportunity cost of holding money. As shown in the bottom left panel of Figure 3, complete inflation stabilization is no longer optimal, though feasible, in such an environment. The monetary authority has to compromise between inflation stabilization and the Friedman rule. It is no longer optimal to stabilize inflation since we know from the cashless model analysis, the nominal rate has to decrease by a bit more to be consistent with constant inflation. Hence, in such a scenario, the monetary authority would completely ignore the welfare cost of nominal interest rate movements. On the other hand, if the nominal rate is completely stabilized as in the Friedman rule, inflation has to vary by more to accommodate movements in the real interest rate associated with consumption movements. Under the optimal policy, the compromise means that the nominal rate responses are smoothed out relative to the cashless case. Inflation is no longer constant: there is a slight deflation on impact but inflation is higher relative to the steady state in subsequent periods. The inflation and nominal rate responses imply that consumption increases by a bit less than 1 percent before subsequently tracking productivity.

[18] Labor in this figure and subsequent ones only include the dynamics of labor used in production. The dynamics of production labor are virtually identical to the dynamics of total labor since labor used in costly credit transaction and in price adjustment activity (under SDP) are small, both in the steady state and in the near steady-state dynamics.

[19] In a linear-quadratic (LQ) framework, the loss function in this two-distortion case is a quadratic function of inflation and an output gap measure.

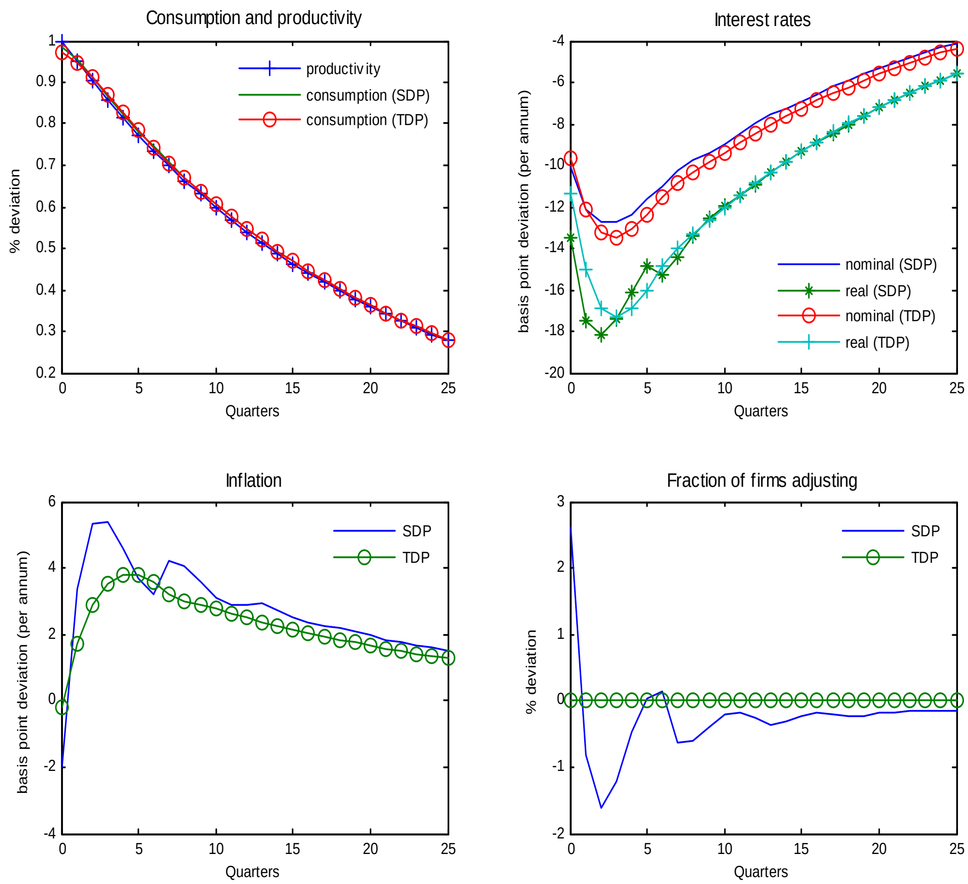
Despite the differences in the dynamics between the TDP case with monetary frictions and the cashless case, the optimal policy can still be considered as an "approximate price stability" rule in a sense the price level remains largely stabilized around its deterministic trend. For example, the largest deviation of inflation is less than 4 basis points deviation (0.04 percent) at the annual rate.

TDP and SDP with "full" distortions In Figure 4, I compare the optimal responses under SDP and TDP when all four sets of distortions are present. The optimal response under SDP is to closely replicate the dynamics under the TDP model counterpart, indicating that approximate price stability is also optimal. It is again optimal to have a slight deflation on impact in response to a productivity shock, although inflation variation is larger under SDP. The monetary authority also smooths out the nominal interest rate movements due to the presence of monetary distortions. As in the TDP case, consumption also increases by a bit less than productivity, although it is closer to the 1-percent increase in productivity. One important difference between the TDP and SDP cases is that there are now movements in the fraction of firms adjusting ($\omega_{0,t}$). Under SDP, on impact, there is an increase in the fraction of firms adjusting in response to higher deflation since firms' relative prices increase more quickly than before. There is a high correlation between the responses of inflation and the fraction of firms adjusting, reflecting the fact that firms' adjustment decision crucially depends on the expected path of the price level.

Quantitatively, it appears that the responses do not differ by much between the two pricing specifications. But it is important to recognize that these quantitatively-small differences occur because the optimal policy under the timeless perspective is close to complete price stability, wherein inflation only slightly deviates from its deterministic path. As shown in the start-up problem analysis in Section 5, however, they can be quantitatively large when inflation response is farther away from its deterministic steady-state path. In spite of these quantitatively-small differences between the TDP and the SDP responses in Figure 4, they still importantly demonstrate the nontrivial effect of the presence of endogenous timing and frequency of price adjustment under SDP for the conduct of optimal policy, as explained below.

Recall that the task of the monetary authority is to optimally balance the costs of variations in various distortions. Since the variation in the additional adjustment cost (menu cost) distortion under SDP is minimized when inflation stays at its steady-state path, one might think that the optimal response should involve a smaller inflation variation under SDP — yet, inflation is actually more variable under SDP than under TDP. This conjecture misses the changing nature of the relative costs of various distortions and the policy tradeoffs between various stabilization goals faced by the monetary authority under SDP. Just as in the steady state, the incorporation of SDP instead of TDP also dynamically alters the policy tradeoffs outside the steady state. The response can instead be understood as follows.

Figure 4: Optimal Ramsey policy responses to a productivity shock
SDP vs. TDP



Notes: (1) the figure is constructed under the assumption that all distortions are present in both TDP and SDP models; (2) the TDP model is constructed under the assumptions that the probabilities of price adjustments ($\{\alpha_j\}_{j=1}^{J-1}$) and firms' distribution ($\{\omega_j\}_{j=0}^{J-1}$) are equal to the steady-state values under TDP, with the same $J = 6$.

First, note that for a given change in inflation, the variation in the relative-price distortion is smaller under SDP than under TDP. Just as in the steady state case above, this smaller variation is due to the fact that firms are able to optimally respond to changing inflation. If inflation deviates farther away from zero or from its trend, additional firms will choose to adjust so that prices are more closely synchronized. This lower relative-price-distortion cost of inflation variation under SDP means that the monetary authority can afford to let inflation vary more and to optimally put more weight on the stabilization of other distortions or variables. This is apparent in the response of the nominal rate, which affect the variations in the two monetary distortions, in the top right panel of Figure 4. The nominal rate varies by less under SDP, indicating that the optimal policy is closer to the Friedman rule. The monetary authority can also afford to stabilize the average markup distortion by more, which is apparent by looking at the consumption response relative to productivity: compared to the TDP case, consumption tracks productivity much closer under SDP, indicating a smaller decrease in the output gap.[20] Overall, however, as mentioned above, this modification to the policy tradeoffs in response to a productivity shock is relatively small since the timeless perspective response is close to price stability. Hence, from the timeless perspective, the optimal response under TDP is a good approximation to the optimal response under SDP.

