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**Ownership, access and sequential  
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# Ownership, access and sequential investment\*

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## Abstract

We extend the property-rights framework to allow for: a separation of the ownership rights of access and veto; and sequential investment. Parties investing first (ex ante) do so before contracting is possible. Parties that invest second (ex post) can contract on (at least some) of their investment costs. Along with this cost-sharing effect, the incentive to invest is affected by a strategic effect generated by sequential investment. Together these effects can overturn some of the predictions of the property-rights literature. For example, the most inclusive ownership structure might not be optimal, even if all investments are complementary.

*Key words:* property rights, access, veto, firm organization, sequential investment, holdup

*JEL classifications:* D23, L22

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# 1 Introduction

When trade requires relationship-specific investments, incomplete contracting can lead to inefficiencies (see Williamson (1975), Klein, Crawford & Alchian (1978) and Grout (1984)). By altering bargaining power, a clever reassignment of property rights can (partially) alleviate the hold-up problem by protecting investors from expropriation at renegotiation. The key finding of the property-rights approach – Grossman & Hart (1986), Hart & Moore (1990) and Hart (1995) – is that when contracts are incomplete, the *residual rights of control* that come with ownership over critical assets become important in determining the bargaining power (and claim on surplus) of the agents. Under these conditions asset ownership is designed to optimize the investment incentives of agents who make key relationship-specific investments, by protecting them from opportunism when renegotiation occurs.

Demsetz (1996) and Rajan & Zingales (1998) have criticized the model of Hart & Moore (1990) for its broad definition of *ownership*. Others, such as Bolton & Scharfstein (1998), have pointed out that the original property-rights model is at best a theory of the entrepreneurial firm as it does not fit the picture of large modern-day corporations where ownership by shareholders is often separated from the day-to-day control of managers.<sup>1</sup>

It follows that a more nuanced definition of property rights is needed to give a clearer understanding of real-world firm structures. Bel (2006) and Bel (2011) unbundle ownership into the right to *access* and use an asset and the right to *veto* access to an asset.<sup>2</sup> Using this refined notion of property rights, and allowing for structures explicitly ruled out in Hart & Moore (1990), Bel examines alternative ownership arrangements such as hybrid organizations, joint ventures and ownership by non-investors.

The possibility of unbundling access and veto rights has far reaching consequences for the optimal allocation of property rights because the requirement that rights of access and veto always be transferred together is often too restrictive to solve complex incentive problems. For example, when all assets are complements at the margin Bel (2006) found that it is optimal for all agents to have access to every asset and that no veto powers should be allocated at all. Thus, there should be a kind of communal access to resources. This ensures that the hold-up problem is minimized because nobody can threaten to withhold these assets from another party.

A second restriction of the standard property-rights model is that specific investments are made simultaneously. A related literature focuses on the timing of investments as a way to overcome holdup (see for example Neher (1999), Smirnov & Wait (2004a), and Smirnov & Wait (2004b)).<sup>3</sup> The key insight of this literature is

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<sup>1</sup>Bolton & Scharfstein (1998) also argued that the property-rights model ignores agency problems because owners are also managers of the firm.

<sup>2</sup>Bel (2006) uses the properties of ownership – access, withdrawal, management exclusion and alienation – defined in Schlager & Ostrom (1992) and groups them into access and veto.

<sup>3</sup>Contracting can be made possible when projects progress from the accumulation of physical assets or collateral (Neher (1999) or because the project itself becomes more tangible, as in Smirnov & Wait (2004a) and Smirnov & Wait (2004b). Also see De Fraja (1999), Che (2000) and Admanti

that sequential investment can alleviate holdup by allowing investment to occur when the contacting environment is more complete.

This paper combines the more refined notion of ownership as well as the possibility to make sequential investment to construct a new model of the optimal allocation of property rights.

With simultaneous investment, as in the standard model, both parties invest *ex ante*, before contracting is possible. For instance, a group of scientists and a large manufacturer of video game consoles are collaborating to develop and bring to market a new graphics processor unit that is to be included in the next generation of consoles. The two tasks (development of a new graphics processor, completed by the scientists, and the establishment of the production process, undertaken by the manufacturer) might need to be completed at the same time to ensure that the product is for the start of the new season, for example. Once both relationship-specific investments have been sunk, the project becomes tangible and the parties renegotiate, where each party's bargaining power depends on the assets they own.

Rather than investing in the two tasks at the same time, it could be the case that the scientific investment must be made first – this situation could arise when it is not possible to start establishing a manufacturing process before the exact nature of the graphics processor is known. For instance, initially the scientists invest in developing the know-how and technologies required to make the new graphics processor unit. While none of these investments could have been adequately described in a contract *ex ante*, as the research proceeds the exact nature of the processor, its specifications and its manufacturing requirements become known and verifiable. It is the scientists' research that makes this possible. At this stage, the parties are in an environment in which contracting is (at least partly) possible, so that the manufacturer can write a cost-sharing agreement prior to sinking her investment. The manufacturer's investment completes the project and both parties receive their negotiated share of the gains from trade.

We analyze the incentives to invest in these alternative timing regimes, extending Smirnov & Wait (2004a) by allowing for an arbitrary number of investors, alternative cost-sharing rules and the possibility of investments to be complementary at the margin. Given this structure, we identify three features that affect both *ex ante* and *ex post* investment. (1) *Cost-sharing*. As a contract can be written to share (at least some) of the *ex post* investor's investment costs, followers will have an enhanced incentive to invest. (2) *Strategic effect*. As the followers observe *ex ante* investments before making their own investment, there is a Stackelberg-type strategic effect that is not present with simultaneous investments; given the complementarity of investments, the strategic effect enhances the incentive to invest for the leaders. (3) *Discounting future payoffs*. If an investment is sequenced, *ex ante* investors need to wait longer to receive their payoffs, dampening their incentive to invest.<sup>4</sup>

The timing of investment also has implication for the optimal allocation of property rights. As noted, Bel (2006) suggests that with non-rival investment all parties in

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& Perry (1991).

<sup>4</sup>This effect was analyzed in Smirnov & Wait (2004a).

the grand coalition should have access to the asset, and no one should have veto rights. This is not necessarily true if investments need to be made sequentially. Given the complementarity of investments, a higher level of ex ante investment increases the ex post incentive to invest. However, it also increases the cost-sharing burden on the ex ante investors, which gives them a reduced incentive to invest. If an ex ante investor makes a relatively large contribution to total surplus and faces a strong disincentive to invest due to the cost-sharing effect, it could be necessary to dampen ex post incentives to invest. This can be achieved by reducing ex post investors property rights – for example giving ex ante investors veto rights over a set of assets. Consequently, the most inclusive ownership structure is no longer second-best optimal. This prediction seems to arise naturally out of the sequencing of investment, and suggests that ex ante agents rely more heavily on property rights to protect their investment returns from holdup than ex post agents who have the option to use alternative means of protection, in this case cost-sharing contracts. These issues are an important consideration in the ownership (and access) structures of real firms. For example, this prediction is consistent with the ownership structure chosen when Daiichi Sankyo, Japan’s third largest drug maker, bought 51% of the Indian generic drug manufacturer Ranbaxy Laboratories Limit. Notably, it was Daiichi Sanko, the party engaged in R&D and drug invention (ex ante investments), that took a controlling stake in the generic pharmaceutical manufacturer (the party making the ex post investment).

