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Limitations of Network Games - a brief discussion

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1 Introduction

I provide an example relating to Proposition 10 of Galeotti et al. (2010), showing how degree independence is crucial for their result [1] The example is given by the network in the diagram below and all discussion in Section 2 refers to this network. The payoff functions used in this example are those of the best-shot public goods described on page 14 of the paper in question. That is:

$$v_{k_i(g)}(x_i, x_{N_i(g)}) = max_{j \in N_i(g) \cup \{i\}}\{x_j\} - cx_i$$

where a player receives a good of value unity if he or any of his neighbours chooses to provide the good (i.e. $x_j = 1$) and incurs a cost of $c \in (0, 1)$ if he chooses to provide the good himself. Strategies $\sigma(t_i)$ give the probability that any given player of type t_i provides the good.

2 Counterexample

Drop the assumption of degree independence. Take the network in our diagram. There are 7 types of players: t_A to t_G. Assume a symmetric decreasing monotonic equilibrium exists. In equilibrium it is impossible that $\sigma(t_B) = \sigma(t_F) = \sigma(t_G) = 0$. As $\sigma(t_B) \geq \sigma(t_G)$ and $\sigma(t_F) \geq \sigma(t_G)$ it follows by monotonicity that $\sigma(t_G) > 0$. If $\sigma(t_G) = 1$ it follows from optimality that $\sigma(t_F) = 0$ and thus that $\sigma(t_B) = 1$. But $t_B \succeq t_A \succeq t_C$ so from monotonicity

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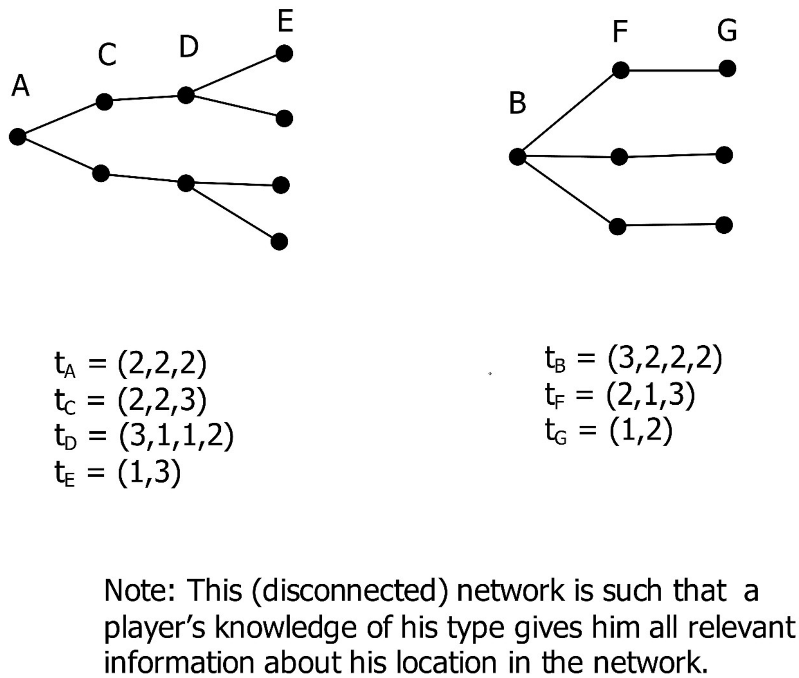
[1] This example was originally formulated as a counterexample to Proposition 9 of an earlier version of the paper, Galeotti et al. (2007).



it must be the case that $\sigma(t_A) = \sigma(t_C) = 1$, but this contradicts optimality of A and C's strategies.

If $0 < \sigma(t_G) < 1$ (i.e. G randomizes over actions) then G is indifferent between providing and not providing the good:

$$1 - c = \sigma(t_F)$$

Player F must also be randomizing or else G would not be randomizing.

$$1 - c = 1 - (1 - \sigma(t_G))(1 - \sigma(t_B)) \geq 1 - (1 - \sigma(t_G)) = \sigma(t_G)$$

The two equations above imply that:

$$\sigma(t_F) \geq \sigma(t_G)$$

but we know from monotonicity that $\sigma(t_F) \leq \sigma(t_G)$ so it must be that:

$$\sigma(t_F) = \sigma(t_G) = 1 - c$$

and

$$\sigma(t_B) = 0$$

Now $t_C \succeq t_G$ so

$$\sigma(t_C) \leq \sigma(t_G) = 1 - c$$

If $\sigma(t_C) = 0$ then by optimality $\sigma(t_A) = 1$ which contradicts monotonicty as $t_A \succeq t_C$. So C must be randomizing. Note also that by monotonicity $\sigma(t_E) \geq \sigma(t_G)$. So by optimality $\sigma(t_D) = 0$. This means that for C to randomize it must be the case that:

$$1 - c = \sigma(t_A)$$

Player A must then also be randomizing so:

$$1 - c = 1 - (1 - \sigma(t_C))^2$$

so

$$\sigma(t_C) = 1 - \sqrt{c} < 1 - c = \sigma(t_A)$$

But from monotonicity we know that $\sigma(t_C) \geq \sigma(t_A)$ so we have a contradiction. This exhausts all possibilities so for this network there does not exist a symmetric decreasing monotonic equilibrium. □

3 The proof

In part (ii) of claim 1 in the proof of proposition 10 it is assumed that monotonicity of strategies implies that the strategies of players with weakly higher degrees first order stochastically dominate the strategies of those with weakly lower degrees. This is true under degree independence but is not necessarily true otherwise. Monotonicity implies the strategies of players with weakly higher *types* first order stochastically dominate the strategies of those with weakly lower *types*. In the counterexample above, player A's neighbours have degrees as high as those of player B but player B's neighbours have higher types than those of player A. Thus, compared to the case when players only have knowledge of their own degree, knowledge of others' degrees can limit what we are able to say about the existence of certain types of equilibria without strong additional conditions such as degree independence.

References

Galeotti, A., Goyal, S., Jackson, M., Vega-Redondo, F., Yariv, L., 2007. Network games. Working Paper .

Galeotti, A., Goyal, S., Jackson, M.O., Vega-Redondo, F., Yariv, L., 2010. Network games. Review of Economic Studies 77, 218–244.