4.2.2 Government purchase shock

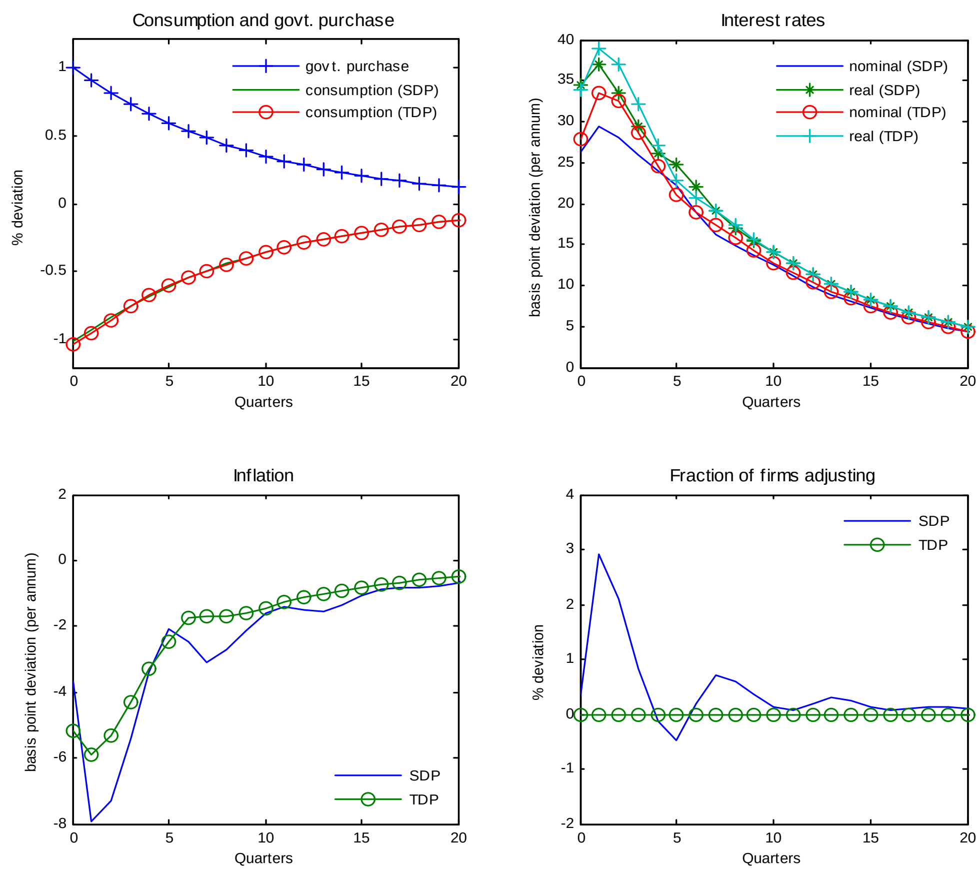
Figure 5 depicts the dynamics when the economy is hit by a 1 percent government purchase shock. A temporary, but persistent, government purchase shock can be thought as a temporary drain on economy's resources from the perspective of the representative agent. Households thus choose to reduce consumption (top left panel) and increase work effort in response to this negative wealth effect. Since the increase in government purchase, and hence, the decrease in consumption, are temporary, the real interest rate is temporarily higher. Inflation movements are generally small so that the price level is largely stabilized around its steady state. These dynamic patterns occur under both the TDP and SDP assumptions. As noted by Schmitt-Grohe and Uribe (2007), the monetary authority chooses an allocation that resembles the allocation in the flexible-price equilibrium case, but with due consideration regarding the presence of predetermined prices and monetary frictions.

The main conclusion on the optimal Ramsey dynamics in the SDP model versus the TDP model in the productivity shock case carries over to the case of a government purchase shock. That is, inflation tends to be more variable and the optimal policy is closer to the Friedman rule under SDP. The latter point is apparent by a more muted nominal interest rate response under SDP as depicted in the top right panel of Figure 6. Once again, the lower cost of inflation variation on the

[20] Government purchase is assumed to be at its zero steady-state level throughout this higher-productivity period, so that consumption is always equal to output.

relative-price distortion under SDP is responsible for this result. As in the productivity shock case, the SDP responses still closely track the responses under TDP due to small inflation deviations from its deterministic path.

FIGURE 5: OPTIMAL RAMSEY POLICY RESPONSES TO A GOVERNMENT PURCHASE SHOCK SDP vs. TDP



Notes: (1) the figure is constructed under the assumption that all distortions are present in both TDP and SDP models; (2) the TDP model is constructed under the assumptions that the probabilities of price adjustments ($\{\alpha_j\}_{j=1}^{J-1}$) and firms' distribution ($\{\omega_j\}_{j=0}^{J-1}$) are equal to the steady-state values under TDP, with the same $J = 6$.

4.2.3 Second moments and the markup shock

To further confirm the conclusion from the impulse response analysis above, I compute the second moments of select variables based on the approximate Ramsey equilibrium solution. Table 3 reports the annualized standard deviations of nominal interest rate, inflation, output gap, and the relative-price distortion measure. The output gap is defined as the log deviation of output from its potential level, defined as the level of output under flexible-price equilibrium with zero credit cost and zero price-adjustment cost.

TABLE 3: STANDARD DEVIATIONS OF SELECT VARIABLES UNDER RAMSEY POLICY (SHUTTING DOWN SELECT EXOGENOUS SHOCKS)

SDP VS. TDP

	σ_R	σ_π	$\sigma_{\tilde{y}}$	σ_Δ
Benchmark				
SDP	0.698	0.168	0.189	0.0084
TDP	0.743	0.135	0.257	0.0080
With only productivity (a_t) shock				
SDP	0.329	0.119	0.104	0.0061
TDP	0.344	0.094	0.127	0.0055
With only government purchase (g_t) shock				
SDP	0.616	0.118	0.158	0.0058
TDP	0.659	0.097	0.224	0.0059
With only markup (ε_t) shock				
SDP	0.0006	0.0015	0.0031	0.0000
TDP	0.0006	0.0005	0.0052	0.0000

Notes: (1) σ_R, σ_π, $\sigma_{\tilde{y}}$, and σ_Δ denote the annualized standard deviations of net nominal interest rate, net inflation, output gap (Y_t/Y_t^*), and relative-price distortion, respectively; (2) potential output, Y_t^*, is defined as the level of output under flexible price equilibrium, with zero credit cost and zero price-adjustment cost; (3) for each case, I set the standard deviations of the other shocks to be zero — e.g. in the case with only productivity shock, I set $\sigma_g = 0$ and $\sigma_\varepsilon = 0$; (4) the standard deviation is measure in percent per year.

Under the benchmark parametrization, the standard deviation of inflation (σ_π) under SDP is about 24 percent higher compared to that under TDP. This 24 percent higher inflation volatility under SDP is associated with only 5 percent higher standard deviation of relative price distortion (σ_Δ=0.0084, versus 0.0080), confirming that the cost of inflation variation on the relative-price distortion is lower under SDP. This in turn makes the policy authority operating in the SDP environment to be able to focus more on other stabilization goals, which is reflected by lower standard deviations of output gap ($\sigma_{\tilde{y}}$) and nominal interest rate (σ_R). Comparing the relative $\sigma_{\tilde{y}}$ and σ_R in the two pricing environments, the gain from lower cost of inflation variation under SDP is mostly absorbed through lower output gap volatility: $\sigma_{\tilde{y}}$ is smaller by 26 percent under

SDP compare to that under TDP (0.189 versus 0.257), while σ_R is only smaller by 6 percent (0.698 versus 0.743). This indicates that the variation in the markup distortion is more important for welfare than the variation in the monetary distortions.