We also discuss how sequencing of investment could affect the predictions of Hart & Moore (1990). The combination of sequencing of investment and the possibility for cost sharing creates the potential for a trade off between encouraging ex post or ex ante investment. When such a tradeoff exists, it could be preferable to discourage ex post investment, particularly if ex ante investment is relatively more important in terms of the surplus it generates. As a consequence, it is no longer necessarily true that an asset idiosyncratic to an ex post investor should be held by that agent (Hart & Moore’s (1990) Proposition 5) or that just one agent have veto rights over an asset (Proposition 4). Moreover, it might be preferred that an indispensable (ex post) investor does not own the asset (Proposition 6) or indeed that complementary assets are not owned together (Proposition 8). The intuition underlying all of these results is the same. Sequencing of investment and cost-sharing can induce followers to overinvest, which in turn can reduce the leaders’ incentive to invest. In this case, it could be better to allocate property rights in such a way so as to reduce the incentives of the followers to invest.

## 2 The Model

The model has two periods, *Date 0* (ex ante) and *Date 1* (ex post). There is no discounting. The economy is populated by a finite set of  $n$  risk-neutral agents. The grand coalition is denoted by  $N$  and can be divided into two mutually exclusive but collectively exhaustive subsets  $N_{ea}$  and  $N_{ep}$  ( $N_{ea} \cap N_{ep} = \emptyset$  and  $N_{ea} \cup N_{ep} = N$ ), such that all agents who invest ex ante are members of  $N_{ea}$  and all agents who invest ex post are members of  $N_{ep}$ .  $N_{ea}$  and  $N_{ep}$  are determined exogenously. There are  $J$  ex

ante agents and  $I$  ex post agents where  $I + J = N$ . The set of productive assets  $\underline{A}$  contains a finite number of  $m$  assets.

Each agent can make a (human capital) investment  $x_i$  that costs  $C_i(x_i)$ , where  $C_i(0) = 0$ ,  $C_i(x_i)$  is twice differentiable as well as strictly increasing and convex in  $x_i$ , where  $x_i$  is a scalar lying in  $[0, \bar{x}_i]$ .<sup>5</sup> Thus, the marginal cost of investment is increasing with the level of investment, as summarized in Assumption 1.

**Assumption 1.** The cost function  $C_i(x_i)$  is non-negative, twice differentiable, strictly increasing in  $x_i$  and strictly convex; i.e.,  $C_i(x_i) \geq 0$ ,  $C_i(0) = 0$ ,  $C'_i(x_i) > 0$  and  $C''_i(x_i) > 0$  for  $x_i \in [0, \bar{x}_i)$  with  $C'_i(0) = 0$  and  $\lim_{x_i \rightarrow \bar{x}_i} C'_i(x_i) = \infty$ .

We adopt the incomplete-contracts framework of Grossman & Hart (1986): *Date 0* investment decisions cannot be specified in a contract as they are too complex or nebulous; ex ante investments are made non-cooperatively; and it is not possible to write a contract specifying the item to be traded ex post or to write a cost-sharing or profit-sharing contract at *Date 0*.

Let  $x = (x_1, \dots, x_n)$ . Consider a coalition  $S$  of agents that control a subset of assets  $A \subseteq \underline{A}$ .<sup>6</sup> This gives us the following definition for the value function  $v$ .

**Definition 1.** Let  $v(S, A | x)$  be the value function of a coalition  $S \subseteq N$  in control of the subset of assets  $A \subseteq \underline{A}$ , where  $x$  is the vector of investments by all agents.

The marginal return to investment of agent  $i$  in coalition  $S$  for a given vector of investments  $x$  is denoted by

$$v^i(S, A | x) \equiv \frac{\partial v(S, A | x)}{\partial x_i}. \quad (1)$$

The value generated by a coalition  $S$  depends on: (i) the agents in  $S$ ; (ii) the assets controlled by  $S$ ; and (iii) the human-capital investments of the agents in  $S$ . We make the following standard assumptions.

**Assumption 2.** The value function  $v(S, A | x) \geq 0$  and  $v(\emptyset, A | x) = 0$  where  $\emptyset$  is the empty set.  $v(S, A | x)$  is twice differentiable in  $x$  and increasing in  $x$ ; i.e.,  $v^i(S, A | x) \geq 0$  for  $x_i \in [0, \bar{x}_i]$ .  $v(S, A | x)$  is also concave.

**Assumption 3.**  $v^i(S, A | x) = 0$  if  $i \notin S$ .

**Assumption 4.**  $v^{ij}(S, A | x) = \frac{\partial v^i(S, A | x)}{\partial x_j} \geq 0$ ,  $\forall j \neq i \in S$ .

**Assumption 5.**  $v(S, A | x) \geq v(S', A' | x) + v(S \setminus S', A \setminus A' | x)$ ,  $\forall S' \subseteq S$  and  $A' \subseteq A$ .

**Assumption 6.**  $v^i(S, A | x) \geq v^i(S', A' | x)$ ,  $\forall S' \subseteq S$ ,  $\forall A' \subseteq A$ .

<sup>5</sup>Note that  $\bar{x}_i > 0$  and it is possible that  $\bar{x}_i = \infty$ .

<sup>6</sup>The specifics of the control structures are detailed below in Section 2.1.

Assumption 2 says that investments increase value but at a decreasing rate. Assumption 3 suggests that  $i$ 's investment only affects coalitions of which she is a member. Assumption 4 indicates that investments are complementary at the margin. The superadditivity of the value function is captured by Assumption 5; assets and agents are always (weakly) complementary, which in turn implies the grand coalition must always produce the largest surplus. Assumption 6 says agents and assets are always complementary at the margin. As in Hart & Moore (1990), Assumptions 5 and 6 together imply that the marginal and total values are positively correlated.

## 2.1 Property rights

Ownership of an asset can involve different rights. Consequently, it is sometimes possible that these different aspects of property rights can be separated and granted to different parties. To capture this our model follows Bel (2006) in assuming that asset ownership can be unbundled into the right to *access* an asset and the right to *veto* others' access to an asset. This section formalizes the definitions of deterministic *access*, *veto* and *control*.

Consider first the rights to *access* an asset. Rights of access are essentially the right to use a particular asset – that is, put it to productive use. To describe this, let  $\gamma(S)$  be the subset of assets that coalition  $S$  can access at *Date 1*. If a sub-coalition ( $S'$ ) can access an asset then the full coalition to which it belongs ( $S$ ) must also be able to access the asset. It follows that the grand coalition can access all assets ( $\gamma(N) = \underline{A}$ ). This discussion is summarized in the following definition. The access structure of the economy is defined as follows:

**Definition 2.** Let the mapping  $\gamma$  from the set of subsets of  $N$  to the set of subsets of  $\underline{A}$  be defined as the *access* structure of the economy. The mapping  $\gamma$  satisfies:

$$\gamma(S') \subseteq \gamma(S) \quad \forall S' \subseteq S \quad \text{and} \quad \gamma(N) = \underline{A}. \quad (2)$$

Next, consider veto rights. A veto, when exercised, allows a party to stop someone else from using a particular asset. Specifically, coalition  $S$  has veto rights with respect to asset  $a_k$  if it can prevent a party who is not a member of  $S$  from using it.  $\chi(S)$  is the subset of assets that the coalition  $S$  has veto rights on at *Date 1*. Following Bel (2006), we assume that if a subset  $S'$  of coalition  $S$  can veto the use of an asset, then the use of that asset can also be vetoed by the whole coalition. The structure of veto rights is defined as:

**Definition 3.** Let the mapping  $\chi$  from the set of subsets of  $N$  to the set of subsets of  $\underline{A}$  be defined as the *veto* structure of the economy. The mapping  $\chi$  satisfies:

$$\chi(S') \subseteq \chi(S) \quad \forall S' \subseteq S \quad \text{and} \quad \chi(N) = \underline{A}. \quad (3)$$

A party's outside option only includes the assets that it can access without the threat of being vetoed by someone else. This idea is captured by a coalition's *control* rights over an asset. Specifically, given the structure of access and veto rights, a

coalition of agents  $S$  is said to *control* an asset  $a$  if and only if  $S$  has access to the asset and no coalition outside of  $S$  has veto over  $a$ . The control structure of the economy is important in determining the investment incentives of the agents because a coalition can only put an asset to productive use (and derive surplus from it) if it controls the asset. Formally:

**Definition 4.** A *control* structure is a mapping  $\beta$  from the set of subsets of  $N$  to the set of subsets of  $\underline{A}$ , such that  $\beta(S) = \gamma(S) \setminus \chi(N \setminus S)$ . The control structure satisfies:

$$\beta(S') \subseteq \beta(S) \quad \forall S' \subseteq S \quad \text{and} \quad \beta(N) = \underline{A}. \quad (4)$$

As noted above, this definition of ownership allows for a greater range of ownership possibilities. For example, a coalition could have access rights to a particular asset or a set of assets, but no veto rights; this is equivalent to a renter of the asset or a tenant who only has the rights to use the asset. Alternatively, a coalition could have veto rights but no access rights to an asset, as would be the case with a landlord. The other two cases are equivalent to the control structures in Hart & Moore (1990); a coalition with both access and veto rights and a coalition with neither the rights of access or veto to a particular asset.

## 2.2 The timing of investment

There are two alternative investment timing regimes. With simultaneous investment all parties invest at Date 0 (ex ante) before contracting is possible, as in the standard property-rights model – that is,  $N_{ea} = N$  and  $N_{ep} = \emptyset$ . At the end of the ex ante period all relationship-specific investments have been made. At this stage, the parties bargain over the surplus (detailed below). The timing is summarized in Figure 1, but it is important to note that investments are not contractible ex ante and a surplus sharing rule is never feasible (that is, surplus is never verifiable).

The alternative is sequential investment. We assume that the subset of agents  $N_{ea}$  invest ex ante. Having observed these investments, the remaining agents – the ex post investors – make their investments. We allow for the possibility that some of the cost of these ex post investments can be shared with all agents through some cost-sharing arrangement.<sup>7</sup> Once all investments have been made, the parties can negotiate over the distribution of surplus. Figure 2 summarizes the timing of the sequential model.

## 2.3 Bargaining over surplus and costs

In the simultaneous investment model the agents bargain over the allocation of surplus in the ex post period. We follow convention and use the Shapley value. Letting  $B_i$

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<sup>7</sup>In some research projects, for example, after some initial investment it becomes clear what additional investments need to be made, even if the final product is still slightly nebulous and unverifiable. In this case, the cost of the new laboratory or the next phase of research could be shared amongst both ex ante and ex post investors.





Figure 1: Simultaneous investment

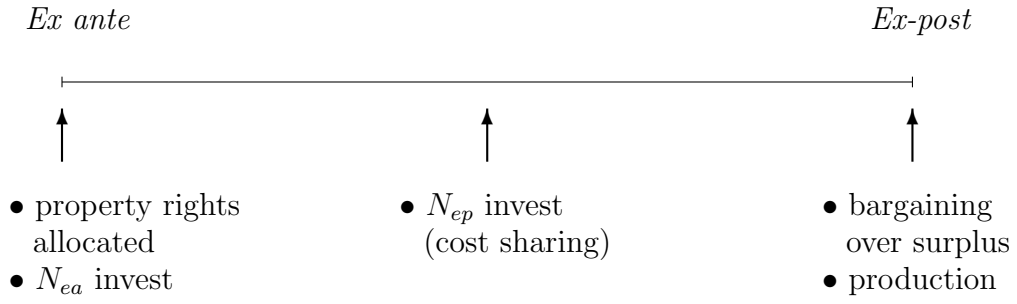


Figure 2: Sequential investment

be agent  $i$ 's share of gross surplus, the following equality must hold:

$$\sum_{i \in N} B_i(\beta | x) = v(N, \underline{A} | x). \quad (5)$$

Equation (5) says that the surplus allocated to all agents sums to the total surplus generated by the grand coalition. As in Hart & Moore (1990), the Shapley value  $B_i$  is defined as:

**Definition 5.** Agent  $i$ 's share of gross surplus  $B_i$  is given by the Shapley value.

$$B_i(\beta | x) = \sum_{S | i \in S} p(S) [v(S, \beta(S) | x) - v(S \setminus \{i\}, \beta(S \setminus \{i\}) | x)], \quad (6)$$

where  $p(S) = \frac{(|S|-1)!(|N|-|S|)!}{(|N|)!}$ .

Now consider sequential investment. Again agent  $i$ 's share of gross surplus  $B_i$  is given by the Shapley value. Sequential investment introduces the possibility of

the sharing the cost of ex post investment amongst all the agents. To capture this let  $\lambda$  be the set of exogenous sharing rules detailing how ex post investment costs will be shared among all agents, where  $\lambda = \{\lambda_{il} \mid i \in N_{ep}, l \in N\}$ . Note given that  $\lambda_{il}$  denotes the proportion of ex post agent  $i$ 's investment cost paid by agent  $l$ , we make the additional requirements that  $\lambda_{il} \in [0, 1] \forall i \in N_{ep}, \forall l \in N$  and that  $\sum_{l \in N} \lambda_{il} = 1, \forall i \in N_{ep}$ . Consequently, the following equality holds:

$$\sum_{l \in N} B_l(\beta \mid x) - \sum_{i \in N_{ep}} \sum_{l \in N} \lambda_{il} C_i(x_i) = v(N, \underline{A} \mid x) - \sum_{i \in N_{ep}} C_i(x_i). \quad (7)$$

Equation (7) gives the gross surplus, minus all ex post investment costs.

To provide some intuition, consider again the research project example,  $\lambda$  determines how much of the cost of the secondary research phase is paid for by ex ante investors, and how much of these costs are borne by the ex post investors themselves. Note also that this general set-up allows for  $\lambda_{ii} = 1 \forall i \in N_{ep}$ , such that each ex post investor pays for all of their own investment costs. This also generalizes the cost-sharing rule in Smirnov & Wait (2004a), in which  $\lambda_{21} = \lambda_{22} = 0.5$ . While we do not explicitly model the cost-sharing rules, they could be determined by various factors and can be thought of as the reduced-form solution of that process.<sup>8</sup>

### 3 Investment incentives

The grand coalition  $N$  generates the largest surplus for any given level of investment (Assumption 5). The superadditivity assumption says that there are always gains from trade and that it is never optimal to deny some productive agents the possibility to trade with each other. As in Hart & Moore (1990, 1127-8), the first-best maximization problem is:

$$\max_x W(x) = v(N, \underline{A} \mid x) - \sum_{i \in N} C_i(x_i). \quad (8)$$

From this, the welfare-maximizing investments (the vector  $x^*$ ) solve:

$$v^i(N, \underline{A} \mid x^*) = C'_i(x_i^*) \quad \forall i \in N. \quad (9)$$

The solution for  $x^*$  exists and is unique given Assumptions 1 and 2. We now turn our attention to the investment outcomes under simultaneous and sequential timing regimes, taking for the moment the allocation of property rights (the control structure  $\beta$ ) as given. First we analyze simultaneous investment, as illustrated in Figure 1. Second, we examine sequential investment, as shown in Figure 2.

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<sup>8</sup>The cost-sharing rule could arise from the relative bargaining strengths of the parties, arising in an extensive-form bargaining game such as Rubinstein (1982) or Binmore, Rubinstein & Wolinsky (1986). In principle the solution concept of this game could also be the Shapley value.

### 3.1 Simultaneous investment

With simultaneous investment all agents invest ex ante. Attempting to maximize their surplus, the first-order condition for each agent is:

$$\frac{\partial B_i(\beta | x)}{\partial x_i} = \frac{\partial C_i(x_i)}{\partial x_i} \quad \forall i \in N \quad (10)$$

where, from Definition 5 and Assumption 3, agent  $i$ 's marginal return is

$$\frac{\partial B_i(\beta | x)}{\partial x_i} = \sum_{S|i \in S} p(S) v^i(S, \beta(S) | x). \quad (11)$$

Once again, the solution for each agent's investment choice exists and is unique given Assumptions 1 and 2.