The rest of Table 3 reports the standard deviations when only one shock is selectively assumed to buffet the economy. For each shock case, I find that the standard deviation of inflation is higher under SDP, ranging from 22 percent higher (under government purchase shock) to as much as 200 percent (under markup shock). And on the flip side, under SDP both $\sigma_{\tilde{y}}$ and σ_R are smaller, ranging from 18 percent smaller (under productivity shock) to 40 percent smaller (under markup shock) in the former and from virtually zero (under markup shock) to 7 percent smaller (under government purchase shock) in the latter. These results indicate that the modification in the policy tradeoffs under SDP does not depend on the type of shocks being assumed, though the modification seems to be more pronounced under the markup shock. Interestingly, in the government purchase shock case, the standard deviation of the relative-price distortion measure is slightly smaller under SDP (0.0058 versus 0.0059) despite of higher volatility of inflation. This again clearly highlights the lower cost of inflation variation on the relative-price distortion under SDP.

4.3 Robustness

I next investigate whether the results above still hold under several different parametrizations of the model. The standard deviations of select variables based on these robustness checks are reported in Table 4, along with the values under the benchmark parametrization.

The first robustness check involves a case where the steady-state degree of price stickiness is higher. Specifically, I calibrate the maximum fixed cost of price adjustment, B, so that the largest period of price fixity, J, is 9 quarters.[21] Other parameter values are set as in Table 1. The median duration of price fixity is 5 quarters, instead of 3 quarters in the benchmark case. Under this higher price rigidity environment, inflation variation is now relatively more important for welfare, prompting the policy authority to dampen the volatility of inflation. This is reflected by lower σ_π compared to the benchmark case, under both SDP (0.106 vs. 0.168) and TDP (0.082 vs. 0135). In turn, this makes both $\sigma_{\tilde{y}}$ and σ_R to be higher under $J = 9$. Comparing now the standard deviations under SDP and TDP, we still find higher inflation volatility and lower output gap and

[21] Other than increasing the average degree of price rigidity, increasing J from 6 to 9 causes the "zigs zags" pattern arising in the SDP model of Dotsey, King, and Wolman (1999) towards the end of each J-cycle to be smoothed out. This robustness check thus ensures that the entire difference between the SDP and TDP responses are not due to this feature of the model.

nominal interest rate volatilities under SDP.

TABLE 4: STANDARD DEVIATIONS OF SELECT VARIABLES UNDER RAMSEY POLICY
(ROBUSTNESS UNDER DIFFERENT PARAMETERIZATIONS)
SDP VS. TDP

	σ_R	σ_π	$\sigma_{\tilde{y}}$	σ_Δ
Benchmark				
SDP	0.698	0.168	0.189	0.0084
TDP	0.743	0.135	0.257	0.0080
Longer period of price fixity ($J = 9$)				
SDP	0.771	0.106	0.234	0.0056
TDP	0.814	0.082	0.370	0.0057
Lower demand elasticity ($\bar{\varepsilon} = 6$)				
SDP	0.690	0.300	0.354	0.0154
TDP	0.772	0.257	0.595	0.0170
Higher elasticity of substitution ($\sigma = 2$)				
SDP	0.700	0.218	0.121	0.0092
TDP	0.730	0.164	0.157	0.0084

Notes: (1) σ_R, σ_π, $\sigma_{\tilde{y}}$, and σ_Δ denote the annualized standard deviations of net nominal interest rate, net inflation, output gap (Y_t/Y_t^*), and relative-price distortion, respectively; (2) for each robustness analysis case, all other parameters are as in Table 1; (3) potential output, Y_t^*, is defined as the level of output under flexible price equilibrium, with zero credit cost and zero price-adjustment cost; (4) the standard deviation is measure in percent per year.

The same conclusion holds under lower demand elasticity ($\varepsilon = 6$) — again, σ_π is higher and both $\sigma_{\tilde{y}}$ and σ_R are lower under SDP than under TDP. In this lower ε environment, inflation is more volatile than in the benchmark case due to lower welfare cost of relative-price distortion. Finally, increasing the consumption intertemporal elasticity of substitution to 2 also produces higher σ_π and lower $\sigma_{\tilde{y}}$ and σ_R under SDP.

4.4 The role of monetary distortions

Optimal monetary policy studies in the literature most commonly assume the presence of only two market distortions: the relative-price distortion and the markup distortion. One may then wonder about the importance of the monetary distortions for the above results.

From the model-solution standpoint, the monetary distortions are needed so that the SDP model used here has a determinate steady-state solution with finite J. Note that in the absence of monetary distortions, the steady-state inflation under the optimal policy is zero. Without some additional, potentially restrictive, assumptions, all firms would then optimally choose not to adjust forever in the steady state, i.e. $J \to \infty$. This would make the model intractable and infeasible to solve quantitatively.

But more importantly, monetary distortions play an important role for the near steady-state dynamics results presented above since these distortions cause the monetary authority to face an additional policy tradeoff. The presence of this additional tradeoff in turn makes it possible for us to observe the change in the policy tradeoffs faced by the monetary authority when firms' pricing decisions are state-dependent. For example, as is familiar and shown previously, there is no tradeoff between inflation and output gap stabilizations if only the relative-price and markup distortions are present when the economy is hit by a productivity shock. The monetary authority can optimally choose the allocation in which both distortions continue to be minimized in both the SDP and the TDP environment by keeping inflation at zero. If inflation is always zero, the SDP model and the TDP model are trivially identical. This two-distortion case with productivity shock is precisely the environment analyzed in Nakov and Thomas (2014), in which they find that the incorporation of SDP instead of TDP does not change the optimal policy responses. To properly observe the change in the policy tradeoffs, however, we need the monetary authority to be faced by a policy tradeoff in the first place and for inflation to deviate from its deterministic path — these are provided by the presence of monetary distortions assumed in the model in this paper.[22]

5 The start-up problem and the optimized policy rule

In this section I analyze two additional important issues. First, I consider a departure from the timeless perspective policy and assume instead the Ramsey planner takes into account that the optimal policy is different in the starting period than that in subsequent periods. The dynamics arising from this "start-up" problem allow us to more clearly observe the change in policy tradeoffs under SDP, due to larger deviation of inflation away from its steady state path. The second issue relates to whether the optimal Ramsey allocation derived above can be supported by an implementable, simple policy rule. Within a class of simple policy rules, it is of interest to investigate how different are the optimal response coefficients under SDP compared to those under TDP.

[22] When the economy is hit by a government purchase or a markup shock, on the other hand, the monetary authority does face a tradeoff between inflation and output gap stabilizations even when monetary distortions are absent. We should still then observe some differences between the impulse responses under SDP and those under TDP, provided we make additional assumptions so that the steady state under the optimal policy is determinate.

5.1 The start-up problem

One should note that the optimal policy from the timeless perspective is not the true Ramsey solution. Under the true solution, the Ramsey planner takes into account that there is no past commitment she must abide by in the starting period during which the optimal state-contingent plan is set and announced. The timeless perspective optimal policy is thus suboptimal, in a sense that it is welfare-dominated by the true Ramsey solution, although it welfare-dominates other time-consistent rules.[23] This so-called "start-up problem" means that the policy rule in the starting period should be different than the timeless-perspectively policy in subsequent periods. In the context of monetary models with monopolistic competition like the model in this paper, this start-up problem takes the form of a monetary authority that generates surprise inflation in the starting period of policy implementation. It is optimal to do so since the economy operates inefficiently due to firms' monopoly power. Generating temporary higher inflation would erode the markup of firms when prices are sticky, and hence, would temporarily bring the economy closer to the first-best allocation.