### 3.2 Sequential timing

With sequential investment we consider ex post investment and ex ante investments in turn.

#### 3.2.1 Ex post investments

Ex post investors choose their investment after having observed ex ante investments. For expositional purposes, assume for the moment that ex ante investments are exogenous. An ex post agent  $i$ 's maximization problem is:

$$\max_{x_i} B_i(\beta | x_{ea}, x_{ep}) - \sum_{k \neq i, k, i \in N_{ep}} \lambda_{ki} C_k(x_{ep,k}) - \lambda_{ii} C_i(x_{ep,i}). \quad (12)$$

The first term is agent  $i$ 's share of gross surplus, the second term is  $i$ 's share of other ex post investment costs, while the last term is  $i$ 's own investment cost. If  $\lambda_{ii} < 1$  agent  $i$  does not pay the full cost of investment. The vector  $x_{ep}$  of ex post equilibrium investments solves the first-order conditions:

$$\frac{\partial B_i(\beta | x_{ea}, x_{ep})}{\partial x_{ep,i}} = \lambda_{ii} \frac{\partial C_i(x_{ep,i})}{\partial x_{ep,i}} \quad \forall i \in N_{ep}. \quad (13)$$

From condition (13), ex post equilibrium investment is determined by the control structure of the economy  $\beta$ , the set of sharing rules  $\lambda$  and the level of ex ante investment  $x_{ea}$ . Following this, we say that the vector of equilibrium investments be governed by the implicit function  $x_{ep} = R(\beta, \lambda | x_{ea})$ , where  $x_{ep,i} = R_i(\beta, \lambda | x_{ea}) \forall i \in N_{ep}$ .

Lemma 1 describes how ex post investment responds to changes in ex ante investment.

**Lemma 1.** *An increase in any ex ante investment (weakly) increases equilibrium investment for all ex post investors – that is, for a given control structure and set of*

sharing rules, an increase in any ex ante investment, *ceteris paribus*, (weakly) increases equilibrium investment of all ex post agents, or that  $\frac{\partial R_i(\beta, \lambda | x_{ea})}{\partial x_{ea,j}} \geq 0 \forall i \in N_{ep}$  and  $\forall j \in N_{ea}$ .

Proof: See Appendix A.

Lemma 1 relies on the assumption that all investments are strictly complementary. If an ex ante agent increases her investment, the marginal productivity of all ex post agents also increase, leading to higher ex post equilibrium investment.

### 3.2.2 Ex ante investments

Consider next the maximization problem for an ex ante agent. Given the Stackelberg-timing of investment ex ante investors incorporate the (implicit) ex post reaction functions  $R = \{R_i : \forall i \in N_{ep}\}$  into their maximization problems. A representative ex ante agent  $j \in N_{ea}$  solves:

$$\max_{x_j} B_j(\beta | x_{ea}, R) - \sum_{i \in N_{ep}} \lambda_{ij} C_i(R_i) - C_j(x_{ea,j}). \quad (14)$$

The first term gives  $j$ 's share of gross surplus, while the middle term specifies how much of the ex post investment costs are paid by the ex ante agent  $j$  (a consequence of the fact that  $\lambda_{ii} < 1$  ex post agents can contract on their investments costs). Finally, the term on the right gives the cost of investing  $x_{ea,j}$ . As shown in Smirnov & Wait (2004a) if an ex ante investor anticipates a negative return they will not invest, potentially accentuating the hold-up problem. However, to focus on other issues, we consider the case when  $B_j(\beta | x_{ea}, x_{ep}) - \sum_{i \in N_{ep}} \lambda_{ij} C_i(x_{ep,i}) - C_j(x_{ea,j}) \geq 0 \forall j \in N_{ea}$ ; that is, all ex ante investors anticipate a positive return at the equilibrium investment levels.

The vector of ex ante equilibrium investments  $x_{ea}$  solves the first-order conditions:

$$\frac{\partial B_j(\beta | x_{ea}, R)}{\partial x_{ea,j}} + \sum_{i \in N_{ep}} \left[ \frac{\partial B_j(\beta | x_{ea}, R)}{\partial R_i} - \lambda_{ij} \frac{\partial C_i(R_i)}{\partial R_i} \right] \frac{\partial R_i}{\partial x_{ea,j}} = \frac{\partial C_j(x_{ea,j})}{\partial x_{ea,j}} \quad (15)$$

$\forall j \in N_{ea}$ , where  $\frac{\partial B_j(\beta | x_{ea}, R)}{\partial x_{ea,j}}$  is simply the marginal return to investment and  $\frac{\partial C_j(x_{ea,j})}{\partial x_{ea,j}}$  is the investor's marginal cost.

Again, to focus on the issues at hand, we assume that there exists a unique solution to the system of equations 13 and 15. Essentially this requires that the direct effect of party  $j$ 's investment on their own benefits and costs is larger than the indirect effect that arises through changes in others' equilibrium investment levels (in response to a change in a  $j$ 's investment). This is summarized in the following Assumption.

**Assumption 7.** The solution to the system of equations 13 and 15 exists and is unique.

Sherali (1984) investigates the conditions required for the existence of a unique solution in a Stackelberg model with multiple leaders and followers, without considering

the added complication of ownership. While these conditions are not our focus here, it is worth noting that a unique solution existed for any examples we constructed that satisfy the assumptions of the model; see the discussion in Example 1.

Comparing condition (10) to (15) reveals that ex ante investment incentives change due to the appearance of the additional term  $\sum_{i \in N_{ep}} \left[ \frac{\partial B_j(\beta|x_{ea}, R)}{\partial R_i} - \lambda_{ij} \frac{\partial C_i(R_i)}{\partial R_i} \right] \frac{\partial R_i}{\partial x_{ea, j}}$ . This new term arises under sequential investment because ex ante agents *internalize* the effect that their investment choices have on ex post investment – we refer to this as the *internalization effect*.

To analyze the *internalization effect*, note that from Lemma 1  $\frac{\partial R_i}{\partial x_{ea, j}} \geq 0 \forall i \in N_{ep}$  and  $\forall j \in N_{ea}$ . Notice that the term inside the summation bracket on the left hand side of equation 15 can be separated into two parts. First, consider  $\frac{\partial B_j(\beta|x_{ea}, R)}{\partial R_i}$ . An increase in ex post equilibrium investment has two opposing effects: (a) it makes all ex post agents more productive, increasing gross surplus; and (b) more productive ex post agents demand a greater share of gross surplus, which reduces the share of surplus for ex ante investors. However, from Assumption 6 the positive effect of (a) weakly dominates the negative incentive for ex ante investment from (b).<sup>9</sup> The second term,  $\lambda_{ij} \frac{\partial C_i(R_i)}{\partial R_i}$ , denotes the impact on ex ante marginal costs. It follows that  $\lambda_{ij} > 0$  increases the marginal cost for an ex ante investor.

To summarize, as  $\frac{\partial B_j(\beta|x_{ea}, R)}{\partial R_i} \geq 0$  and  $\lambda_{ij} \frac{\partial C_i(R_i)}{\partial R_i} > 0$ , the sign of the *internalization effect* is ambiguous. However, if  $\lambda_{ij} = 0$ , the *internalization effect* is non-negative, as summarized in Proposition 1.

**Proposition 1.** *For a given control structure all investments will (weakly) increase relative to the simultaneous timing regime provided  $\lambda_{ii} = 1 \forall i \in N_{ep}$ .*

Proof: See Appendix A.