A question of interest is whether the start-up inflation would be more muted under the SDP assumption. An implication of SDP is that inflation is more variable since firms can change their prices more freely due to endogenous timing of adjustment. This property of SDP means that the monetary authority has less leverage over real activity—or from a standpoint of the New Keynesian Phillips curve, the slope of the Phillips curve is steeper under SDP than under TDP. Hence, the conjecture that the start-up inflation would be lower under SDP is a reasonable one to make. As we see below, this conjecture is incorrect for the very reason that the policy tradeoffs are altered under SDP.

The dynamics of the start-up problem can be analyzed in the context of the present model by setting the lagged policy multipliers to zero since these multipliers essentially represent the monetary authority's past commitments.[24] I assume that the economy was at the steady state prior to $t = 0$ and there is zero stochastic shock. After the starting period, the monetary authority is assumed to follow the timeless-perspective policy rule. Furthermore, to avoid from having to fluctuate from negative to positive inflation and back, the optimal policy dynamics presented below,

[23] One can view the optimal policy from the timeless perspective as time consistent since if the policy itself is assumed to have been implemented since time immemorial, there is no reason for the monetary authority to deviate from this behavior now or in the future.

[24] Specifically, I assume that the optimal policy's starting period is at $t = 0$ and set the vector of state variables $s_0^{(2)}$ to zero, which reflect the fact that there is no past commitment in this starting period.

both under TDP and SDP, are approximated around a steady-state consistent with a small positive inflation (0.25% per quarter).[25] We can then look at the response of the economy based on these assumptions.

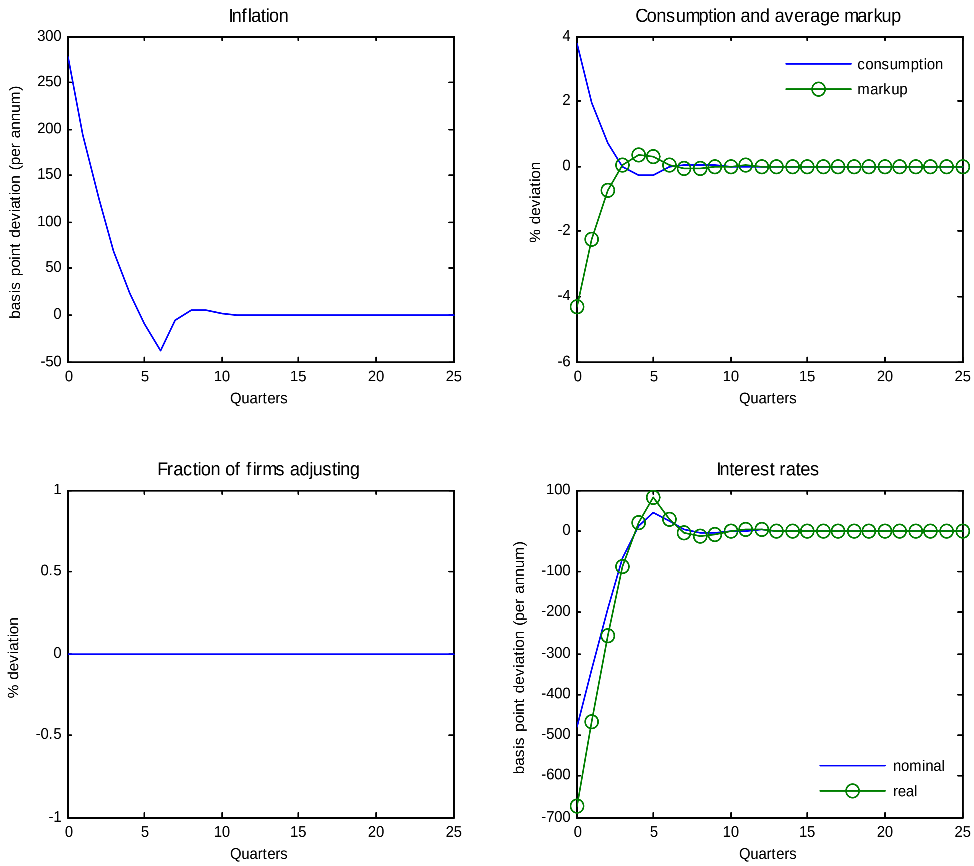
The start-up problem under TDP Consider first the start-up problem dynamics under TDP, depicted in Figure 6. As discussed above, the monetary authority optimally chooses to generate higher inflation in the starting period of the precommitment policy. Inflation goes up initially by more than 250 basis points, and this temporarily higher inflation lasts for about five to six quarters. The transition speed of inflation back to its steady-state level mainly depends on the probabilities of price adjustment (α_j), which are assumed to be constant under TDP. The top right panel of Figure 6 depicts the motivation behind this surprise inflation: The temporary increase in inflation temporarily erodes firms' average markup, stimulating consumption and output. Cumulatively over the entire stimulation periods, consumption is higher relative to its steady state by about 6 percent. Both the real and nominal interest rates decrease during these early periods, but the nominal rate decreases less on average due to expected inflation.

Note that the size of the start-up inflation depends on several factors. First, it depends on the size of firms' monopoly power and hence, the size of the average markup distortion, in the economy. The higher is the markup distortion, the higher the start-up inflation is.[26] Second, it depends on the short-run inflation-output tradeoff, facilitated by the presence of predetermined prices. Other things equal, the start-up inflation is lower if prices are more flexible as the ability of the monetary authority to stimulate real activity in the short run is more limited. Third, it depends on the cost of inflation variation since the decision of the optimizing monetary authority to generate this surprise inflation and to erode the average markup is *not* a free lunch. In this case, higher inflation increases the size of the relative-price distortion, which affects welfare. In determining the size of the start-up inflation, the monetary authority has to take into account these various factors. The last two factors are especially important when we consider the start-up problem under the corresponding SDP specification below.

[25] One can assume that the economy starts at the steady-state associated with the optimal policy. However, since the optimal steady-state inflation is negative and since the start-up problem involves an increase in (net) inflation up to the positive region (so that the markup distortion is temporarily eroded), the start-up problem dynamics are not so clear cut to analyze. Approximating the dynamics around a (small) positive steady-state inflation mitigates this complication without too much loss in generality. That is, one can view that the optimal policy dynamics are measured in terms of deviations from a point consistent with 0.25% inflation per quarter.

[26] If there were no markup distortion so that market-generated output level is efficient, there would be no need for the monetary authority to generate this surprise inflation.

Figure 6: The start-up problem under TDP

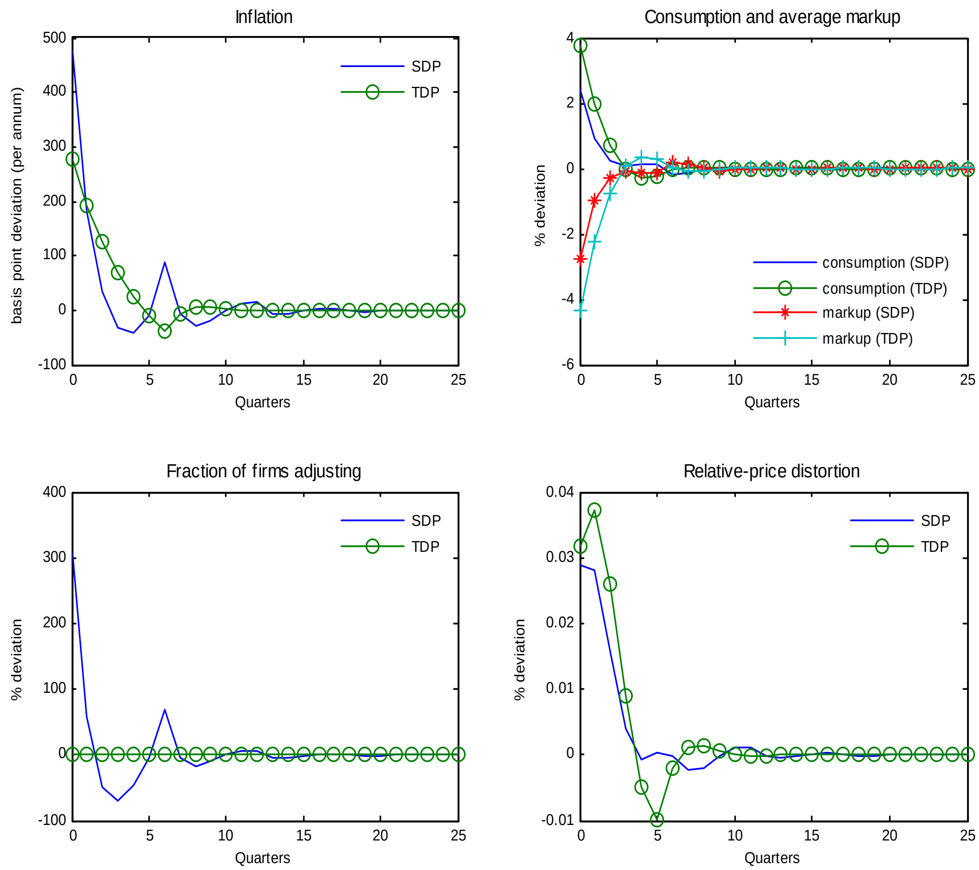


Notes: (1) the figure is constructed under the assumption of zero exogenous shock; (2) the starting point prior to the optimal policy implementation is the deterministic steady state consistent with 0.25% inflation per quarter; (3) all market distortions are assumed to be present.

The start-up problem under SDP Figure 7 compares the start-up problem dynamics in the SDP and TDP models. We can see that it is still optimal for the monetary authority to generate surprise inflation in the first period under the SDP assumption. Yet, there are two important differences regarding the dynamics. The first difference is in terms of the transition of inflation back to its steady-state level: under SDP, inflation is *less* persistent than under TDP. This is intuitive since more firms choose to adjust their prices in response to higher inflation in the SDP case, as can be seen in the bottom left panel of Figure 7.

The second, more important and interesting, difference involves the size of the start-up inflation at $t = 0$. Under SDP, the surprise inflation is much higher: inflation jumps by about 500 basis

Figure 7: The start-up problem under SDP vs. TDP



Notes: (1) the figure is constructed under the assumption of zero exogenous shock; (2) the starting point prior to the optimal policy implementation is the deterministic steady state consistent with 0.25% inflation per quarter; (3) all market distortions are assumed to be present.

points relative to the steady state. This result can be understood as follows. The economy under SDP can be thought as one with relatively more flexible prices due to firms' ability to change their timing of price adjustment. An implication of this lower degree of nominal rigidity is that the monetary authority has less leverage over real activities. This is apparent if we look at the relative consumption and the average markup dynamics in the top right panel of Figure 7. Under SDP, the average markup is eroded by *less* and consumption increases by *less* than in the TDP case — these facts occur in spite of the higher increase in inflation under SDP. Yet, despite this lower leverage over real activities, it is optimal to generate a higher start-up inflation under SDP precisely because the cost of inflation variation on the relative-price distortion is lower under SDP. This feature is apparent from the bottom right panel of Figure 7: the relative price distortion under SDP increases by less on average even though the surprise inflation is higher. That is, even though the decrease in the monetary authority's leverage over real activities means that there is less incentive to generate surprise inflation, the changing nature of the policy tradeoffs involving the relative-price distortion makes it optimal to increase inflation by more.

5.2 The optimized policy rule

I now ask whether the Ramsey equilibrium allocation can be supported and implemented by way of a simple policy rule. Specifically, I consider the following two, Taylor-rule-like, interest rate feedback rules,

$$\ln(\mathbb{R}_t/\bar{\mathbb{R}}) = \alpha_R \ln(\mathbb{R}_{t-1}/\bar{\mathbb{R}}) + \alpha_\pi \ln(\Pi_t/\bar{\Pi}) + \alpha_y \ln\left(Y_t/\bar{Y}\right), \text{ and} \tag{25}$$

$$\ln(\mathbb{R}_t/\bar{\mathbb{R}}) = \alpha_R \ln(\mathbb{R}_{t-1}/\bar{\mathbb{R}}) + \alpha_\pi \ln(\Pi_t/\bar{\Pi}) + \alpha_y \ln\left(Y_t/Y_t^*\right). \tag{26}$$

where $\bar{x}$ is the steady state level of variable x_t. $\mathbb{R}_t$ is the gross nominal interest rate, Π_t is the gross inflation rate, and Y_t^* is the potential output level. The sole difference between the two rules is in terms of what measure of output deviation the policy authority is responding to. In (25), the authority sets the interest rate in response to the log deviation of output from its constant steady state level, while in the second rule in (26) it is in response to the output gap, defined as the log deviation of output from its potential level.[27] I define the optimized policy rule, based on either (25) or (26), as the rule in which the feedback coefficients, $\{\alpha_R,\, \alpha_\pi,\, \alpha_y\}$, are set to maximize the unconditional expectation of lifetime utility,

$$V_t = E_t \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)\ .$$

As with the equilibrium solution, I search for the set $\{\alpha_R,\, \alpha_\pi,\, \alpha_y\}$ that maximizes the second-order approximation to the welfare function above. All three feedback coefficients are restricted to be nonnegative. And as in Schmitt-Grohe and Uribe (2006), I restrict α_π to have a maximum value of 3. Each of the resulting optimized rules delivers a unique equilibrium solution.

Table 5 reports the resulting optimized policy rule under SDP, along with its diagnostics. Under the first rule (Rule 1), the optimized rule features a highly responsive feedback on inflation deviation (α_π is at its maximum restricted value of 3) and a muted response to output deviation, which are consistent with the finding in Schmitt-Grohe and Uribe (2006, 2007) under a Calvo, time-dependent, pricing environment. The interest rate smoothing coefficient, α_R, however, is above unity, implying

[27] The first policy rule, (25), is also considered in Schmitt-Grohe and Uribe (2006).

a superinertial rule.[28] That is, the monetary policy authority is forward looking in responding to deviations of inflation and output from their target levels. If the second rule (Rule 2) is imposed instead, the feedback response to output deviation is somewhat larger ($\alpha_y = 0.632$, versus 0.004 in the first rule), but still not as large as the feedback coefficient on inflation deviation. The larger value of α_y under Rule 2 is perhaps an indication that it is the fluctuations of output around its time-varying flexible-price equilibrium level, rather than around its constant steady state level, that are important for welfare. As in Rule 1, the second rule also features a superinertial rule, with a slightly larger α_R. The much larger α_π coefficient, compared to α_y and α_R, in both rules is consistent with the finding in Section 4 that the optimal policy closely resembles a price-stability or an inflation-targeting rule.

Table 5: Diagnostics of optimal monetary policy under SDP
(Optimal Policy Rule, Welfare Costs, and Second Moments)

Policy Rule 1: $\ln(\mathbb{R}_t/\bar{\mathbb{R}}) = \alpha_R \ln(\mathbb{R}_{t-1}/\bar{\mathbb{R}}) + \alpha_\pi \ln(\Pi_t/\bar{\Pi}) + \alpha_y \ln\left(Y_t/\bar{Y}\right)$
Policy Rule 2: $\ln(\mathbb{R}_t/\bar{\mathbb{R}}) = \alpha_R \ln(\mathbb{R}_{t-1}/\bar{\mathbb{R}}) + \alpha_\pi \ln(\Pi_t/\bar{\Pi}) + \alpha_y \ln\left(Y_t/Y_t^*\right)$

	α_π	α_y	α_R	Conditional Welfare Cost ($\lambda^c \times 100$)	Unconditional Welfare Cost ($\lambda^u \times 100$)	σ_R	σ_π	$\sigma_{\tilde{y}}$
Ramsey Policy	—	—	—	—	—	0.698	0.168	0.189
Optimized Rule								
Rule 1	3.00	0.004	1.600	0.0001	0.0001	0.653	0.173	0.192
Rule 2	3.00	0.632	1.883	0.0001	0.0001	0.649	0.175	0.186
Taylor Coefficients								
Rule 1	1.50	0.50	—	3.7535	3.8809	20.34	23.30	3.77
Rule 2	1.50	0.50	—	0.0211	0.0220	2.30	1.59	0.30

Notes: (1) $\mathbb{R}_t$, Π_t, Y_t, and Y_t^* denote the gross nominal interest rate, gross inflation, aggregate output, and potential output, respectively; (2) σ_R, σ_π, and $\sigma_{\tilde{y}}$ are the annualized standard deviations of net nominal rate, net inflation, and output gap (Y_t/Y_t^*), respectively; (3) for all the optimized rules I set $\max(\alpha_\pi) = 3$; (4) potential output, Y_t^*, is defined as the level of output under flexible price equilibrium, with zero credit cost and zero price-adjustment cost; (5) the standard deviation is measure in percent per year.