It is important to note that as all the functions are continuous, one can slightly decrease some  $\lambda_{ii}$  and still have the result that there is an increase in all investments.

Proposition 1 outlines the conditions for which sequential investment (weakly) improves the under-investment problem due to holdup, generalizing the result of Smirnov & Wait (2004b). While in their model they include discounting (favoring simultaneous investment) and cost sharing (which favors sequential investment), they do not allow for the possibility of complementarity between ex ante and ex post investments.<sup>10</sup> In the model presented here, provided  $\lambda_{ii}$  is sufficiently high – so the imposition of follower costs on ex ante agents is relatively small – all investments weakly increase. Our assumption of the complementarity between investments drives this result. If  $\lambda_{ii}$  is small enough for some  $i$ , ex ante investors could reduce their investment relative to their choice with the simultaneous regime because of the potential increase in costs they face. The intuition of Proposition 1 is highlighted in the following Example.

<sup>9</sup>To see this, first note that from Definition 5  $\frac{\partial B_j(\beta|x_{ea}, R)}{\partial R_i} = \sum_{S|j \in S} v^i(S, \beta(S)|x_{ea}, R) - v^i(S \setminus \{j\}, \beta(S \setminus \{j\})|x_{ea}, R)$ . From Assumption 6  $v^i(S, \beta(S)|x_{ea}, R) \geq v^i(S \setminus \{j\}, \beta(S \setminus \{j\})|x_{ea}, R)$ , meaning that the overall sign is non-negative.

<sup>10</sup>Essentially, their Assumption 2 reduces the *internalization effect* to zero.

## Example 1

Assume there are only 2 agents and 2 assets, where each asset is controlled by a different agent. The first agent invests ex ante, while the second agent invests ex post. The coalition containing only the first agent generates a surplus of  $\ln(x_1)$ , while the coalition containing only the second agent generates a surplus of  $\ln(x_2)$ . The coalition containing both agents generates a surplus of  $\ln(x_1) + \ln(x_2) + 0.3 * x_1 * x_2 + 2$ . The costs are assumed to be  $C_i(x_i) = \frac{1}{2}x_i^2 \forall i$ . One can check that for values  $x_1 < 1.7$  and  $x_2 < 1.7$  Assumptions 1-6 are satisfied. For values higher than 1.7 an alternative specific functional form could apply to ensure Assumptions 1-6 are satisfied. In the first-best case, investments are  $x_1 = x_2 = \sqrt{1/0.7} \approx 1.195$ , and  $W = 1.3567$ .<sup>11</sup>

The following Shapley values for both players can be derived  $B_1 = \ln(x_1) + 0.15 * x_1 * x_2 + 1$  and  $B_2 = \ln(x_2) + 0.15 * x_1 * x_2 + 1$ . From the simultaneous case FOC (10), investments are  $x_1 = x_2 = \sqrt{2/1.7} \approx 1.0847$ , and  $W = 1.339$ .

Now let us consider two different sequential regimes. The first regime is when  $\lambda_{22} = 1$  and the second regime is when  $\lambda_{22} = \lambda_{21} = 0.5$ . The first regime results in the following system of equations:

$$\begin{cases} \frac{2}{x_1} + 0.3x_2 + (0.3x_1 - x_2) \frac{0.15x_2^2}{1+x_2^2} = 2x_1, \\ \frac{2}{x_2} + 0.3x_1 = x_2. \end{cases} \quad (16)$$

The solutions are  $x_1 \approx 1.0914$ ,  $x_2 \approx 1.0852$  and  $W = 1.3401$ . These results suggest that both investments increase after the switch from the simultaneous regime to the first sequential regime. Similarly, the second regime results in  $x_1 \approx 1.0677$ ,  $x_2 \approx 1.5834$  and  $W = 1.2087$  which suggests that only ex post investment increases after the switch from the simultaneous regime to the second sequential regime. Note that the incentives of the ex post agent has improved so much that he overinvests in comparison with the first best level of investment.

Note that Assumption 7 – that a unique solution exists to the system of equations 13 and 15 – holds in this example. This is because the direct effect of the first firm’s investment on their own benefits and costs is significantly larger than the indirect effect from a change in firm 2’s investment that comes as a result of a change in firm 1’s investment.

This example highlights the tradeoff present in our model. Assuming complementarity, sequential investment without cost sharing raises the incentive to invest for both ex post and ex ante investors. However, the possibility of cost-sharing can create a trade off. With cost sharing ex ante investors will temper their investment, understanding the flow-through effect of ex post investment and the additional costs they have to bear. Cost sharing can also create perverse incentives for ex post agents, as this example shows. Consequently depending on the sharing rules used, the total welfare can both go up or down when the regime is switched from simultaneous to

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<sup>11</sup>Given Assumptions 1 and 2, any derived solution has to be unique. If the solution involves  $x_1 < 1.7$  and  $x_2 < 1.7$  then specifying functions for other values is not critical.

sequential.<sup>12</sup>

## 4 Property rights with sequential investment

Previously, optimal ownership has been analyzed when investments are made simultaneously (see Hart & Moore (1990) and Bel (2006) amongst others). Less is known about the allocation of property rights when there is sequential investment. In order to do this we utilize the following definition.

**Definition 6.** Control structure  $\beta$  is said to be more *inclusive* than control structure  $\beta'$  ( $\beta'$  is more *exclusive* than  $\beta$ ) if and only if  $\beta'(S) \subseteq \beta(S)$ ,  $\forall S \subset N$  and  $\beta'(S) \subset \beta(S)$ , for at least one  $S \subset N$ .

Definition 6 can be interpreted as follows; if either the set of assets controlled by some coalitions increase or if the number of coalitions who control some set of assets increases – *ceteris paribus* – the control structure is said to be more inclusive. Conversely, if the set of assets controlled by some coalitions decrease or if the number of coalitions who control some set of assets decreases – *ceteris paribus* – the control structure is said to be more exclusive.

The two property rights access and veto have conflicting effects on the control structure. Allocating access rights is said to be inclusive because increasing the number of assets accessed by a coalition potentially increases the number of assets it controls without diminishing the control rights of other coalitions. The allocation of veto rights, on the other hand, is exclusive because increasing the number of assets for which a coalition has veto rights has no effect on the number of assets it controls but it potentially reduces the number of assets controlled by other coalitions.

In Bel (2006) under simultaneous investment the most inclusive control structure is optimal if all assets are complementary at the margin. Specifically, he finds that the property rights assignment requires that each agent *individually* accesses all the assets while only the agents of the grand coalition *jointly* veto all assets. Formally:  $\gamma(i) = \underline{A}$ ,  $\chi(i) = \emptyset$ ,  $\forall i \in N$  and  $\chi(N) = \underline{A}$ . Allocating access and veto rights in this way means that the control structure is the most inclusive because every agent controls every asset.

Sequential investment complicates the analysis of property rights because of the potential interplay between *ex ante* and *ex post* investments. First, we analyze Bel's (2006) result that the most inclusive ownership structure is (second-best) optimal.

### Optimality of inclusive structures

Let Assumption 4 hold with equality; so that Lemma 1 is amended to state that  $\frac{\partial R_i(\beta, \lambda | x_{ea})}{\partial x_{ea, j}} = 0$ .<sup>13</sup> Consequently, the marginal return of an agent  $j$  is independent of

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<sup>12</sup>As noted above, a third factor, not modeled here, is the discount factor which will always tend to favor simultaneous over sequential investment (See Smirnov & Wait (2004b)).