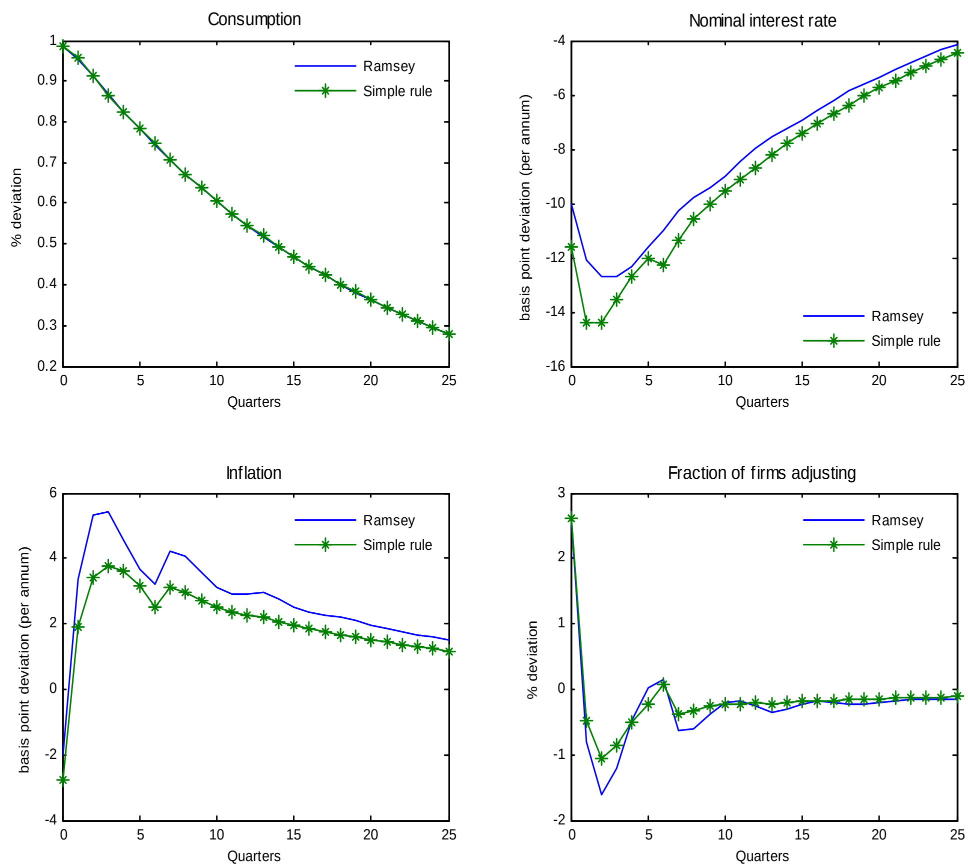
The last three columns of Table 5 report the standard deviations of nominal interest rate, inflation, and output gap, based on the optimized rule. The standard deviations of the same three variables under the Ramsey policy are also reported as the benchmark values. It is apparent that under both rule types, the optimized policy rules can deliver nearly identical fluctuations of these

[28]Schmitt-Grohe and Uribe (2006) instead find $\alpha_R < 1$.

three important variables as those under the Ramsey policy. The conditional welfare cost, λ_c, of adopting either of these rules instead of the Ramsey policy is a negligible 0.0001 percent of consumption per year.[29] A similar result is attained when the unconditional welfare cost measure, λ_u, is used instead. Figure 8 compares the impulse responses to a productivity shock based on the second optimized rule ((26)) to those under the Ramsey policy. The similarity of the dynamics arising from the Ramsey policy and the optimized rule is striking.

Figure 8: Responses to a productivity shock

Ramsey policy vs. Optimized simple rule



Note: (1) the optimized simple rule is $\ln(\mathbb{R}_t/\bar{\mathbb{R}}) = \alpha_R \ln(\mathbb{R}_{t-1}/\bar{\mathbb{R}}) + \alpha_\pi \ln(\Pi_t/\bar{\Pi})$ $+\alpha_y \ln(Y_t/Y_t^*)$, with coefficients $\alpha_R = 1.883$, $\alpha_\pi = 3.00$, and $\alpha_y = 0.632$; (2) $\mathbb{R}_t$, Π_t, Y_t, and Y_t^* denote the gross nominal interest rate, gross inflation, aggregate output, and potential output, respectively.

[29] The fact that both rules deliver the same volatility and welfare cost implications despite of differing output deviation measures and feedback coefficients is perhaps also an indication that it is mostly price stability that matters for welfare. As long as α_π is large enough, the welfare cost is minimal.

To appreciate how small the welfare cost of adopting the optimized rule is, the last two rows of Table 5 considers both types of rules with the conventional Taylor coefficients, $\alpha_\pi = 1.5$, $\alpha_y = 0.5$, and $\alpha_R = 0$. The application of these conventional feedback coefficients delivers a much larger welfare cost compared to when the optimized coefficients are used. This is especially true when the first rule is employed: the unconditional welfare cost is now 3.88 percent of consumption per year. Using the 2012 nominal U.S. per capita personal consumption expenditure figure of $35,498, this amounts to a welfare cost of $1,378, compared to merely 3.5 cents per capita per year under the optimized rule. This high welfare cost is also reflected by much higher inflation, output gap, and nominal interest rate volatilities.

How different are the optimal feedback coefficients under SDP compared to those under TDP? Table 6 reports the optimized policy rule under TDP, based on the same two simple rules in (25) and (26). As in the SDP model, the optimized policy rule under either Rule 1 or Rule 2 delivers virtually the same volatilities of inflation, output gap, and nominal interest rate as those arising from the Ramsey policy. The welfare cost, both the conditional and the unconditional measure, is also essentially zero in either rule. As in SDP, the optimum α_π is reached at its allowable maximum value of 3. However, both the α_y and α_R coefficients are now somewhat different. Specifically, under Rule 2 where the interest rate responds to the output gap measure, α_y and α_R are both larger under SDP (Table 5) than those under TDP (Table 6). This is a clear representation of the change in the policy tradeoffs under SDP highlighted in Section 4. Here, under SDP, since the welfare cost of inflation variation is smaller, it is now optimal to respond more to output gap and nominal interest rate fluctuations, compared to when firms' pricing decision is time dependent.

Comparing the standard deviation numbers in Table 5 and Table 6 under Rule 2, it is apparent that the optimized policy rule under SDP deliver higher inflation volatility and lower output gap and nominal interest rate volatilities. The standard deviation of inflation is higher by 47 percent (0.175 versus 0.119 under TDP) and the standard deviations of the output gap and the nominal rate are lower by 48 percent (0.186 versus 0.359) and 1 percent (0.649 versus 0.657), respectively. Hence, as in Table 3 under the Ramsey policy, the gain from lower cost of inflation variation under SDP is mostly absorbed through lower output gap volatility. Interestingly, under Rule 1, α_y is actually slightly higher under TDP, though α_R is still lower.[30] However, the volatility implications between SDP and TDP are similar to those under Rule 2.