<sup>13</sup>To verify, let Assumption 4 hold with equality and substitute it into the proof of Lemma 1 to get  $\frac{\partial x_{ep, i}}{\partial x_{ea, j}} = 0$ .

the investments of all other agents. It follows that the *internalization effect* drops out of the ex ante first-order conditions. It also means that the first-order condition for each agent's investment is independent of all other investments, so that

$$\frac{\partial B_j(\beta | x_{ea,j})}{\partial x_{ea,j}} = \frac{\partial C_j(x_{ea,j})}{\partial x_{ea,j}} \quad \forall j \in N_{ea}. \quad (17)$$

The same reasoning applies to an ex post investor's maximization problem, which becomes

$$\frac{\partial B_i(\beta | x_{ep,i})}{\partial x_{ep,i}} = \lambda_{ii} \frac{\partial C_i(x_{ep,i})}{\partial x_{ep,i}} \quad \forall i \in N_{ep}. \quad (18)$$

The following proposition summarizes the impact of a more inclusive control structure on investment incentives when Assumption 4 holds with equality, all assets are complementary at the margin and  $\lambda_{ii} = 1 \forall i \in N_{ep}$ .

**Proposition 2.** *Assume that all assets are complementary, investments are neutral at the margin and  $\lambda_{ii} = 1 \forall i \in N_{ep}$ . The optimal ownership structure is the most inclusive control structure.*

Proof: See Appendix A.

Sequencing generates a potential interplay between ex ante investments and the choice of ex post investments that follow. Making the assumption of investment neutrality at the margin removes this channel. Our model effectively replicates the simultaneous investment model of Bel (2006). As a consequence, the most inclusive ownership structure is second-best optimal. Furthermore, utilizing continuity, one can determine that this result can hold provided  $\lambda_{ii}$  is sufficiently close to one.

### Non-optimality of inclusive ownership structures

When  $\lambda_{ii} < 1$  for some  $i \in N_{ep}$  at least some ex post investors do not consider their full marginal cost when investing. This creates a potential externality, and can encourage followers to make inefficiently large investments (as in Example 1). Consequently, a more inclusive property-rights regime might further accentuate this ex post over-investment problem. Rather, a more exclusive regime might act as a countervailing means to offset the cost-sharing rule. Thus, the possibility of cost sharing that arises due to sequencing of investment means that the most inclusive ownership need not be the optimal ownership structure. This point is illustrated in the following Example.

#### Example 2

Assume there are only 2 agents and 3 assets. The first agent invests ex ante and controls one asset, while the second agent invests ex post and controls either one or two assets. If the ex post agent controls only one asset, then the third asset is controlled by the coalition containing both agents. The coalition containing only the first agent



generates a surplus of  $\ln(x_1)$ , while the coalition containing only the second agent generates a surplus of  $\alpha \ln(x_2)$ , where the value of  $\alpha \leq 1$  depends on how many assets ex post agent controls. The coalition containing both agents generates a surplus of  $\ln(x_1) + \ln(x_2) + 0.3 * x_1 * x_2 + 2$ . The costs are assumed to be  $C_i(x_i) = \frac{1}{2}x_i^2 \forall i$ . One can check that for values  $x_1 < 1.7$  and  $x_2 < 1.7$  Assumptions 1-6 are satisfied. For values higher than 1.7 these functions are assumed to be continuously replaced by functions that satisfy Assumptions 1-6.<sup>14</sup> The following Shapley values for both players can be derived  $B_1 = \ln(x_1) + \frac{1-\alpha}{2} \ln(x_2) + 0.15 * x_1 * x_2 + 1$  and  $B_2 = \frac{1+\alpha}{2} \ln(x_2) + 0.15 * x_1 * x_2 + 1$ . Consider the case when there is cost sharing, specifically  $\lambda_{22} = \lambda_{21} = 0.5$ . From the sequential case FOC (13) and (15), one can derive the following system

$$\begin{cases} \frac{2}{x_1} + 0.3x_2 + \left(0.3x_1 + \frac{1-\alpha}{x_2} - x_2\right) \frac{0.3x_2^2}{1+\alpha+2x_2^2} = 2x_1, \\ \frac{1+\alpha}{x_2} + 0.3x_1 = x_2. \end{cases} \quad (19)$$

When  $\alpha = 1$  the equilibrium investments are  $x_1 \approx 1.0677$  and  $x_2 \approx 1.5834$  with  $W = 1.2087$ , while when  $\alpha = 0.5$  the equilibrium investments are  $x_1 \approx 1.0776$  and  $x_2 \approx 1.397$  with  $W = 1.3043$ . One can see that when  $\alpha$  decreases the ex ante investment  $x_1$  increases, while the ex post investment  $x_2$  decreases.

The intuition underlying this example is that a more inclusive control structure increases ex post investment incentives, meaning that for a given level of ex ante investment all ex post agents invest more in equilibrium. Increased ex post investment, of course, increases the costs borne by ex ante agents. Ex ante agents internalize this anticipated increase. This means that it is possible that ex ante investment can fall with a more inclusive structure if the *internalization effect* is negative.

Let us refer back to the example in the introduction in which a group of scientists and a large video games console manufacturer collaborate on a joint research project. Suppose now that property rights are reallocated so that some critical assets that were previously under the control of the scientists are now controlled jointly (i.e., a more inclusive control structure is implemented). This clearly increases the incentives of the manufacturer because the scientists can no longer threaten to withhold these assets. However, this change has potentially made investment more costly for the scientists because they realize that every extra dollar they invest into the project will increase the value of the manufacturer's investment by more than it did with the previous ownership structure. In turn, this can increase the manufacturer's investment but also the cost borne by the scientist. If this negative effect is sufficiently large, a more inclusive structure can reduce investment incentives for the scientist, potentially reducing overall surplus.

## 5 Concluding Remarks

In this paper we augment the standard property-rights approach in two ways. First, we allow for a more refined notion of ownership, allowing for separate rights of access

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<sup>14</sup>See the reasoning in Example 1.

and veto. Second, we consider the consequences when investment needs to be completed sequentially. The introduction of sequential investment creates three additional incentive effects no present in the simultaneous model: (1) a cost-sharing effect; (2) a strategic (Stackelberg) effect; and (3) a discounting of future payoffs effect. Focussing on the first two effects, we show that it is possible that the sequencing can increase both ex ante and ex post investments when investments are complementary and cost sharing of the followers' costs is not too large.

We also examine the affect sequencing has on the optimal allocation of ownership. Bel (2006) suggests all parties should have access to the asset and no coalition should have the right to veto anyone else's access. However, once we allow for sequential investment this need no longer be the case. Specifically, it might be advantageous to dampen the incentives to invest of ex post investors when: higher ex post investment provides a disincentive for the ex ante investors due to sharing of the followers' investment costs; and when ex ante investment is relatively important. Consequently, Bel's (2006) result that all parties should have access and no one should have veto rights no longer necessarily holds – rather, it could be the case that surplus increases when ex post investors can have their access vetoed (reducing their outside option, and their incentive to invest).

A similar point can be made in relation to the predictions in Hart & Moore (1990), as sequencing and cost sharing can create a potential tradeoff between encouraging investment by the leaders or by the followers. For instance, if it is optimal to dampen ex post investment incentives (to encourage investment by ex ante agents) it may no longer be optimal to have: an asset that is idiosyncratic to an ex post agent to be held by that agent (Proposition 5); just one agent have veto rights over an asset (Proposition 4); an indispensable (ex post) agent to own the asset (Proposition 6); or, indeed, to have complementary assets owned together (Proposition 8).