[30] This perhaps indicates that to gauge the change in the policy tradeoffs through changes in the optimized feedback coefficients of a simple policy rule, it is more appropriate to use a rule that responds to output deviation from its potential level, instead of deviation from its constant steady-state level.

Table 6: Diagnostics of optimal monetary policy under TDP
(Optimal Policy Rule, Welfare Costs, and Second Moments)

Policy Rule 1: $\ln(\mathbb{R}_t/\bar{\mathbb{R}}) = \alpha_R \ln(\mathbb{R}_{t-1}/\bar{\mathbb{R}}) + \alpha_\pi \ln(\Pi_t/\bar{\Pi}) + \alpha_y \ln\left(Y_t/\bar{Y}\right)$
Policy Rule 2: $\ln(\mathbb{R}_t/\bar{\mathbb{R}}) = \alpha_R \ln(\mathbb{R}_{t-1}/\bar{\mathbb{R}}) + \alpha_\pi \ln(\Pi_t/\bar{\Pi}) + \alpha_y \ln\left(Y_t/Y_t^*\right)$

	α_π	α_y	α_R	Conditional Welfare Cost ($\lambda^c \times 100$)	Unconditional Welfare Cost ($\lambda^u \times 100$)	σ_R	σ_π	$\sigma_{\tilde{y}}$
Ramsey Policy	—	—	—	—	—	0.743	0.135	0.257
Optimized Rule								
Rule 1	3.00	0.010	1.233	0.0002	0.0002	0.677	0.137	0.299
Rule 2	3.00	0.079	1.498	0.0003	0.0003	0.657	0.119	0.359

Notes: (1) $\mathbb{R}_t$, Π_t, Y_t, and Y_t^* denote the gross nominal interest rate, gross inflation, aggregate output, and potential output, respectively; (2) σ_R, σ_π, and $\sigma_{\tilde{y}}$ are the annualized standard deviations of net nominal rate, net inflation, and output gap (Y_t/Y_t^*), respectively; (3) for all the optimized rules I set $\max(\alpha_\pi) = 3$; (4) potential output, Y_t^*, is defined as the level of output under flexible price equilibrium, with zero credit cost and zero price-adjustment cost; (5) the standard deviation is measure in percent per year.

6 Conclusion

In this paper I study optimal monetary policy under precommitment with the assumption that firms' pricing decisions are state dependent. The main finding from the analysis is that the presence of endogenous timing of price adjustment alters the policy tradeoffs faced by the monetary authority, due to lower cost of inflation variation on the relative-price distortion under state-dependent pricing relative to that under time-dependent pricing. It is thus desirable for the monetary authority to put less weight on inflation stabilization, relative to other stabilization goals.

Compared to under the standard time-dependent pricing assumption, the optimal Ramsey policy under state-dependent pricing involves greater inflation volatility, but with smaller output gap and nominal interest rate volatilities. These facts are apparent from both the dynamics in response to exogenous shocks and from the calculated second moments. The optimal response dynamics, however, can still be characterized as an approximate price stability rule, in a sense that the price level is still largely stabilized around its deterministic path.

The analysis under the true Ramsey policy—the so-called start up problem—further demonstrates that the implication of the change in the policy tradeoffs can be nontrivial when inflation deviates farther away from its deterministic path. There, I show that even though the monetary authority has less leverage over real activities under state-dependent pricing, the optimal start-up problem policy involves a much higher surprise inflation. Finally, I show that the optimal policy can be implemented by way of a simple policy rule. In the optimized rule, the change in the policy tradeoffs is manifested in higher feedback response coefficients on the output gap and the lagged nominal interest rate deviation under the state-dependent pricing environment.

The finding in this paper has important implications on the literature on optimal monetary policy design based on medium-scaled, highly-realistic estimated DSGE models. Virtually all studies in this area assume some kind of time-dependent pricing mechanisms. Future research in this area should be careful in deciding whether it is more appropriate to use a state-dependent pricing assumption instead. This issue is especially relevant for studies on countries in which the inflation rates are historically high and very volatile. Another promising future research avenue involves the use a higher-order approximation method in characterizing the dynamics of the optimal policy. Although the second-order approximation method used in this paper allows for a second-order accurate computation of welfare, the impulse response dynamics are still virtually identical to those arising from the standard first-order approximate solution, even under the state-dependent pricing assumption. This method thus appears to be inadequate in capturing the state-dependence nature of this pricing mechanism. A third-order approximation to the equilibrium solution may be warranted.

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Appendix

Appendix A: Details of households' intertemporal optimization problem

Households choose the amount of final goods consumption (c_t) and leisure (l_t) to maximize the lifetime utility function,

$$U_t = \sum_{h=0}^{\infty} \beta^{t+h} \left[\frac{1}{1-\sigma} c_{t+h}^{1-\sigma} + \chi \frac{1}{1-\phi} l_{t+h}^{1-\phi} \right] ,$$

subject a budget constraint which I describe next. In each period households receive income from their total labor labor effort (n_t) with nominal wage W_t, nominal one-period bond (B_t) from the last period, and dividends from their ownership of the intermediate-goods firms (Z_t). Households also inherit their previous period's portfolio of intermediate-goods firms (x_t). The (pre-dividend) value of the portfolio of firms is V_t. They then pick the amount of consumption for that period, buy current period bonds (B_{t+1}), and may buy more claims on the ownership of the intermediate goods firms. As mentioned in the main text, households optimally choose to finance some portions of consumption goods using money and the other portions using credit. Money is used since credit is costly, as in the standard transaction cost model. Some goods are bought using credit because there is an opportunity cost of holding money (the nominal interest rate). Households thus accumulate debt from the credit use. This debt is assumed to be paid in the next period without any interest.

Let ξ_t be the proportion of credit goods at time t. If we let $\bar{P}_t$ as the nominal price of final consumption goods at time t, then the amount of nominal money holding is $M_t = (1 - \xi_t)\bar{P}_t c_t$. The amount of nominal debt that must be paid in the next period is then $D_{t+1} = \xi_t \bar{P}_t c_t$. There is also a lump-sum tax T_t that must be paid to the government. Hence, the budget constraint at time t is given by

$$M_t + \frac{1}{1+R_t} B_{t+1} + x_{t+1}(V_t - Z_t) \leq x_t V_t + B_t + W_t n_t - D_t + T_t .$$

Note that one can make the nominal budget constraint into a real one by dividing it by the aggregate price level P_t — in the case of the CES aggregator,

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} ,$$

where $P_t(i)$ is the nominal price of intermediate good i. On the price of final consumption goods, $\bar{P}_t$, the assumption is that the final-goods producers (retailers or households) must borrow to finance the production with interest rate R_t. Since the final-goods producers are perfectly competitive and receive zero profits, it follows that $\bar{P}_t = (1 + R_t)P_t$.

As in Khan, King, and Wolman (2003), I assume that the each of the (mass) of final consumption goods has a random fixed time cost in terms of labor unit. (Although it is common to assume the representative household is assumed to purchase a single final consumption good, one can think of the final good as a continuum of final goods in a unit interval). Let $F(.)$ be the cumulative distribution function of the time cost. The largest time cost that households are willing to pay is then $F^{-1}(\xi_t)$. A good would be purchased with credit if its time cost is below this largest time cost, while money would be used to purchase a good if its time cost is above this value. The total transaction time at time t in labor unit is then given by

$$h_t = \int_0^{F^{-1}(\xi_t)} v dF(v) .$$

Since labor must also be used in production (n_t^y) and in price adjustment activity (n_t^p), the total labor time is then

$$n_t = n_t^y + n_t^p + h_t = 1 - l_t.$$

We can then form a Lagrangian to solve for the optimization problem described above. Let λ_t be the multiplier attached to the budget constraint. Solving for the Lagrangian and rearranging the first order necessary conditions lead to equations (8), (9), (10), (11), and (12) in the main text.

Appendix B: The Lagrangian of the Ramsey problem

In the Lagrangian below, several of the private sector efficiency conditions described in the main text have been modified in order to simplify the first order conditions. Firstly, I define the difference in firms' values between $j = 0$ (price-adjusting firms) and $j \geq 1$ as

$$A_{j,t} \equiv \lambda_t \left(v_{0,t} - v_{j,t}\right) \ ,$$

for $j = 1, ..., J-1$. The value differences above are in terms of marginal utility unit (hence, multiplied by λ_t). The value difference, rather then the level of values for each j ($v_{j,t}$), is what matters to firms for deciding whether to adjust price.

Next, the optimal price in (5) needs to be recursively written. This is facilitated by representing it in terms of marginal value recursion,

$$\begin{aligned}
0 &= -\lambda\frac{\partial z_{0,t}}{\partial p_{0,t}} - \beta E_t(1-\alpha_{1,t+1})\frac{1}{1+\pi_{t+1}}m_{1,t+1}) \ , \\
m_{j,t} &= \lambda_t\frac{\partial z_{j,t}}{\partial p_{j,t}} + \beta E_t(1-\alpha_{j+1,t+1})\frac{1}{1+\pi_{t+1}}m_{j+1,t+1}, \text{ for } j = 1, ..., J-2, \\
m_{J-1,t} &= \lambda_t\frac{\partial z_{J-1,t}}{\partial p_{J-1,t}} \ ,
\end{aligned}$$

where I define

$$m_{j,t} \equiv \frac{\partial(\lambda_t v_{j,t})}{\partial p_{j,t}},$$

and where profits are given by

$$z_{j,t} = [p_{j,t} - \frac{w_t}{a_t}] \cdot p_{j,t}^{-\varepsilon_t}(c_t + g_t) \ .$$

Lastly, I rewrite the aggregate pricing labor expression in (14) into

$$n_t^p = \sum\nolimits_{j=0}^{J-2} \omega_{j,t-1}\boldsymbol{\Xi}\left(\frac{A_{j+1,t}}{w_t\lambda_t}\right) + (1 - \sum\nolimits_{j=0}^{J-2}\omega_{j,t-1})\Xi(B),$$

so that I do not have to carry $\omega_{J-1,t-1}$ as part of the state variables.

Following the approach of Marcet and Marimon (2011), the recursive Lagrangian is given by:

$$
\begin{aligned}
L(s_t) \;=\; & \min_{\{\mathbf{\Lambda}_t\}}\max_{\{\mathbf{d}_t\}}\{\; u(c_t,l_t)+\beta E_t L(s_{t+1}) \\
& +\sum_{j=1}^{J-1}\phi_{jt}*\left[A_{j,t}-\lambda_t\left(z_{0,t}-z_{j,t}\right)\right] \\
& +\sum_{j=1}^{J-1}\phi_{j,t-1}*\left[(1-\alpha_{1,t})A_{1,t}-(1-\alpha_{j+1,t})A_{j+1,t}+\lambda_t w_t\left(\Xi\left(\frac{A_{1,t}}{w_t\lambda_t}\right)-\mathbf{\Xi}\left(\frac{A_{j+1,t}}{w_t\lambda_t}\right)\right)\right] \\
& +\kappa_{0,t}*\left[-\lambda_t\frac{\partial z_{0,t}}{\partial p_{0,t}}\right]+\sum_{j=1}^{J-2}\kappa_{j,t}*\left[m_{j,t}-\lambda_t\frac{\partial z_{j,t}}{\partial p_{j,t}}\right]+\sum_{j=0}^{J-2}\kappa_{j,t-1}*\left[-(1-\alpha_{j+1,t})\frac{1}{1+\pi_t}m_{j+1,t})\right] \\
& +\kappa_{J-1,t}*\left[m_{J-1,t}-\lambda_t\frac{\partial z_{J-1,t}}{\partial p_{J-1,t}}\right] \\
& +\varphi_t*\left[\frac{\lambda_t}{1+R_t}\right]-\varphi_{t-1}*\left[\frac{\lambda_t}{1+\pi_t}\right] \\
& +\eta_{1t}*\left[a_t n_t^y-\sum\nolimits_{j=0}^{J-1}\omega_{j,t}p_{j,t}^{-\varepsilon}(c_t+g_t)\right] \\
& +\eta_{2t}*\left[1-n_t^y-n_t^p-l_t-\int_0^{F^{-1}(\xi_t)}vdF(v)\right] \\
& +\eta_{3t}*\left[n_t^p-(1-\sum\nolimits_{j=0}^{J-2}\omega_{j,t-1})\Xi(B)-\sum\nolimits_{j=0}^{J-2}\omega_{j,t-1}\mathbf{\Xi}\left(\frac{A_{j+1,t}}{w_t\lambda_t}\right)\right] \\
& +\gamma_t*\left[1-(\sum\nolimits_{j=0}^{J-1}\omega_{j,t}p_{j,t}^{1-\varepsilon})\right] \\
& +\nu_t*\left[u_c(c_t,l_t)-\lambda_t(1+R_t(1-\xi_t))\right] \\
& +\tau_t*\left[u_l(c_t,l_t)-w_t\lambda_t\right] \\
& +\varrho_t*\left[\xi_t-F\left(\frac{R_t c_t}{w_t}\right)\right] \\
& +\sum_{j=1}^{J-1}\varsigma_{j,t}*\left[p_{j-1,t-1}\frac{1}{1+\pi_t}-p_{jt}\right] \\
& +\rho_{0,t}*\left[1-\sum\nolimits_{j=0}^{J-1}\omega_{j,t}\right] \\
& +\sum_{j=1}^{J-1}\rho_{j,t}*\left[(1-\alpha_{jt})\omega_{j-1,t-1}-\omega_{j,t}\right] \\
& +\sum_{j=1}^{J-1}\delta_{j,t}*\left[G\left(A_{j,t}/(w_t\lambda_t)\right)-\alpha_{j,t}\right]\;\}
\end{aligned}
$$

The elements of vector of endogenous variables, $\mathbf{d}_t$, and policy multipliers, $\mathbf{\Lambda}_t$, are

$$
\begin{aligned}
\mathbf{d}_t &= \left[c_t,\, l_t,\, \xi_t,\, \lambda_t,\, R_t,\, \frac{1}{1+\pi_t},\, w_t,\, ny_t,\, np_t,\, \{\alpha_{j,t}\}_{j=1}^{J-1},\, \{A_{j,t}\}_{j=1}^{J-1},\, \{m_{j,t}\}_{j=1}^{J-1},\, \{\omega_{j,t}\}_{j=0}^{J-1},\, \{p_{j,t}\}_{j=0}^{J-1}\right], \\
\mathbf{\Lambda}_t &= \left[\{\phi_{j,t}\}_{j=1}^{J-1},\, \{\kappa_{j,t}\}_{j=0}^{J-1},\, \varphi_t,\, \eta_{1t},\, \eta_{2t},\, \eta_{3t},\, \gamma_t,\, \nu_t,\, \tau_t,\, \varrho_t,\, \{\varsigma_{j,t}\}_{j=1}^{J-1},\, \{\rho_{j,t}\}_{j=0}^{J-1},\, \{\delta_{j,t}\}_{j=1}^{J-1}\;\right].
\end{aligned}
$$

The vector of state variables contains $s_t = (s_t^{(1)}, s_t^{(2)})$, with

$$
\begin{aligned}
s_t^{(1)} &= \left[\{p_{j,t-1}\}_{j=0}^{J-2},\, \{\omega_{j,t-1}\}_{j=0}^{J-2},\, a_t,\, g_t,\, \varepsilon_t\right] \ , \\
s_t^{(2)} &= \left[\varphi_{t-1},\, \{\phi_{j,t-1}\}_{j=1}^{J-1},\, \{\kappa_{j,t-1}\}_{j=0}^{J-2}\right] \ .
\end{aligned}
$$