## A Proofs

### Proof of Lemma 1

The vector of Nash equilibrium ex post investments  $x_{ep}$  is characterised by the system of equations (13):

$$\frac{\partial B_i(\beta | x_{ea}, x_{ep})}{\partial x_{ep,i}} = \lambda_{ii} \frac{\partial C_i(x_{ep,i})}{\partial x_{ep,i}} \quad \forall i \in N_{ep}.$$

Totally differentiating gives:

$$\mathbf{J}_{ep} \begin{bmatrix} dx_{ep,1} \\ \vdots \\ dx_{ep,I} \end{bmatrix} + \begin{bmatrix} \sum_{j \in N_{ea}} \frac{\partial^2 B_1}{\partial x_{ep,1} \partial x_{ea,j}} dx_{ea,j} \\ \vdots \\ \sum_{j \in N_{ea}} \frac{\partial^2 B_I}{\partial x_{ep,I} \partial x_{ea,j}} dx_{ea,j} \end{bmatrix} = \begin{bmatrix} \frac{\partial C_1}{\partial x_{ep,1}} d\lambda_{11} \\ \vdots \\ \frac{\partial C_I}{\partial x_{ep,I}} d\lambda_{II} \end{bmatrix}. \quad (20)$$

Where  $\mathbf{J}_{ep}$  is the  $I \times I$  matrix:

$$\mathbf{J}_{ep} = \begin{bmatrix} \frac{\partial^2 B_1}{\partial x_{ep,1}^2} & \cdots & \frac{\partial^2 B_1}{\partial x_{ep,1} \partial x_{ep,I}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 B_I}{\partial x_{ep,I} \partial x_{ep,1}} & \cdots & \frac{\partial^2 B_I}{\partial x_{ep,I}^2} \end{bmatrix} + \begin{bmatrix} -\lambda_{11} \frac{\partial^2 C_1}{\partial x_{ep,1}^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\lambda_{II} \frac{\partial^2 C_I}{\partial x_{ep,I}^2} \end{bmatrix}. \quad (21)$$

By Assumptions 1 and 2  $\mathbf{J}_{ep}$  is negative definite; that means  $\mathbf{J}_{ep}$  is invertible. By Assumption 4 and equation 11, the off-diagonal elements of  $\mathbf{J}_{ep}$  are nonnegative. So the inverse  $\mathbf{J}_{ep}^{-1}$  is a nonpositive matrix (see, e.g. Takayama (1978), p. 393, theorem 4.D.3 [III"] and [IV"]).

Pre-multiplying both sides of (20) by the inverse  $\mathbf{J}_{ep}^{-1}$  gives:

$$\begin{bmatrix} dx_{ep,1} \\ \vdots \\ dx_{ep,I} \end{bmatrix} = \mathbf{J}_{ep}^{-1} \begin{bmatrix} -\sum_{j \in N_{ea}} \frac{\partial^2 B_1}{\partial x_{ep,1} \partial x_{ea,j}} dx_{ea,j} + \frac{\partial C_1}{\partial x_{ep,1}} d\lambda_{11} \\ \vdots \\ -\sum_{j \in N_{ea}} \frac{\partial^2 B_I}{\partial x_{ep,I} \partial x_{ea,j}} dx_{ea,j} + \frac{\partial C_I}{\partial x_{ep,I}} d\lambda_{II} \end{bmatrix}. \quad (22)$$

The sharing rules are constant, hence set  $d\lambda_{ii} = 0 \forall i \in N_{ep}$ . Further, to isolate the impact of a change in investment by only one representative ex ante agent  $j$ , set  $dx_{ea,k} = 0 \forall k \neq j \in N_{ea}$ , which gives:

$$\begin{bmatrix} dx_{ep,1} \\ \vdots \\ dx_{ep,I} \end{bmatrix} = \mathbf{J}_{ep}^{-1} \begin{bmatrix} -\frac{\partial^2 B_1}{\partial x_{ep,1} \partial x_{ea,j}} dx_{ea,j} \\ \vdots \\ -\frac{\partial^2 B_I}{\partial x_{ep,I} \partial x_{ea,j}} dx_{ea,j} \end{bmatrix}. \quad (23)$$

Consequently,

$$\begin{bmatrix} \frac{\partial x_{ep,1}}{\partial x_{ea,j}} \\ \vdots \\ \frac{\partial x_{ep,I}}{\partial x_{ea,j}} \end{bmatrix} = \mathbf{J}_{ep}^{-1} \begin{bmatrix} -\frac{\partial^2 B_1}{\partial x_{ep,1} \partial x_{ea,j}} \\ \vdots \\ -\frac{\partial^2 B_I}{\partial x_{ep,I} \partial x_{ea,j}} \end{bmatrix}. \quad (24)$$

By Assumption 4 and equation 11,  $\frac{\partial^2 B_k}{\partial x_{ep,k} \partial x_{ea,j}} \geq 0 \forall k \in N_{ep}$  and  $\forall j \in N_{ea}$ . Given that  $\mathbf{J}_{ep}^{-1}$  is a nonpositive matrix, the right hand side of (24) must therefore be (weakly) positive (i.e.,  $\frac{\partial x_{ep,i}}{\partial x_{ea,j}} \geq 0$ ). Hence, an increase in investment by any ex ante agent increases ex post equilibrium investment of all ex post agents. *Q.E.D.*

### Proof of Proposition 1

(i) For a given control structure  $\beta$ , when investments are simultaneous the *internalization effect* is zero by assumption  $Y_j = 0 \forall j \in N_{ea}$  but when investments are sequential it can be negative, zero or positive; equation 15 can be presented in the

following way

$$\frac{\partial B_j(\beta | x_{ea}, R)}{\partial x_{ea,j}} + Y_j = \frac{\partial C_j(x_{ea,j})}{\partial x_{ea,j}} \quad \forall j \in N_{ea}. \quad (25)$$

Let us also represent equation 13 in a similar way:

$$\frac{\partial B_i(\beta | x_{ea}, x_{ep})}{\partial x_{ep,i}} + Y_i = \frac{\partial C_i(x_{ep,i})}{\partial x_{ep,i}} \quad \forall i \in N_{ep}, \quad (26)$$

where a similar variable is introduced for ex post investments  $Y_i = (1 - \lambda_{ii}) \frac{\partial C_i(x_{ep,i})}{\partial x_{ep,i}} \quad \forall i \in N_{ep}$ . Note that by construction it is always true that  $Y_i \geq 0$ .

Now let us combine all ex ante and ex post variable into one set of variables  $k=1 \dots N$  and represent system of equations 25 and 26 as

$$\frac{\partial B_k}{\partial x_k} + Y_k = \frac{\partial C_k}{\partial x_k} \quad \forall k \in N. \quad (27)$$

We want to show that when all  $Y_k \geq 0 \quad \forall k \in N$  all equilibrium investments are the same or higher. Therefore, if the *internalization effect* for sequential investments is positive  $Y_j \geq 0 \quad \forall j \in N_{ea}$  then both ex ante and ex post equilibrium investment can only be higher under sequential investment than under simultaneous investment.

Totally differentiating the above equation and rearranging gives

$$\mathbf{J} \begin{bmatrix} dx_1 \\ \vdots \\ dx_N \end{bmatrix} = \begin{bmatrix} -dY_1 \\ \vdots \\ -dY_N \end{bmatrix}. \quad (28)$$

Where  $\mathbf{J}$  is the  $N \times N$  matrix:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial^2 B_1}{\partial x_1^2} & \dots & \frac{\partial^2 B_1}{\partial x_1 \partial x_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 B_N}{\partial x_N \partial x_1} & \dots & \frac{\partial^2 B_N}{\partial x_N^2} \end{bmatrix} + \begin{bmatrix} -\frac{\partial^2 C_1}{\partial x_1^2} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & -\frac{\partial^2 C_N}{\partial x_N^2} \end{bmatrix}. \quad (29)$$

By Assumptions 1 and 2  $\mathbf{J}$  is negative definite; that means  $\mathbf{J}$  is invertible. By Assumption 4 and equation 11, the off-diagonal elements of  $\mathbf{J}$  are nonnegative. So the inverse  $\mathbf{J}^{-1}$  is a nonpositive matrix (see, e.g. Takayama 1985, p. 393, theorem 4.D.3 [III"] and [IV"]).

Pre-multiplying both sides of (28) by the inverse  $\mathbf{J}^{-1}$  gives:

$$\begin{bmatrix} \frac{\partial x_1}{\partial Y_1} \\ \vdots \\ \frac{\partial x_N}{\partial Y_N} \end{bmatrix} = -\mathbf{J}^{-1} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}. \quad (30)$$

Given that  $\mathbf{J}^{-1}$  is a non-positive matrix, the right hand side of (30) must therefore

be (weakly) positive (i.e.,  $\frac{\partial x_k}{\partial Y_k} \geq 0$ ). Hence, a higher *internalization effect* of any ex ante agent increases equilibrium investment of all ex ante and ex post agents.

When  $\lambda_{ii} = 1 \forall i$  it is clear that all  $Y_k \forall k \in N$  are non-negative, which means that in this case sequential investment leads to higher equilibrium investments of all ex ante and ex post agents. This final observation ends the proof. *Q.E.D.*

## Proof of Proposition 2

Define function

$$g(\beta, \lambda|x) = \sum_S p(S)v(S, \beta(S)|x) - \sum_{j \in N_{ea}} C_j(x_{ea,j}) - \sum_{i \in N_{ep}} \sum_{l \in N} \lambda_{il} C_i(x_{ep,i}).$$

Note that the first order conditions 17 and 18 are equivalent to  $\nabla g(\beta, \lambda|x) = 0$ . Let  $\beta$  be a more inclusive control structure than  $\beta'$ . For a representative ex ante agent  $j \in N_{ea}$  define the function

$$f(\alpha, x_{ea,j}) = \alpha g(\beta, \lambda|x_{ea,j}, x_{-j}) + (1 - \alpha)g(\beta', \lambda|x_{ea,j}, x_{-j})$$

for  $\alpha \in [0, 1]$ , where  $x_{-j} = \{x_{ep}, x_{ea,1}, \dots, x_{ea,j-1}, x_{ea,j+1}, \dots, x_{ea,J}\}$  is exogenous and let  $x_{ea,j}(\alpha)$  solve

$$\frac{\partial f(\alpha, x_{ea,j}(\alpha))}{\partial x_{ea,j}} = 0. \quad (31)$$

Totally differentiating (31) and taking  $\alpha$  as the exogenous variable, gives

$$\frac{dx_{ea,j}}{d\alpha} = - \frac{\frac{\partial^2 f(\alpha, x_{ea,j}(\alpha))}{\partial x_{ea,j} \partial \alpha}}{\frac{\partial^2 f(\alpha, x_{ea,j}(\alpha))}{\partial x_{ea,j}^2}}, \quad (32)$$

where  $\frac{\partial^2 f(\alpha, x_{ea,j}(\alpha))}{\partial x_{ea,j}^2} = \alpha \left[ \frac{\partial^2 B_j(\beta)}{\partial x_{ea,j}^2} - \frac{\partial^2 C_j}{\partial x_{ea,j}^2} \right] + (1 - \alpha) \left[ \frac{\partial^2 B_j(\beta')}{\partial x_{ea,j}^2} - \frac{\partial^2 C_j}{\partial x_{ea,j}^2} \right] < 0$  by Assumptions 1 & 2, while the inequality  $\frac{\partial^2 f(\alpha, x_{ea,j}(\alpha))}{\partial x_{ea,j} \partial \alpha} = \left[ \frac{\partial B_j(\beta)}{\partial x_{ea,j}} - \frac{\partial C_j}{\partial x_{ea,j}} \right] - \left[ \frac{\partial B_j(\beta')}{\partial x_{ea,j}} - \frac{\partial C_j}{\partial x_{ea,j}} \right] = \frac{\partial B_j(\beta)}{\partial x_{ea,j}} - \frac{\partial B_j(\beta')}{\partial x_{ea,j}} \geq 0$  follows from Assumption 6 and equation 11. Hence,  $\frac{dx_{ea,j}}{d\alpha} \geq 0 \forall j \in N_{ea}$  and  $x_{ea,j}(1) \geq x_{ea,j}(0)$  or  $x_{ea,j}(\beta) \geq x_{ea,j}(\beta') \forall j \in N_{ea}$ .

For ex post agents, the proof that  $x_{ep,i}(\beta) \geq x_{ep,i}(\beta') \forall i \in N_{ep}$  is similar and will be omitted. Thus, a more inclusive control structure always increases ex ante and ex post investment incentives and consequently have higher equilibrium investment. Therefore, the optimal control structure is the most inclusive control structure. *Q.E.D.*

## References

- Admanti, A. & Perry, M. (1991), 'Joint projects without commitment', *Review of Economic Studies* **58**(2), 259–276.
- Bel, R. (2006), Access, veto and ownership in the theory of the firm. School of Economics Working Papers: <http://ses.library.usyd.edu.au/handle/2123/7630>.
- Bel, R. (2011), Access, veto and ownership in the theory of the firm. mimeo.
- Binmore, K., Rubinstein, A. & Wolinsky, A. (1986), 'The Nash bargaining solution in economic modelling', *The RAND Journal of Economics* **17**(2), 176–88.
- Bolton, P. & Scharfstein, D. (1998), 'Corporate finance, the theory of the firm, and organizations', *Journal of Economic Perspectives* **12**(4), 95–114.
- Che, Y.-K. (2000), 'Can a contract solve hold-up when investments have externalities? a comment on de Fraja (1999)', *Games and Economic Behavior* **33**(2), 195–205.
- De Fraja, G. (1999), 'After you Sir, hold-up, direct externalities, and sequential investments', *Games and Economic Behavior* **26**(1), 22–29.
- Demsetz, H. (1996), 'Ownership and control: A review', *International Journal of the Economics of Business* **3**(1), 107–112.
- Grossman, S. & Hart, O. (1986), 'The costs and benefits of ownership: A theory of vertical and lateral integration', *Journal of Political Economy* **94**(4), 691–719.
- Grout, P. (1984), 'Investment and wages in the absence of binding contracts: A Nash bargaining approach', *Econometrica* **52**(2), 449–460.
- Hart, O. (1995), *Firms, Contracts, and Financial Structure*, London: Oxford University Press.
- Hart, O. & Moore, J. (1990), 'Property rights and the nature of the firm', *Journal of Political Economy* **98**(6), 1119–1158.

- Klein, B., Crawford, R. & Alchian, A. (1978), 'Vertical integration, appropriable rents, and the competitive contracting process', *Journal of Law and Economics* **21**(2), 297–326.
- Neher, D. (1999), 'Staged financing: An agency perspective', *Review of Economic Studies* **66**(2), 11–21.
- Rajan, R. & Zingales, L. (1998), 'Power in a theory of the firm', *Quarterly Journal of Economics* **113**(2), 387–432.
- Rubinstein, A. (1982), 'Perfect equilibrium in a bargaining model', *Econometrica* **50**(1), 97–109.
- Schlager, E. & Ostrom, E. (1992), 'Property-rights regimes and natural resources: A conceptual analysis', *Land Economics* **68**(3), 249–62.
- Sherali, H. (1984), 'A multiple leader Stackelberg model and analysis', *Operations Research* **32**(2), 390–404.
- Smirnov, V. & Wait, A. (2004a), 'Hold-up and sequential specific investments', *RAND Journal of Economics* **35**(2), 386–400.
- Smirnov, V. & Wait, A. (2004b), 'Timing of investments, holdup and total welfare', *International Journal of Industrial Organization* **22**(3), 413–425.
- Takayama, A. (1978), *Mathematical Economics*, 2 edn, Hinsdale: Dryden.
- Williamson, O. (1975), *Market and Hierarchies: Analysis and Antitrust Implications*, New York: The Free Press.